Kolloquium

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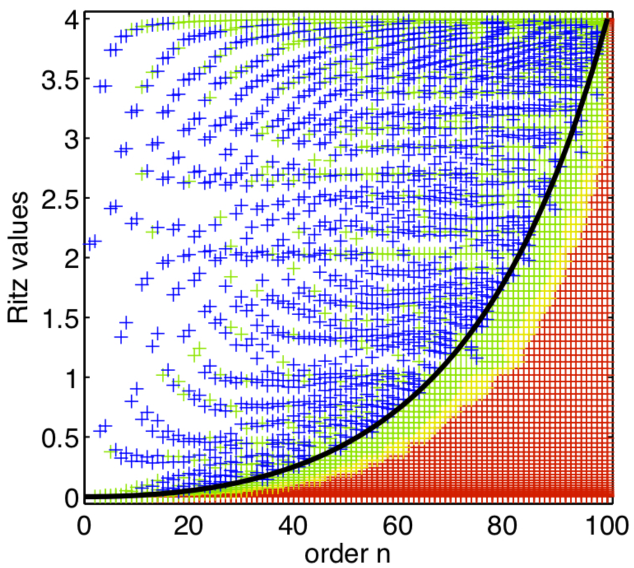
On the approximate computation of the exponential of a matrix times a vector

The computation of $y = f(A)b$ for a (large and sparse) matrix $A \in \mathbb{C}^{n\times n}$, a vector $b \in \mathbb{C}^n$, and entire functions f like the exponential function are of major importance for several tasks in numerical analysis. They are for instance the building block of so-called exponential Runge-Kutta methods. In this talk, after giving a quick overview of the state of the art, we will concentrate on the approximation $y_m = Vf(V^*AV)V^*b$ through a projection matrix $V \in \mathbb{C}^{n\times m}$ with $m \ll n$ orthogonal columns, where only a function of a small size matrix has to be computed. In particular, we will be interested in a priori bounds as a function of m.



Estimates for matrix functions typically involve tools like the numerical range or some other K-spectral set $\mathbb{E} \subset \mathbb{C}$ attached to our matrix A. Using tools from approximation theory in the complex plane such as Faber series, we show that for (polynomial) Krylov projection the above approximation error can be bounded in terms of best polynomial approximation of f on E, the rate being explicit.

In typical applications applications the above set $\mathbb{E}$ is large or even unbounded and thus often rational Krylov spaces like the shift-and-invert technique are used. We discuss the link with best rational approximation on $\mathbb{E}$ with a fixed pole, which for $\mathbb{E} = (-\infty, 0]$ goes back to Saff, Schönhage, Varga, Andersson, Borwein, and others.

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