Tübingen Team
Arbeitsgemeinschaft Funktionalanalysis
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Lecture 12

Exercise 2:

The stability function of the Crank-Nicolson scheme is $r(z) = \frac{1+\frac{z}{2}}{1-\frac{z}{2}}$.

We show that the respective stability region $S(r) = \{z \in \mathbb{C} : \|r(z)\| \leq 1\}$ is the left half plane of $\mathbb{C}$, so the Crank-Nicholson scheme as a rational approximation is called A-stable.

$$\begin{aligned}\left|\frac{1+\frac{z}{2}}{1-\frac{z}{2}}\right| &= \left|\frac{\frac{2+z}{2}}{\frac{2-z}{2}}\right| \\ &= \left|\frac{2+z}{2-z}\right| \\ &\leq \left|\frac{2+z}{2+z}\right| \\ &= 1\end{aligned}$$

for Re z ≤ 0

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