Talk:Lecture 9

From 15th ISeminar 2011/12 Operator Semigroups for Numerical Analysis

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Comments

Hello, so I guess I have some typos and a remark. I begin with some typos.

I think in Prop. 9.8 the θ and θ' have to be an index.

In the proof of Prop. 9.15, considering the equation in the middle of the page, which begins with $T(z)f - f$, there is one f too much in the central term of the equality.

In the proof of Cor. 9.22 b) you refer to Remark 7.10, which does not exist. I think you either might mean Inequality (7.10) or Remark 7.24.

Now, I have got a remark concerning the proof of Proposition 9.14 c), where you prove the semigroup property of T. In the last line of page 103 you state that

$$\frac{1}{2\pi i}\int_{\tilde{\gamma}}\frac{e^{\mu w}}{\lambda-\mu}d\mu = 0 \text{ and } \frac{1}{2\pi i}\int_{\gamma}\frac{e^{\lambda z}}{\lambda-\mu}d\lambda = e^{\mu z},$$

where $\tilde{\gamma}$ is an admissible curve **to the right** of γ. But this does not fit very well with Cauchy's Theorem, because if μ is a point on $\tilde{\gamma}$, then this point lies on the right hand side of γ, consequently we get

$$\frac{1}{2\pi i}\int_\gamma \frac{e^{\lambda z}}{\lambda-\mu}d\lambda = 0,$$

by Cauchy's Theorem. Conversly, if λ is a point on γ, we would obtain by Cauchy's Theorem

$$\frac{1}{2\pi i}\int_{\tilde{\gamma}} \frac{e^{\mu w}}{\lambda-\mu}d\mu = -\frac{1}{2\pi i}\int_{\tilde{\gamma}} \frac{e^{\mu w}}{\mu-\lambda}d\mu = -e^{\lambda w}.$$

Sincerely PatrickTolksdorf 21:00, 13 December 2011 (CET)

Some comments

p.101, (9.1): I think, the domain of $\gamma_{\eta,r,3}$ should be $[r,\infty)$ instead of $(-\infty,-r]$.
Prop. 9.14: Here some words are missing: in the statement "the following _assertions_ are true", in the proof "in (9.3) these _estimates_ yield" and on p. 104, "_we_ have seen"
Prop. 9.15, proof, seventh line from below: "for all λ that lie on" (not lies)
Cor. 9.16, (ii): You here talk of a bounded holomorphic semigroup, but everywhere else you call it a bounded analytic semigroup.
Prop. 9.17, proof, second calculation: I think in the third expression you lost a factor $\frac{1}{2\pi}$, which came from the absolute value of $\frac{1}{2\pi i}$.
In the same expression you write $\sup\limits_{z\in\theta}$, but you mean $\sup\limits_{z\in\gamma}$. Then you estimate $\sup\limits_{z\in\gamma}\|T(z)\| \leq \sup\limits_{z\in\Sigma_{\theta'}}\|T(z)\|$. The problem that I see here, is that, according to my calculation, γ is not a subset of $\Sigma_{\theta'}$ but only of $\overline{\Sigma_{\theta'}}$. Does the claimed estimate still hold by a continuity argument? (But the mapping $z\mapsto\|T(z)\|$ doesn't need to be continuous, does it?) (Furthermore, you wrote Σ'_θ instead of $\Sigma_{\theta'}$.)
In the same proof you don't mention that if *A* generates a bounded analytic semigroup, we have $ranT(t)\subseteq D(A)$, what you have claimed. Why, and why is this the case?
Prop. 9.21, fifth line from below: "From this _we_ can"
Cor. 9.22, b): Maybe you could add that Rem 7.24 (or inequality (7.10), see Patrick's comment) proves the statement for $\alpha\in(0,1)$. Later you suppose at one point that α > 1. Two lines later you suppose $\alpha\geq 1$, which sounds a bit strange, especially because you have already dealt with the case α = 1 several lines before.
JohannesEilinghoff 15:33, 13 December 2011 (CET)

Dear Johannes,

concerning the proof of Proposition: You are right - in the proof of first implication a proof of the fact that $ranT(t)\subseteq D(A)$ for *t* > 0 is missing. You can obtain that result by the following observation: $t\mapsto T(t)$ is differentiable on the intervall $(0,\infty)$ (with respect to the operator norm) and hence the limit that has to exist in order to bring *T*(*t*) into the domain of our generator exists.

MoritzEgert 18:09, 15 December 2011 (CET)

Dear Johannes,

concerning `(But the mapping $z\mapsto\|T(z)\|$ doesn't need to be continuous, does it?)': We are dealing with an analytic semigroup, and therefore the mapping $z\mapsto T(z)$ is continuous on Σ_θ as an $\mathcal{L}(X)$-valued mapping.

JurgenVoigt 22:52, 21 December 2011 (CET)

Proof of Proposition 9.19

Dear Isem-Team,

I don't see why the last equality in the last estimate in the proof of Proposition 9.19 should hold, for λ and α have nothing to do with each other. I suggest to use the previously obtained estimates instead. For instance, in the case $Im(\lambda) \le 0$ one has

$$Re(\lambda)\cos(\alpha) - Im(\lambda)\sin(\alpha) = |Re(\lambda)|\cos(\alpha) + |Im(\lambda)|\sin(\alpha) \ge \min(\cos(\alpha), \sin(\alpha))(|Re(\lambda)| + |Im(\lambda)|)$$

which gives the desired estimate.

MoritzEgert 18:17, 15 December 2011 (CET)

comments

p.97: Had $H^2(\mathbb{R})$ been defined? (Or is it considered general education?)

p.100, line 1: Why not $\theta'' \in (\theta', \theta)$? ($\frac{1}{2}(\frac{\pi}{2} + \theta')$ might be larger than θ, anyway.)

Definition 9.9: I think you should allow $\delta = \frac{\pi}{2}$. (See Engel-Nagel.) Then, in Example 9.10, you get that M_m is of angle $\frac{\pi}{2}$.

Remark 9.11 (a) I do not understand why you use δ' and not η, here. (After all the estimate is used later always for η.) (b) I find this rather confusing. Keeping to the figure on p. 101, I would expect angles δ' < η < δ, possibly with $\delta - \eta \le \eta - \delta'$, and then the estimates for all λ with $|\arg\lambda| = \frac{\pi}{2} + \eta$ and all z with $|\arg z| \le \delta'$.

p.103, line7: ...these **estimates** yield ... And three lines below: If **we** specialise ...

p.103, last line: I think that Patrick is right with his remark above. The reason is that one has to supplement arcs on the left, for the application of Cauchy's theorem. Additionally: Typo in the first integral, it should be $d\mu$. And maybe one should supplement the first of the integrals with `for all λ on γ', and the second correspondingly.

p.104, line 7 from below: for all λ lying on ...

p.106, middle: Replace $T^n(t / n)$ by $T(t / n)^n$.

p.106, line 7 from below: ... These **considerations** yield ... And in the next line: Discard 'uniformly'.

p.107: It is not clear, what an `analyitic semigroup of type (M,ω)' should be. The ω might mean something like in Prop. 9.20; but what is M, then? Or just analytic, and the restriction to $[0, \infty)$ is of type (M,ω)?

p.109, line 3: $t > 0$

p.110: Is L^p_* the weak L^p-space? Is everybody assumed to be familiar with this?

Greetings, JurgenVoigt 16:12, 16 December 2011 (CET)

Dear Jürgen,

on p.110 I am sure that L^p_* is not the weak L^p-space but $L^p(0, \infty; d/dt)$ as, e.g. in Triebel's book on interpolation theory. PeerKunstmann 14:59, 19 December 2011 (CET)

Dear Jürgen,

indeed, L^p_* was defined in Section 8.6. But you are right, we should have made a reference to it. We will correct this in the revised version. IsemTeam 16:01, 19 December 2011 (CET)

It should have read $L^p(0, \infty; dt/t)$, of course. PeerKunstmann 18:18, 19 December 2011 (CET)

Dear Peer, thanks! (I had already wondered what to make of your measure.) Anyway, I definitely dislike the misleading notation L^p_* (even if the great Triebel introduced or used it), and I do not see a reason why to use it here, if there is the self-explanatory notation $L^p((0, \infty), dt/t)$. I understand now that the * seems to be there in order to indicate that one uses the Haar measure for the multiplicative group $(0, \infty)$, but then it does not belong to the L. Greetings, JurgenVoigt 10:00, 20 December 2011 (CET)

Dear Jürgen, we will change the notation in the updated version. We have chosen it because Triebel and Lunardi and many others in interpolation theory use it, but we also do not like it very much. IsemTeam 10:07, 20 December 2011 (CET)

Comments on Propositions 9.17. and 9.23.

Dear Isem-Team,

I think in Proposition 9.17 you have to assume that A generates a bounded semigroup, since in the second step of the proof you use this fact and I do not see, how to derive this from the given assumptions.

And a typo in Proposition 9.23: There should be no power p in the inner integral.

Best regards

SaschaTrostorff 12:51, 19 December 2011 (CET)

Dear Sascha, you are absolutely right. We have to assume boundedness. IsemTeam 16:05, 19 December 2011 (CET)

Concerning Prop.9.21

The last line of the statement should be

$$X_\alpha = \{f \in X; \lim_{t\to\infty} t^{1-\alpha} AT(t)f = 0\}.$$

(Or else, in view of one of my remarks on Lecture 8,

$$X_\alpha = \{f \in X; \lim_{t\to\infty} t^{1-\alpha} AT(t)f \text{ exists}\}.$$

In this form it would also be true for α = 1.)

In the first line of (9.8), the second ` = ' should be a `$\leqslant$'.

And in the last two sentences of the proof it might be more precise to refer to the beginning of the estimates (9.7) and (9.8) for the two implications.

JurgenVoigt 23:16, 21 December 2011 (CET)