Talk:Lecture 8

From 15th ISeminar 2011/12 Operator Semigroups for Numerical Analysis

Contents

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Comments

Some typing errors

I've found some typing errors:
p. 85, second line: "a given semigroup T." (the word "semigroup" is twice)

Exa. 8.1: $J_n(y_0, \ldots, y_{n-1})(x)$ (the "(x)" is missing)
p. 86, after Def. 8.2: "context which semigroup" (small letters)
Prop. 8.4, proof: You define a sequence (x_m), but you want to call it (f_m) because you go on calculating with f_m.
at the same place, p. 87: "if $n \in \mathbb{N}$ _is_ sufficiently"
p. 87, after Rem. 85: The word "in" after "decay" is twice. (Don't we better say "decay at infinity"?)
Prop. 8.6, proof, at the end: You obtain the inequality in the second to the last displayed line not by (8.1), as you write, but by the definition of the $|||\cdot|||_{F_\alpha}$-norm.
Section 8.3, first line: "$D(A^k)$ for" (there is an extra f)
Section 8.5, third line: "In the first two" (not "to")
JohannesEilinghoff 22:05, 5 December 2011 (CET)

Dear Johannes,

thank you for pointing out these typos.

IsemTeam 13:41, 12 December 2011 (CET)

Definition of X_1 missing

The definition is given for $\alpha \in (0,1)$, but the subsequent propositions and theorems deal with X_1. The literal extension of the definition gives $X_1 = \ker(A) \neq D(A)$, in contrast to the first line of the Proof of Proposition 8.9. Is $X_1 := D(A)$ with graph norm of A?

ChristophSchumacher 15:28, 7 December 2011 (CET)

Dear Christoph,

the space X_1 has already been defined in previous lectures, see e.g. 7.1., as $X_1 := D(A)$ with graph norm of A.

MoritzEgert 23:11, 10 December 2011 (CET)

some comments

In Prop.8.4 I suggest to include the inequality $\|T(t)\|_{\mathcal{L}(F_\alpha)} \leq \|T(t)\|$ in the statement of the proposition. After all, this is proved anyway.

In the sentence before Prop.8.10 I suggest to add a comma after the first F_α.

p.89, line 4 from below: I am not sure what the `again' refers to. If it refers to a similar argument you used in Lecture 2, I repeat my comment from Lecture 2, that the equality of the integral $\int_0^T T(s) f\, ds$ as elements of F_α and of X is a consequence of Prop.2.33 c), applied to the continuous embedding of F_α into X (as the operator T in Prop.2.33 c)). (No Riemann sums needed!)

Definition 8.13: Typo in the last line, $X_\alpha = X \ldots$.

Example 8.17: What you call `uniformly bounded variation' is just `bounded variation', commonly (see Rudin, Real and complex analysis, e.g.). In the first sentence of the last paragraph one should say `... at most countably many discontinuity points, ...'. And the following statement is just erroneous: If you modify the function $1_{\{0\}}$, which has total variation 2, to its left continuous version, it becomes the zero function. And the `more precisely' in the parentheses is misleading; the parentheses should read `(i.e., $g$ is in the $L^1$-equivalence class of $f$)'. Note also the missing `in' in the text!

Best wishes, JurgenVoigt 21:29, 12 December 2011 (CET)

Dear Jürgen,

thanks for the comments.

"again" refers to the fact that you are right again about those Riemann sums.

About bounded variation: We were arguing about BV or UBV, but about the modification of a BV function you are right. Sorry for that error. The term "more precisely" may be misleading, but is it not so that f as an L1 "function" is the equivalence class of the function g? (If you interpret f as an eqivalence class and not a representative of that. Of course, you may say then that f=g a.e. does not make any sense, rather g\in f should have been written.) Anyway, in the revised version this paragraph will be rewritten.

Again, many thanks for the careful reading!

IsemTeam 22:32, 12 December 2011 (CET)

Proposition 8.10

It seems to me that the proof given in the first part only covers the cases α < 1. In order to give a proof covering the whole range of α's, one may define the slightly larger `Hölder space' $\hat{X}_\alpha := \{f \in F_\alpha;\ \lim_{t\to 0} \frac{1}{t^\alpha}(T(t)f - f) \text{ exists}\}$. It may come as a surprise that then this `larger' space turns out to be equal to the Hölder space, for α < 1. (We proved this in a discussion with Ralph Chill.) Because it is somewhat cumbersome to include it here, I refer to http://www.math.tu-dresden.de/~voigt/isem11/ where you can find it under prop-8-10. (Incidentally, the proof given there for the fact that the larger Hölder space is contained in the space of strong continuity is shorter than the proof given in the lecture.)

And a typo(?): In the last line of the proof, the set in the middle should start by $\{f \in F_\alpha \ \ldots$, as in the formulation of the Proposition itself.

And a typo on line 2 from below on p.91: completes.

Greetings, JurgenVoigt 10:36, 13 December 2011 (CET)

Dear Jürgen,

yes, the first part of proof works for α < 1 only. Thanks for the comments and the link.

IsemTeam 12:50, 13 December 2011 (CET)

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