Talk:Lecture 7

From 15th ISeminar 2011/12 Operator Semigroups for Numerical Analysis

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Comments

Remark on Exercise 1.a

At my home page http://wwwhome.math.utwente.nl/~zwarthj/counterexamples/ there is an example showing that AB need not to be densely defined.

HansZwart 08:51, 25 November 2011 (CET)

Dear Hans, thank you for the great example! This is an extremly important remark. We hope, however, that this does not affect the solvability of Exercise 1.a. IsemTeam 11:40, 25 November 2011 (CET)

Remark on Example 7.2

I do not agree with the formulation "The minus sign here is only a matter of convention", or maybe I don't get its meaning. Since $-n^2$ is negative you would have to resort to complex logarithms to give a meaning to $(-n^2)^\alpha$. If you use the usual conventions for real powers, you have to put the minus sign. In fact, in my opinion the example would be a nice motivation for putting the minus sign.

On the other hand, you are right. One could use other branches of the logarithm to avoid the minus sign (as Simon Brendle did in the first Engel/Nagel book). But is this relly meant by the remark? PeerKunstmann 15:33, 25 November 2011 (CET)

Dear Peer, you are right, the formulation is not very lucky. To our defence, some of the lecturers were batteling a lot about which branch of the logarithm to take; it is really annoying to write out the minus sign eevry time if you work with semigroup generators. IsemTeam 15:39, 25 November 2011 (CET)

Definition of admissible curves on p.76

I think you are way too sloppy here.

1) Going from $\infty e^{i\theta}$ to $\infty e^{-i\theta}$ can be understood intuitively, but for making this precise you have to do something (e.g., use a radial compactification of the complex plane), and I am not really sure if everybody can do so right away.

2) You have to make sure that the length of the curves does not grow too fast for $t \longrightarrow \pm\infty$, otherwise you will not get the right estimates for contour integrals. Curves might highly oscillate with oscillations getting wilder when approaching the "limit points" at $t = \pm\infty$.

3) It is essential that the curves do not intersect $(-\infty, 0]$!! Since $\lambda \mapsto \lambda^z$ is not holomorphic there. Which is the reason why you take $a > 0$ in Example 7.7!!

I guess that it would do no harm to stick to the curves in Example 7.7. Then the estimate in the proof of Lemma 7.8 is OK, and you are using them in the proof of 7.9 anyway. The formulation "We may assume ..." on p.77, line 1, cannot be accepted. PeerKunstmann 15:53, 25 November 2011 (CET)

Dear Peer,

your are right about all your remarks. Thank you. With 1) one could argue that it is intuitively clear what is meant. But you are right: This intuition should not have been made into a definition. But your remarks 2) and 3) are absolutely crucial. In the revised version, we shall clear that up.

Thanks again IsemTeam 11:41, 28 November 2011 (CET)

Some typings

Some typings that I have found:
Exa. 7.7, at the beginning: Let (capital letter)
The curve γ_1 in Figure 7.2 has the orientation of the curve $-\gamma_1$ of the text.
Equation (7.5): It should be $\sin(\pi\alpha)$, not $\sin(\pi a)$ in the numerator and this should hold for $\alpha \in (-1,0)$, not $a \in (-1,0)$.
Prop. 7.14: There is a minus sign on the right-hand side of the inequality missing (also in the proof).
Cor. 7.15, proof, first calculation: The *f* is missing in the integral.
also there, second calculation: After the first estimate, the *M* shouldn't be there, but after the second estimate, it should be there.
Prop. 7.18, proof, c), last line: $A^{z-1}f \in D(A)$ (the *f* is missing)
Prop. 7.19 a): The full stop is missing at the end of the sentence.
Prop. 7.19, proof, a), at the end: not $x \in D(A^z)$, but $f \in D(A^z)$
same proof, b), first expression in the calculation: $A^{-n+w-z}A^z f$, not $A^{-n+w-z}A^w f$
also there, last line: $A^z f \in D(A^{w-z})$ (the *f* is missing)
Prop. 7.20 c), proof: I don't see why A^z and A^w are bounded and hence I wonder if $\|A^z\|$ and $\|A^w\|$ exist. If my doubts are reasonable, I suggest the estimate
$\|f\|_{A^z} \le C_1\|A^z f\| \le C_1\|A^{z-w}A^w f\| \le C_1\|A^{z-w}\|\cdot\|A^w f\| \le C_1C_2\|A^{z-w}\|\cdot\|f\|_{A^w}$, where the constants $C_1,C_2 > 0$ exist by b).
Prop. 7.21: The second and the third expression in the formula in the statement are exactly the same. Probably you want to put the *A* from the place in front of the integral to the place directly in front of the *f* in the third expression, as you did in the proof.
Rem. 7.22: Here the second and the third expression in the formula in the statement are also exactly the same. Maybe you also want to put the A^β in the second expression not directly in front of *f*, but in front of the integral as in the proposition before, but the proof of Prop. 7.23 indicates that you want to put it directly in front of $(s + A)^{-1}$.
p. 84: ... question, to which ...
JohannesEilinghoff 22:34, 26 November 2011 (CET)

Dear Johannes,

thanks for the comments. Your doubts are reasonable: The mentioned operators in the proof of Prop.7.20.c) are not bounded, but some *f*s are missing from the estimate.

IsemTeam 11:46, 28 November 2011 (CET)

Some Comments

Dear ISEM-Team!

During our seminar the following questions/comments arose:

1) Proof of 7.5: The final computation is done for $|\arg \mu| \in]\theta_0, \pi[$. In this computation one obtains M^{k+1}.

2) Proof of 7.9: In the formula one gets $(-A)^{-(k+1)}$. Maybe it should be explained a bit more explicit, why this computation works (CIF); for the first equality one needs $|\lambda| \leqslant 1/\|A^{-1}\|$, but later one wants to consider $r \to \infty$.

3) Proof of 7.14: In the first line one selects α such that $\mathrm{Re}\ \alpha \in]-1,0[$; at least the proof is for complex α, but I don't understand the last sentence "For real α the assertion follows from this trivially". It would be nice, if you could make a comment on this.

4) Proof of 7.15: In the first chain of equations one gets $\sin(-\pi t)$ (2 times).

5) Proof of 7.20.(c): In the comment above, Johannes gave a proof for 7.20.(c). If $\|\cdot\|_{A^z}$ in his proof denotes the norm defined in 7.20.(b), then his proof simplifies to

$$\|f\|_{A^z} = \|A^z f\| = \|A^{z-w}A^w f\| \leqslant \|A^{z-w}\|\|A^w f\| = \|A^{z-w}\|\|f\|_{A^w}.$$

If $\|\cdot\|_{A^z}$ denotes the graph norm, then at least $C_2 = 1$ can be simplified.SvenWegner 14:24, 1 December 2011 (CET)

Dear Sven,

to your 2nd point: Indeed, the power of A should be $-(k+1)$. The computations works, since for the calculation of the residues you may use any curve, i.e., arbitrary $r > 0$.

To your 3rd remark: Prop. 7.14 contains two assertions, the second one (starting with "In particular...") concerns real α only. We wanted to refer to that statement, but agree that should have be written up more clearly.

Thanks for your comments!

IsemTeam 14:00, 12 December 2011 (CET)

Dear Isem Team,

one comment on the final calculation in the proof of 7.5. As far as I can see, the constant M1 is obtained by the finiteness of the geometric series $\sum_{k=0}^{\infty}\left(\frac{M|Im(\mu)|}{1+|Re(\mu)|}\right)^k$. But on the sector chosen in the proof of 7.5, the quotient $\frac{M|Im(\mu)|}{1+|Re(\mu)|}$ can get arbitrarily close to 1. By your estimates, one therefore obtains the desired bounds for the resolvent only on compactly contained subsectors of your sector, as far as I can see (which doesn't change anything, of course).

MoritzEgert 23:04, 1 December 2011 (CET)

Dear Moritz,

you are certainly correct. I wouldn't call the subsectors `compactly contained', though, but rather `properly contained'.

JurgenVoigt 23:19, 4 December 2011 (CET)

Dear Moritz and Jürgen,

of course you are right. The θ that appears in the proof is exactly because of this. We have just forgotten to say that.

Thanks!

IsemTeam 14:04, 12 December 2011 (CET)

some comments

p.73, line 5 from below: `much more(!) is true'. I cannot interpret this. A remark like that does not seem helpful to me.

p.79, Prop. 7.14, last sentence: Wouldn't it sufficient (and better) to just say that the mapping is bounded. (Or what does `uniformly' refer to?

p.79, Prop. 7.14 and the proof: It was mentioned above that minus signs are missing. One could also say that some absolute value signs are missing.

p.79, middle: Concerning the equality starting by A^{-t}f-f = ... on might mumble something like `second resolvent equation'? (Also, in the middle term an `f' is missing.)

p.80, second display line in the proof of Prop.7.18: It might be helpful to supplement a term in the chain of equalities: $AA^z = \frac{1}{2\pi i}\int_\gamma \lambda^z AR(\lambda, A)\, d\lambda = \dots$

Prop.7.20 a): This is a bit awkward. The graph norm \|\cdot\|_B of an operator B had already been defined in Lecture 2 (and is somehow standard notation, \|x\|_B:\|x\|+\|Bx\|). (Also, the graph norm can not be equivalent `for all f'!)

Typos and style:

Assumption 7.4: Suppose _that_ for ...

Prop.7.5: ... Then there _are_ ...

Lemma 7.8: For an admissible curve \gamma in ...

p.77, line 3: Usually, `hence' is used if one concludes something from previous arguments. This does not seem the case here.

p.79, middle: It follows _that_ ...

p.80, last sentence in the proof of a): From these _properties_ the assertion follows.

p.81, last line: two missing f's.

p.82, in (7.7): missing ds at the end.

JurgenVoigt 22:22, 4 December 2011 (CET)

Dear Jürgen

we appreciate your comments. Thank you!

IsemTeam 13:44, 12 December 2011 (CET)

Proof of Prop. 7.18 c)

Dear ISemTeam,

in the first line of the proof, I think one has to use the equality $A^{-n} = A^{z-n}A^{-z}$ instead of the equality indicated in the lecture, in order to obtain the further conclusion.

JurgenVoigt 00:01, 7 December 2011 (CET)

Dear Jürgen,

you are right. Thank you!

IsemTeam 14:10, 12 December 2011 (CET)

Proof of Proposition 7.21

Dear Isem Team,

at the end of the proof of Proposition 7.21 you use Proposition 7.19 a) to conclude $A^{\alpha}f = AA^{\alpha-1}f$. However, I think you have to refer to Proposition 7.19 a) and Proposition 7.18 b) to deduce the equality. Indeed, you just get by Proposition 7.19 a) that $A^{\alpha}f = A^{\alpha-1}Af$ and applying Proposition 7.18 b) to the latter expression would yield the assertion.

Best regards

SaschaTrostorff 16:52, 7 December 2011 (CET)

Dear Sascha,

that is right. Thank you!

IsemTeam 14:15, 12 December 2011 (CET)

Proof of Proposition 7.11

Dear Isem Team,

Last line of the formula at the end of the proof of Proposition 7.11 should be

$$= 0 + \frac{1}{2\pi i} \int_\gamma \lambda^z \lambda^w R(\lambda, A)\, d\lambda = A^{w+z}$$

OrifIbrogimov 01:11, 14 December 2011 (CET)

Some further comments

Dear ISem team,
here two points that have been found during the discussion at my univerity, and, I think, have not been mentioned yet:
Prop. 7.23, proof: In the second to the last expression in the long calculation there are factors missing, that appear because of the integrals are computed.
Prop. 7.23 and Cor 7.26: You seem to prove part b) of the Corollary by using Prop. 7.23 a) with the real number α and β = 1. But this is at least not directly possible, since *Re*β = 1 has to be strictly less than one; α = 1, which is allow according to the statement in the Corollary, is not strictly less than *Re*β = 1; and you want to get the result for all $f \in D(A^\alpha)$, but Prop. 7.23 only holds for $f \in D(A^\beta) \subseteq D(A^\alpha)$.
JohannesEilinghoff 15:16, 3 January 2012 (CET)