Talk:Lecture 6

From 15th ISeminar 2011/12 Operator Semigroups for Numerical Analysis

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Comments

closed graph theorem

In the proof of Theorem 6.2 c), one needs: If B is a bounded operator, then B is closed if and only if D(B) is closed. This is _not_ the closed graph theorem, but a rather easy consequence of the definition of `closed'.

JurgenVoigt 17:35, 17 November 2011 (CET)

Right... IsemTeam 22:03, 24 November 2011 (CET)

Proof of Proposition 6.5

I think the proof of the converse direction is made to complicated, and the reason is that you tried to do the same idea as in the general case. Indeed, in the Hilbert space case things are easier (as usual): If A is dissipative and ||f|| = 1, then for \lambda=n we obtain

$$n^2 \le \|nf - Af\|^2 = n^2\|f\|^2 - 2n\mathrm{Re}\langle Af|f\rangle + \|Af\|^2 = n^2 - 2n\mathrm{Re}\langle Af|f\rangle + \|Af\|^2,$$

hence $\mathrm{Re}\langle Af|f\rangle \le \frac{1}{2n}\|Af\|^2 \to 0$ if $n \to \infty$.

AlexanderUllmann 10:05, 18 November 2011 (CET)

Thank you! This is nice, and we include it in the corrected manuscript. IsemTeam 22:05, 24 November 2011 (CET)

Some typing errors

Prop. 6.2, proof, b): You mean $R(\lambda,A)$ instead of $R(\lambda,A_n)$ (two times).
Exa. 6.8.1, def. of X: "vanishes"
p. 69, middle: "that for $f \in L^p(\mathbb{R})$" (without "the")
p. 70, first line: Maybe you could add "$M \geq 0$" behind "a constant".
Thm. 6.16 c): "if and only if" (the second "if" is missing)
JohannesEilinghoff 16:16, 19 November 2011 (CET)

Definition of quasi-dissipative operators

Dear Isem Team,

I'm wondering whether in the definition of quasi-dissipativity, one should require $Re\langle Af, j(f)\rangle \leq w\|f\|^2$ instead of $Re\langle Af, j(f)\rangle \leq w$. For instance for the delay operator from exercise 2 I'm not able to show the required estimate without some $\|f\|^2$ on the righthand side.

MoritzEgert 12:17, 20 November 2011 (CET)

Dear Moritz, you are right, it is missing. Thank you. IsemTeam 22:05, 24 November 2011 (CET)

Comments

Prop. 6.6: The element j(f) pops out of nothing and looks like the image under a mapping j, which, however, was never mentioned before. It should be better to simply take \phi instead of j(f). This would also remove the little problem that, presently, you say that (6.4) should be valid for all \phi: This is meaningless, as there is no \phi in (6.4).

In (6.5): missing \|f\|^2 on the rhs, as Moritz pointed out above.

p:66, middle: In the sentence starting with `We note here without proof ...' the `i.e.' is not quite correct. If \Omega is a measure space consisting of two unit masses, then L_1(\Omega) and L_\infty(\Omega) are reflexive (and in these spaces the duality set is not a singleton).

p.67, line 1: Is this really `Clearly'? One has to convince oneself that for the generalised derivatives one can carry out integration by parts. Has this been done somewhere?

p.67, line 4: `generalisations'? I do not understand in what sense these examples generalise the previous results.

p.68, bottom: In version a) of the proof there is again j(f) popping up which I cannot interpret.

p.69, middle: You say `cf. Example 6.9' for the dissipativity of \Delta. It is not quite clear what the reader should think here. For the heat semigroup on \R, the contractivity is known anyway, whereas in Example 6.9 only the interval (0,\pi) is treated, and one would have to argue that the proof carries over to \R.

p.69, line following (6.11): I do not understand the `if'. It was stated earlier that q \ge p (\ge 1) is supposed. Correspondingly on the last line of this page: The assumption is that q\ge p\ge1.

p.70, The Banach-Alaoglu Theorem: I would prefer to see a sequence (\phi_n) and not single elements. And \phi_n is an element, which cannot be convergent. Also, even if the definition of weak*-accumulation point is correct, I would prefer the formulation that ... there is a subsequence such that \langle f,\phi_{n_k}\rangle \to \langle f,\phi\rangle as k\to\infty.

p.71, Exercise 2: Replace the last line by `Show that A is dissipative, but that its closure does not generate a semigroup'.

p.71, Exercise 4: One might add the information that, additionally assuming that B is dissipative, it follows that A+B generates a contraction semigroup. And: That one can show this also if a\in[0,1); maybe with a reference.

Style and misprints

p.62, middle: Passing ... yields ...: Dangling participle.

p.63, last sentence: `Fortunately ...'. What do you want to express here?

p.64, line 10: ... on _the_ right hand ... And for the whole sentence: I do not think that one can have a colon in a sentence and then continue the sentence started before the colon. If you omit `following' on line 10 and also omit the colon on line 11, then - for my taste - everything would be OK.

p.66, line 5: `contains all (weighted) point measures ...' (a suggestion for a slightly different formulation)

p.67, sentences following Ex.6.10: This `however' would require commas before and after. However, it doesn't make sense to me anyway, in this introduction for the following example.

p.69, line 2: I suggest you should decide whether to use \subset or \subseteq for containment of sets. (For my taste, it is clear that, analogously to < and \le, the sign \subset means proper containment, but I know that this is not generally accepted.)

p.69, middle: ... and here we have to suppose _that_ q \ge p. (I think that here one cannot omit the `that'; Hendrik V. has educated me in this respect.)

Prop.6.15: The formulation `Define B ... operator' is grammatically simply not possible.

p.71, Exercise 2: Why not say D(A):=C_c^2(\Omega)? (Or at least replace `the support of f is compact' by `\supp f compact'.)

Exercise 4, second line: `There _are_ a ...' or `There _exist_ a ...'.

Exercise 5: ... Show that A generates ...

Greetings, JurgenVoigt 17:46, 20 November 2011 (CET)

Dear everybody,

I think that Jürgen's comment about `Clearly` on the top of the page 67 shoud be explained.

I have noticed only one (not already mentioned) small typo: p. 70. at the third centered formula there should be $\varphi(f)$ and not $\varphi(x)$.SandorKelemen 22:41, 6 December 2011 (CET)

one thing.

dear professors: one thing: on page#68,on the proof of theorem 6.13,one paragraph above the last paragraph,named part (a):,i can see that A+B-||B|| is dissipative,which is OK.but,why we can involke theorem 6.3 in this case,since we donot have that A+B is dissipative yet,please help me to see that. thank you. sincerely your student wei WeiHe 22:25, 24 November 2011 (CET)Wei He

Dear Wei He,

you just have to make one more step: As you already agreed, the Operator C:=A+B-||B|| is dissipative, and also all other assumptions of the Lumer-Phillips theorem are fulfilled, hence C generates a contraction semigroup T. Now b:= ||B|| is just a non-negative real number, and it is elementary to show that also A+B = C+b (= C + b*Id) generates a semigroup S given by S(t) = exp(bt)T(t) (to prove this you can use canons as the Hille-Yosida theorem, or simply varify the conditions from the definition of a generator). Here you can also see why the type of S is (M,\omega+b).

Greetings, AlexanderUllmann 00:59, 25 November 2011 (CET)

Another remark: In the beginning of the proof the "renorming procedure" was applied so for me ||B|| is the operator norm with respect to the "new" norm. If one switches back to the "original" norm, I think that S is actually of type (M, \omega + M ||B||). Andreas Geyer-Schulz 14:11, 26 November 2011 (CET)

Dear Andreas,

you are absolutely correct. This type of the perturbed semigroup also appears in other sources, like Kato's book, Theorem 2.1 in Chapter IX. Thanks!

JurgenVoigt 09:16, 7 December 2011 (CET)