Talk:Lecture 4

From 15th ISeminar 2011/12 Operator Semigroups for Numerical Analysis

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Comments

Derivative, Yosida approximants

Prop. 4.5: It is rather unusual, and a bit audacious, to use the notation F'(0) for a strong (!) derivative on a subset. (After all, you never used the notation T'(0) for the generator A.) I think that one should write F'(0) only if F is differentiable at 0.

Prop. 4.8: In b), I would rather write \lambda A R(\lambda,A) f, in order to make it clearer that the operator is bounded.

And on p. 43: The Yosida approximants (of A!) are \lambda A R(\lambda,A) (an A is missing!) for \lambda > \omega (not 0).

JurgenVoigt 13:14, 28 October 2011 (CEST)

I agree with Jurgen's comment on the use of F'(0), which, in my opinion, should rather be written like [*F*(.)*f*]'(0), for instance. To stick to the notation of Thm 2.31 (product rule) one could perhaps also write $F'(0)\cdot f$. FelixSchwenninger 15:05, 28 October 2011 (CEST)

Corollary 4.10

Dear Lecturers, I guess the compact set in Corollary 4.10. should be $\ldots \cup \{Af\}$ instead of "$\ldots \cup \{0\}$". Moreover, it is Theorem 2.30 that we apply instead of Proposition 2.30. Greetings, HannesMeinlschmidt 15:50, 28 October 2011 (CEST)

This is probably true, but even then I do not see how it helps to prove the claim at hand. In order to show that S(t)(1/t)(T(t)f-f) tends to Af, I just compute S(t)(1/t)(T(t)f-f)=S(t)((1/t)(T(t)f-f)-Af)+S(t)Af, and from this decomposition one sees the assertion.

JurgenVoigt 00:02, 29 October 2011 (CEST)

< *math* > *InsertformulahereInsertformulahere*</math> Maybe once more concerning this convergence, in order to put it into a more fundamental context: If (S_n) is a seqence in L(X), converging in the strong operator topology to S\in L(X) (and therefore (S_n) is bounded, by the uniform boundedness theorem), and (f_n) is a sequence in X converging to f\in X, then S_nf_n \to Sf. (Proof: S_nf_n - Sf = S_n(f_n - f) + (S_nf - f) \to 0.) In my post above I have just written this out in the special context.

JurgenVoigt 14:50, 2 November 2011 (CET)

This is only a wild guess, but the compactness/Theorem 2.30.-idea probably comes from the more-general brother of Theorem 2.30., which says that strong continuity for a function $F:[0,t_0]\to\mathcal{L}(X)$ is equivalent to uniform continuity of $[0,t_0]\times K \ni (t,x) \to F(t)x$ for every compact $K\subset X$ (one may replace [0,t_0] by some compact subset of $\mathbb{R}$ in both occurences); e.g. Lemma 5.2 in the first chapter in the book of Engel&Nagel. However, the more specified version of this equivalence, as stated in Theorem 2.30., certainly doesn't help too much here and I agree that the way you proposed it is probably the natural and 'easiest' to obtain the desired convergence.

HannesMeinlschmidt 08:46, 4 November 2011 (CET)

Dear all, Although this discussion was some time ago (I just remembered it while coming up with a similar thing again in this week's lecture and therefore come across this discussion again), I have a remark following the very first remark of Hannes, that the set united with $\ldots \cup \{Af\}$ is compact.

If you consider the function (fixed f) $J_f : [0, h_0] \to X$, that maps

$h \mapsto \frac{1}{h}(T(h)f - f)$ if $h > 0$

and $J_f(0) = Af$ if $h = 0$,

then, this J_f is continuous and hence, the image of $[0, h_0]$ is compact.

Since we use this argument quite often, I thought it would be good to have it written somewhere (however, I also prefer the argumentation as Jurgen mentioned above)

greetings, FelixSchwenninger 17:21, 17 January 2012 (CET)

Questions

Dear ISem Team, in example 4.3, don't we have to consider λ > ω in this case, or isn't that the growth bound? On page 41 in the definition of C, I think it has to be $s \in [0, t_0]$. On page 45, last line before the footnote, don't it has to be $(\frac{1}{h}R(\frac{1}{h}, A) - T(h))g$?MartinAdler 18:54, 29 October 2011 (CEST)

Dear Martin,

ω should be greater than the growth bound in order to ensure that the resolvent $R(\omega,A)$ exists. This is needed because we want to define a strongly continuous function on $[0, \infty)$. On the other hand, to have $F(h)$ defined for all $h \geq 0$ is for notational convenience only. Otherwise, if we have a strongly continuous function $F : [0, h_{\max}] \to \mathcal{L}(X)$, then everytime when we write $F(\frac{t}{n})$, we have to make sure that n is large enough, i.e., $\frac{t}{n} \in [0, h_{\max}]$.

Thanks for pointing out the other two typos.

IsemTeam 09:45, 31 October 2011 (CET)

Def. 4.1 and Example 4.3

Dear IsemTeam,

in Def. 4.1 b), wouldn't it be nicer (and sufficient for the purpose) to require: ... stable, if there _exist_ t_0 and M \ge 1 such that ...? (I think it then follows automatically that the condition is also satisfied with `for all'.)

In Def. 4.1 a), I think that the first line is not correct English. (Well, if you put in two commas, then it works.)

In Example 4.3, I think you should require that \omega > 0; otherwise the interval (0,1/\omega] doesn't make sense.

JurgenVoigt 23:03, 29 October 2011 (CEST)

Dear Jürgen,

thank you for the comments. You are right: Stability is nicer with "exists", and indeed it follows then with "for all".

IsemTeam 09:31, 31 October 2011 (CET)

Question concerning proof of Thm.4.6

Dear IsemTeam,

I have a question concerning the proof of Thm 4.6: After having chosen the subsequences properly you want to make use of the convergence property of F. But in order to do so, we have to assure that $h_k n_k \in [0,t]$ holds (not only $h_k n_k \in [0,t_0]$ since $h_k n_k$ converges to t and not to t_0). I don't see why this should hold. For example $(h_k n_k)_k$ could be a decreasing sequence?

MoritzEgert 11:22, 31 October 2011 (CET)

Dear Moritz,

it seems to me that you have a fine point here. At least, I did not succeed to show the property under the condition formulated as stability, in the lecture. Bringing the issue to a narrower formulation, one would have to show that, if one has consistency (which did not help in this part anyway) and convergence, then for any sequences (n_k), (h_k) with n_k h_k \to 0, the sequence (F(h_k)^{n_k}) is bounded. (Which does not really help because I cannot show it.)

Anyway, looking at the Lax-Richtmyer paper, I see that the convergence property is not formulated as in the lecture, but rather the property should be true for all t \in [0,t_0] and any sequences (n_k), (h_k) such that n_k h_k \to t (and it doesn't say anything where n_k h_k should be lying).

This may clear it up, that convergence should be defined differently. (Indeed there is no problem to prove the more general kind of convergence from consistency and stability, as far as I can see.)

JurgenVoigt 22:14, 1 November 2011 (CET)

Dear Jürgen,

unfortunately there has been no response explaining how to fix the proof if one takes for granted the definition of convergence given in the lectures, so far. But I agree that everything works well with the more general definition of convergence from the Lax-Richtmyer paper.

MoritzEgert 15:17, 12 November 2011 (CET)

Dear Isem team,

I think Moritz's comment and Jürgen's reaction are crucial. Please clear up the definition of convergence of numerical scheme. Thank you. SandorKelemen 14:30, 14 November 2011 (CET)

Dear All, of course Jürgen is right: there was a typo when writing the definition. Thank you for correcting it. IsemTeam 21:44, 24 November 2011 (CET)

Strong continuity in Prop. 4.5

This may be nitpicking, but the wording of Prop. 4.5 suggests to me that strong continuity of T on Y is automatic from the invariance of Y under T. This is clearly not the case, as can be seen by considering the heat semigroup on $L^2(0,1)$ with Dirichlet boundary conditions and taking Y as the space of differentiable functions with Lipschitz continuous derivatives subject to zero boundary conditions equipped with its natural norm. Since T(t) maps $L^2(0,1)$ into the space of twice continuously differentiable functions for every t > 0, the semigroup cannot be strongly continuous on Y. Some additional comment, for what it is worth: since Y is isomorphic to $L^\infty(0,1)$, there are in fact no strongly continuous semigroups on Y except for those with bounded generator by a result due to Lotz.

But since you only applied the proposition to Y = D(A) up to now, there is no problem checking this additional assumption. So I would be perfectly happy with the rewording "Suppose that it is invariant under the semigroup T and that the restriction is a strongly continuous semigroup again." By the way, there is an f missing in the proof's second formula line.

RobinNittka 12:34, 31 October 2011 (CET)

Dear Robin,

thanks for your contribution, and thanks for the example! It gives me the opportunity to point out that one has to be careful to formulate unambiguously. The `so that' really can mean two things: 1. `and with the property that', 2. `which automatically implies that'. When I read this sentence, feeling that the second does not hold, I interpreted it as 1. (But, as I said, one should by all means avoid ambiguous formulations.)

JurgenVoigt 13:35, 31 October 2011 (CET)

Miscellaneous

Proof of Proposition 4.5, second formula line: missing f in the very last term.

Proof of Thm 4.6, second part. Regardless whether you will have to change the definition of `convergent' or not, I suggest to prove the stronger version, and therefore start this part of the proof as follows: Let t_0>0. Fix t\in[0,t_0] and take sequences ... with h_kn_k\in[0,t_0] and ...

p. 42, line 10 should start: holds for all s\in[0,t_0) and k\ge N. (No t should be mentioned, because t is fixed!)

Definition 4.11, preamble: Throw in some commas, please!

Definition 4.11 a): Discard `there is a subspace ... , so that'.

Cor. 4.15. Decide whether you want C or K.

p. 45, last line. Starting from here, there are some typos. In this line, the upper limit in the integrals should be s, and the integration variable should be r (for instance). Also missing in the first of the integrals: dr.

p.46, line 4. The upper limit in the integral should be s, and the integration variable r. (Then the second term in the estimates make sense again!)

And here is a suggestion for a triple norm macro (Exercise 1 a)): \newcommand\triplenorm{|\hskip-1pt|\hskip-1pt|}.

(We had our session on this lecture today; so, this is the collection which came up.)

JurgenVoigt 14:34, 2 November 2011 (CET)

Lie-Trotter product formula - stability

Dear Isem team

My comment deals with the proof of the Corollary 4.10. I've tried to get the stablility of the scheme *F* from the assumption $\left\|\left(S\left(\frac{t}{n}\right)T\left(\frac{t}{n}\right)\right)^n\right\| \leq M\mathrm{e}^{\omega t}$ but I haven't succeed. May I ask for help? Thank you. SandorKelemen 12:02, 15 November 2011 (CET)

Dear Sandor,

in order to prove stability of the scheme *F*, fix $t_0 > 0$.

For $h \geq 0$ and $n \in \mathbb{N}$ with $hn \leq t_0$ you can use the assumption in the Lie-Trotter product fromula you metioned (it should hold for all with $t \geq 0$ and all $n \in \mathbb{N}$ though this is not explicitely written there) with the choice $t = hn$ in order to estimate as follows:

$$\|F(h)^n\| = \|(S(h)T(h))^n\| = \|(S(\frac{hn}{n})T(\frac{hn}{n}))^n\| \leq M\mathrm{e}^{hnw}$$

If $w \leq 0$, the rightmost term is bounded by *M*. If $w > 0$, you get $M\mathrm{e}^{t_0 w}$ as a bound. This gives you stability of *F*.

MoritzEgert 14:03, 16 November 2011 (CET)

That's fine! Thank you Moritz. SandorKelemen 12:38, 17 November 2011 (CET)