Talk:Lecture 3

From 15th ISeminar 2011/12 Operator Semigroups for Numerical Analysis

Contents

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Comments

Typo in the definition of the "hat" functions in Example 3.4

Dear ISEM team. As I look from Figure 3.1, the function $B_{k,n}(x)$ should be $n\left(x-\frac{(k-1)}{n}\right)$ if $x\in\left[\frac{k-1}{n},\frac{k}{n}\right)$ instead of $n\left(x-\frac{k}{n}\right)$ in the first line of the definition. ParinyaSa 04:21, 21 October 2011 (CEST)

Dear Parinya Sa,

thanks for pointing that out.

IsemTeam 10:04, 21 October 2011 (CEST)

Just a short question/remark: In the line following (3.3) with 'equvialent' you mean that $\|A_ny_n-P_nAg\|\longrightarrow 0$ is equivalent to $\|J_nA_ny_n-Ag\|\longrightarrow 0$?! (which follows from the assumptions made in Ass. 3.2). But this does not imply $\|y_n-P_ng\|\longrightarrow 0$(in general). $X = X_n$,$J_n = P_n = I$ and $A_n = A$ with noninvertible A should serve as a counter example. FelixSchwenninger 10:24, 21 October 2011 (CEST)

Dear Felix,

yes, we mean exactly what you are saying. It should have been written so: "the second convergence in the above is equivalent to".

Thank you!

IsemTeam 11:44, 21 October 2011 (CEST)

"Mistake" in Example 3.4 (finite difference)

Dear ISEM team,

With the definition of the hat functions Bn,k mentioned in page 30, it is easy to verify that: Bn,0+Bn,1+...+Bn,n-1=0, so Jn(1,1,...,1)=0 which contradicts the fact that Jn is injective (see the second point of Assumption 3.2)RachidSaij 20:01, 21 October 2011 (CEST)

Dear Rachid, thanks for your observation. Indeed, as already pointed out by ParinyaSa above, we made a typo in the definition. IsemTeam 21:22, 21 October 2011 (CEST)

Dear Isem Team, I have a small question on page 36, Lemma 3.13, $(i)\Rightarrow(ii)$: Don't you have to define g_n: =

$(\lambda - A_n)f_n$? And in the inequalities in the last norm, it has to be $\|f_n - P_n f\|$. MartinAdler

comments 2

Dear Isem Team,

thanks for this lecture! Some remarks:

In Proposition 3.8. the constant C' is depending on t (or on the compact set from which t is taken). Therefore, I would name it C'_t?!

Chapter 3.2. Apparently, it is assumed that A^{-1} and A_n^{-1} exist, as e.g. for exponentially stable semigroups. As it is indicated in Remark 3.11 this isn't really a restriction. But let us assume for a moment that the semigroups T_n and T are exponentially stable, so the ω can be negative. Then, one has to be a bit more careful with the estimates of T_n,T for t in $[0,t_0]$ (for instance on p.36 first inequality). Alternatively, one can just take positive \omegas by assumption. However, this is only about constants. Is this a general guideline that you assume that A^{-1} exists?

In the proof of Lemma 3.13 it might be good to mention that it is sufficient to show the convergence on $(\lambda - A)Y$ because $R(\lambda,A_n)$ is norm bounded by assumption (Thm 2.30).

In the Trotter Kato Theorem, I would in addition assume that $\|T(t)\| \leq Me^{t\omega}$ (for the sameω as in the stability condition) since otherwise it is, for instance, not clear that the resolvents of A and A_n exist for all $\lambda > \omega$. But since T is a semigroup, this is just a technical assumption.

Some typos:

p.32: there went something wrong in the definition of the modulus of uniform continuity(I would replace the second sup by ':').

$D(A^2)$: there should be $f(1)$,$f'(1)$ and $f''(1) = 0$ instead of the evaluation at 0.

p.36 (as mentioned above) g_n should be defined as $(\lambda - A_n)f_n$

p.37: For the sake of completeness P_n should be inserted in the first formula of the proof(right hand side of inequality). The second inequality from below is true, but probably can be left out anyway. In the line above an 'f' (or two brackets) is missing. FelixSchwenninger 16:19, 22 October 2011 (CEST)

Transpose in Appendix A

Dear ISem Team, another question to the appendix: Isn't the definition of the matrix A_m in the Galerkin method the transposed of what we need? MartinAdler 16:47, 23 October 2011 (CEST)

Dear Martin, you are of course right, thank you for your remark. IsemTeam 00:15, 24 October 2011 (CEST)

Little mistake in Taylor's formula

Dear ISEM-team,

I just wanted to mention a little mistake in Example 3.7. First, for the sake of correctness, I think you have to define $y_n = P_n f$ and not $y = P_n y$ at the first line of page 32. The actual mistake concerns the application of Taylor's formula in the middle of page 32. Both, the definition of $(A_n P_n f - P_n A f)_k$ and Taylor's formula require the centered term to be

$\frac{f(\frac{k+1}{n}) - f(\frac{k}{n})}{\frac{1}{n}} - f'(\frac{k}{n})$ instead of $\frac{f(\frac{k+1}{n}) - f(\frac{k}{n})}{\frac{1}{n}} + f'(\frac{k}{n})$.

PatrickTolksdorf 10:30, 24 October 2011 (CEST)

And yet another remark to that particular equation: It should only be $\leq f''(\xi_k)\frac{1}{2n}$, since we apply Mean Value theorem twice and cannot know what the distance is between the first ξ and k/n is.

Katharina Schade 10:50, 25 October 2011 (CEST)

Hi Katharina, I agree that one could also arrive at the sup-norm estimate in the following line by using the Mean Value Theorem for differentiation twice, although without the 2 in the denominator (which still yields first order convergence). However, the Taylor's formula as used here is usually (at least as far as I know) derived using the Mean Value Theorem for _integration_, which indeed gives the stated equality. Cheers :) - HannesMeinlschmidt 13:30, 25 October 2011 (CEST)

Question to Proposition 3.8

In proposition 3.8 we assume that Y, what is by assumption 3.5 a dense subset of $D(A)$, is a Banach space, with regard to the norm of the Banach space X? In this case Y would be closed in X. Moreover the denseness of $D(A)$ in X would imply $X = Y$. As a consequence we would obtain that A is bounded and this coveres very less of all possible operators A, we want to approximate.

PatrickTolksdorf 11:03, 24 October 2011 (CEST)

@Patrick: No, the norm for Y is not the same as for X in general (I would say). This is indicated by $\|.\|_Y$. In example 3.7 you can see that it is not an equivalent norm. But I think it might be good to emphasize this. In connection to this, I'm a bit confused since in Ass. 3.5 we assume Y to be a subSET, and in Prop. 3.8 we want that is even a space(which we need there). But is the completeness of this space really important?FelixSchwenninger 11:24, 24 October 2011 (CEST)

Ah yes! Thanks! It was just an over reaction by myself ;-) PatrickTolksdorf 21:37, 24 October 2011 (CEST)

Dear Felix, subSET or subSPACE may be confusing; however, if Ass. 3.5 is valid with a subset Y, then it is also valid with \lin Y. (So, one may as well formulate Ass. 3.5 with a subspace.) JurgenVoigt 10:03, 28 October 2011 (CEST)

Small typo

Dear Isem team, I noticed a small typo on page 1001 of appendix A. A few lines from the bottom, in the section on Galerkin methods, a reference is made to problem (A.1). I believe this should be (A.4). JanRozendaal

Dear Jan, thank you for calling our attention to this misleading typo. IsemTeam 23:59, 24 October 2011 (CEST)

Proof of Theorem 3.14

Dear IsemTeam,

I have the impression that there are many P_n's missing. Am I correct?

Jürgen Voigt (Dresden)

(Stupid question: How do I get my identity in the signature blue?)

Dear Jürgen, you have to use 4 tildes (~) to produce a signature. Greetings - HannesMeinlschmidt 14:57, 25 October 2011 (CEST)

Dear Jürgen, sometimes it was intended that the P_n-s are missing. In the proof of Theorem 3.14 we forgot to mention that, as in the proof of Proposition 3.10, we only carry out the calculations in the case $X_n = X$, $P_n = J_n = I$. If there are other places, please, point them out, it would be great to have them all corrected. IsemTeam 16:58, 25 October 2011 (CEST)

Five Things

+ dear professors:five things:1)in the proof of proposition 3.10,we need w to be positive,not any real number,since we want exp(wt)<=exp(wt0).so,since t<=t0,and if w <0,we have exp(wt)>=exp(wt0). + 2):in lemma 3.13,proof (i)=>(ii):gn:=(lemda-An)fn is what we want,not gn:=(lemda-A)gn. + 3):in theorem3.14:proof (iii)=>(ii):for lebesgue dominated convergence theorem,we need a L^1 function,but,we have the integrand to be locally L^1. + 4):forgive me that i need help to see:proposition3.8: + how does the variation of constant formula work. + 5):also,please help me to see:how does theorem 3.14 part (iii)=>(ii) the integral representation of resolvant work. + thank you very much. + sincerely your student weiWeiHe 21:31, 26 October 2011 (CEST)

Dear Wei, concerning your points:

-1)yes, the estimates have to be done a bit differently, but for $\omega < 0$, you can always say $Me^{\omega t} < M$ for $t > 0$. Note that the right hand side of the inequality should read

$$\int_0^\infty e^{-\lambda t} \|T(t)P_n f - P_n T_n(t) f\| dt.$$

-2)of course, as already pointed out above,

-3) for T holds: $\|T(t)\| \leq Me^{\omega t}$ (concerning the ω see my remark above), hence the integrand can be estimated by an L1 function (we consider $\lambda > \omega$!)

-4)As mentioned in the proof, $s \mapsto T_n(t-s)T(s)f$ is differentiable(let us denote it by $v(s)$) by Thm 2.31 and also from this Thm we know how the derivative can be calculated:

$v'(s) = -A_nT_n(t-s)T(s)f + T_n(t-s)AT(s)f.$

Since $[T()f]'(s) = AT(s)f = T(s)Af$ (and analogously for T_n), we obtain

$v'(s) = -T_nA_n(t-s)T(s)f + T_n(t-s)AT(s)f = T_n(t-s)(A-A_n)T(s)f.$

Now, as mentioned in the proof, the fundamental theorem of calculus says

$v(t) = v(0) + \int_0^t v'(s)ds$, which yields the 'variation of constants formula' on p.33 ($T_n(0)$,$T(0) = I$).

-5)See Prop.2.26a). Therefore we have

$$R(\lambda, A_n)P_nf - P_nR(\lambda, A)f = \int_0^\infty e^{-\lambda t}T_n(t)P_nf dt - P_n\int_0^\infty e^{-\lambda t}T(t)f dt.$$

Since P_n is a bounded operator, it follows that $P_n\int_0^\infty e^{-\lambda t}T(t)fdt = \int_0^\infty e^{-\lambda t}P_nT(t)fdt$ (see Prop. 2.33). Hence,

$$\|R(\lambda, A_n)P_nf - P_nR(\lambda, A)f\| \leq \int_0^\infty e^{-\lambda t}\|T(t)P_nf - P_nT_n(t)f\|dt,$$

which gives us the desired convergence by Lebesgue Dominated convergence(see -1)).

Hope, this clarifies a bit. Greetings, Felix FelixSchwenninger 09:02, 27 October 2011 (CEST)

Examples concerning Proposition 3.10

Dear Isem-Team,

I have some problems in connecting Proposition 3.10 with the following examples in 3.12 which should somehow show that the results of Proposition 3.10 are far away from being sharp.

For the heat-semigroup as well as for the Schroedinger-semigroup you estimate $\|J_nT_n(t)P_nf - T(t)f\|_X$. This however is not the expression we get an estimate for in the conclusion of Proposition 3.10. If we look at $\|T_n(t)P_nf - P_nT(t)f\|_{X_n}$ this expression is just identically zero for T being the heat- or Schroedinger-semigroup.

Your examples raise the question whether it wouldn't be more interesting to get an estimate for the order of convergence of $\|J_nT_n(t)P_nf - T(t)f\|$ in Proposition 3.10.? Such an estimate does not follow from the estimate for $\|T_n(t)P_nf - P_nT(t)f\|_{X_n}$ because we don't know anything about convergence rates for the strong convergence $J_nP_n \to Id$

Regards MoritzEgert 13:38, 27 October 2011 (CEST)

Proposition 3.10

I have a question regarding the final step of the proof of Proposition 3.10. Combining all the previous estimates you arrive at:

$$\|T_n(t)g - T(t)g\| \leq \|A^{-1} - A_n^{-1}\|C(\|A^2g\| + \|Ag\|)$$

In which way can i establish that this is bounded from above by a constant times the $\|\cdot\|_{A^2}$ -Norm of g which is derfined as $\|g\|_{A^2} := \|g\| + \|A^2g\|$?

FlorianMueller 13:49, 27 October 2011 (CEST)

Dear Florian, let me answer this for the case that 0\in\rho(A). (Otherwise you have to replace A by A-\lambda, for some \lambda\in\rho(A).) In this case, you can use as graph norm on D(A^2) simply |||g|||:=||A^2g|| (because ||g|| = ||A^{-2}A^2g|| \le ||A^{-2}|| ||A^2g||). And then ||Ag|| = ||A^{-1}A^2g|| \le ||A^{-1}|| ||A^2g|| = ||A^{-1}|| |||g|||.

JurgenVoigt 10:57, 28 October 2011 (CEST)

Two typing errors on page 34(?)

I think I have found two typing errors on page 34 that not have been mentioned yet:
In Remark 3.11 1.: $J_nA_n^{-1}P_n$ instead of A_n^{-1}
In the footnote 2: $\|A^2g\|$ instead of $\|A^2f\|$ JohannesEilinghoff 19:51, 27 October 2011 (CEST)

I think it should be noted that Remark 3.11. 1. basically kills two birds with one stone: On one hand, it would be equivalent to require $\|R(\lambda, A_n)P_n - P_nR(\lambda, A)\| \leq \frac{C}{n^p}$ for some $\lambda \in \rho(A_n) \cap \rho(A)$ in Proposition 3.10.. In fact, and this was pointed out already in an earlier comment on this page, Proposition 3.10. (the way it is currently stated) basically assumes that $0 \in \rho(A) \cap \rho(A_n)$. However, this is without loss of generality, since once may rescale the semigroup appropriately to guarantee its boundedness and shift 0 into it the resolvent set of its generator (this could be emphasized a bit more if you ask me, since it is heavily used in the lectures and not mentioned explicitly, the only thing hinting at it being Exercise 3.b of Lecture 2). Also, the stability condition (3.2) guarantees that this procedure is possible for the approximating semigroups (and their generators) as well. On the other hand, the condition $\|J_nR(\lambda, A_n)P_n - R(\lambda, A)\| \leq \frac{C}{n^p}$ is _stronger_ than $\|R(\lambda, A_n)P_n - P_nR(\lambda, A)\| \leq \frac{C}{n^p}$. So, of course, we could also assume $\|J_nR(\lambda, A_n)P_n - R(\lambda, A)\| \leq \frac{C}{n^p}$ in Proposition 3.10. and still obtain the desired result. However, this is a stronger assumption and _not_ equivalent. Maybe this should be explained a bit further (which is obviously related to the comment of Moritz above). HannesMeinlschmidt 08:58, 28 October 2011 (CEST)

Miscellaneous

Prop. 3.8: I think that as convergence rate one could also allow any p > 0. In fact, instead of using the function n

\mapsto 1/{n^p} one could allow any function \alpha: \N \to (0,\infty) (need not even tend to zero).

p.33, second formula line from below: I do not see a reason, why s \mapsto \|T(s)f\|_Y should be measurable. Therefore, one should insert immediately the estimate from the last line on p. 32, and everything works.

Concerning Prop. 3.10: Assumption 3.2 consists of two parts (the common stability condition and the approximation condition), and only the first of these conditions is used in the proof, as far as I can see. Also: Nothing tells us that A_n or A are invertible. So, in order to write this, one should add the corresponding hypothesis.

Remark 3.11. 1: There is no specification of the relation between \lambda and n. (One can guess that the same \lambda is meant for all n, because that is the only sense one could try to make out of the statement.)

Last estimate before (3.5): It is simply false if \omega < 0. (The same happens in the proof of Theorem 3.14.)

Lemma 3.13 (i): For the first line, wouldn't it be easier (and more convincing) to say that Y is a core for A. (Same remark concerning part (i) of Theorem 3.14. Anyway, in (i) of Theorem 3.14 one has to take \lambda > \omega; under the assumption that (3.2) holds also for T.)

Theorem 3.14 (iii): I would prefer to see the convergence stated as \|J_n T_n(t) P_n f - T(t)f\| \to 0 ... (After all, one wants to approximate T!) This is equivalent to the statement as given now.

Proof of Theorem 3.14 (iii) => (ii): One might add the observation that for this implication one only needs convergence for all t > 0; no uniformity is needed.

Proof of Theorem 3.14 (ii) => (iii): In the first paragraph, one does not use the uniform boundedness theorem, but rather what I would like to provide with the name `strong-convergence lemma', meaning that a bounded sequence of operators converging on a dense set converges strongly. And it also would be nice to indicate the dense set, the {range of R(\lambda,A)}=D(A), on which one wants to show convergence. (Maybe you want it to be the set D(A^2); but this is not really needed: It is shown that the middle term is a bounded sequence converging on the dense set D(A), and therefore strongly.)

still Proof of Theorem 3.14 (ii) => (iii): `independently of t' cannot be said. Replace it by '... converges uniformly for (or `in', as you say it later) t \in [0,t_0] ...' or '... converges uniformly on [0,t_0] ...'

JurgenVoigt 15:47, 28 October 2011 (CEST)

Mean Value Theorem

Hi. I think that the application of the Mean Value Theorem in a way on the page 32 works only for *real-valued functions* $f : [0,1] \to \mathbb{R}$. As a counterexample for complex-valued function (where Mean Value Theorem fails) we have $t \in [0,1] \to f(t) := \mathrm{e}^{2\pi i t}$. Here we get $\left|\frac{df}{dt}(t)\right| = 2\pi$ for all $t \in [0,1]$ so the equation $f(1) - f(0) = \frac{df}{dt}(\xi)$ cannot be satisfied for any $\xi \in [0,1]$.

The estimation of $||J_n A_n P_n f - Af||_\infty$ can be done also for complex valued function, but we have to use the *integral form* of the Mean Value Theorem: $\frac{f(\frac{k+1}{n}) - f(\frac{k}{n})}{\frac{1}{n}} = \int_0^1 f'(\frac{k+s}{n})ds.$

The same note corresponds to the Taylor's formula in the estimation of $||A_nP_nf - P_nAf||$.

Thank you for answer. SandorKelemen 22:40, 29 October 2011 (CEST)

Dear Sandor,

you are definitely correct with your mistrust in the mean value theorem for \C-valued function. However, there is a good replacement for the mean value theorem, i.e., the mean value inequality. The proof of the desired inequality is minimally longer, but the inequality in the second line of the estimate on p. 32 is correct again. Because I found it too cumbersome to insert it as a post, I put the proof on my home page under http://www.math.tu-dresden.de/~voigt/isem11/ . (As a service, I have also included a proof of the mean value inequality on the page indicated above.)

JurgenVoigt 18:55, 30 October 2011 (CET)

Dear Jürgen,

we appreciate your efforts very much! Let us add one more comment: In this very problem, one can use the mean value theorem for the real and imaginary parts separately, to conclude the convergence for each of them (since Re and Im both commute with all the occuring operators).

IsemTeam 10:03, 31 October 2011 (CET)

Thank you. SandorKelemen 10:40, 31 October 2011 (CET)

Trotter - Kato Theorem

Dear ISEM team,

I would like to ask you for some clarification related to Lemma 3.13 (i). Do you require denseness of the subset Y regarding to the norm in X or the graph norm? I guess you mean the first one, but I am not sure. And could it be that there es some typing error in Thm. 3.14? Maybe it would be more convenient to ask λ to be λ > ω instead of λ > 0? Thank you very much.

Best regards, RebekkaBurkholz 17:28, 31 October 2011 (CET)RebekkaBurkholz

I think Rebekka is right. I would like to add the following: for the part $(ii) \Rightarrow (i)$ of Lemma 3.13 the density of $D(A)$ in X is also needed. SandorKelemen 13:39, 1 November 2011 (CET)

Dear Rebekka and Sandor,

concerning Rebekka's first question: The lecturers assume Y to be a subset of D(A), to be dense in X, and (\lambda - A)Y to be dense in X. These properties are equivalent to requiring that Y is dense in D(A) with respect to the graph norm (or equivalently, that Y is a core for A, under the assumption that Y is a subspace).

And I agree with Sandor: In order that the Lemma becomes correct, one should also assume that D(A) is dense in X, I think (or at least assume this additionally in (ii)). (Otherwise Y:=D(A) in step (ii) => (i) will not be dense in X!)

Concerning Rebekka's second question: Yes, I think that \lambda should be > \omega (supposing that the exponential bounds for T are the same as for the T_n's (see the posts above!).

JurgenVoigt 23:23, 1 November 2011 (CET)

Example 3.9

Dear ISEM Team, can you please make some comments on the first sentence in Example 3.9 (p. 33)? - In the situation of Example 3.6, the first condition in Proposition 3.8 is always satsfied (if I see it correctly); can you please expand on what you mean with "difficult and unnatural" in this context? Thanks in advance! SvenWegner 14:24, 2 November 2011 (CET)

Example 3.12

Dear ISEM Team, at the end of Example 3.12 you state that in the setting of this example convergence of order $p = 2$ holds for $g \in D(B)$. Is it true that this is sharp? - We selected $g^{(n)}$ to be the n-th unit vector times suitable constants to get $\|J_nP_nS(t)g - S(t)g\| = \frac{1}{(n+1)^2}$ for any n which should show that the estimate in the last computation of Example 3.12 cannot be true for any g and all large n with $\frac{1}{(n+1)^4}$ replaced by $\frac{1}{p^2}$ with $p > 2$. Is this correct? Thanks in advance! SvenWegner 14:41, 2 November 2011 (CET)

Sorry, I do not understand your question. You compare the decay 1/(n+1)^2 with 1/(n+1)^4. Did you, maybe, overlook that there is a square at the term on the left hand side on p.35, line 7? And why do you want to replace 1/(n+1)^4 by 1/p^2? (There is no n in the last term.)

Anyway, as Moritz already pointed out above, it is not clear what the relation between Prop. 3.10 and Example 3.12 should be. (Because for the semigroups considered in Example 3.12, the expressions estimated in Prop. 3.10 are = 0 anyway.)

JurgenVoigt 21:58, 2 November 2011 (CET)

Thanks for your comment. I don't overlook the square; but the square is the reason for the confusion: In the text above I first considered

$$\|J_nP_nS(t)g - S(t)g\| = \frac{1}{(n+1)^2}$$

(where I claimed that this holds for certain $g = g_n$) but then I refered to the computation in the lecture as it is, i.e. with the squares,

$$(\star) \quad \|J_nP_nS(t)g - S(t)g\|^2 \leqslant \cdots \leqslant \frac{1}{(n+1)^4}\|Bg\|^2.$$

Therefore, I wrote "$\frac{1}{(n+1)^4}$" instead of "$\frac{1}{(n+1)^2}$".

In the last sentence there is of course a mistake, thanks for pointing this out: I want to know if $p = 2$ is sharp, i.e. if convergence of order $p > 2$ can hold, or not. And convergence of order $p > 2$ would require that

$$\|J_nP_nS(t)g - S(t)g\| \leqslant \frac{1}{n^p}\|Bg\|$$

is valid for every g and all large n. In $(\star)$ this would mean that it is possible to replace $\frac{1}{(n+1)^4}$ by $\frac{1}{n^{2p}}$. SvenWegner 09:10, 3 November 2011 (CET)

Dear Sven, now that you clarified, you mention an important thing we omitted from the lecture. Thank you for making sure the estimate cannot be improved. IsemTeam 00:17, 4 November 2011 (CET)

Two remarks

Two things that have been mentined during the discussion at my university:
Example 3.4: When you define the function $J_n(y_0, \ldots, y_{n-1})$ you should either add an (x) on the left-hand side or leave the one on the right-hand side away.
Example 3.7: When we define D(A) as you have done, can we be sure that $(Af)(1) = f'(1)$ ist 0, which is necessary for Af beeing in X? JohannesEilinghoff 16:00, 5 November 2011 (CET)

Dear Johannes,

you are absolutely right; this is not the correct domain of the generator. The domain should be D(A)=\{f\in C^1([0,1]): f(1)=f'(1)=0\}. (Anyway, the formulation `define the generator' is misleading, because the generator 'defines itself'; one can only find out what it is.) Incidentally, I would denote the space C_{(0)}([0,1]) by C_0([0,1)), which means the continuous functions on [0,1) vanishing at \infty of the locally compact space [0,1).

JurgenVoigt 21:30, 6 November 2011 (CET)