Talk:Lecture 13

From 15th ISeminar 2011/12 Operator Semigroups for Numerical Analysis

Contents

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Here you can discuss the material of Lecture 13.

- **To make comments you have to log in.**
- You can add a new comment by clicking on the plus (+) sign on the top of the page.
- Type the title of your comment to the "Subject/headline" field.
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- You can write mathematical expressions in a convenient way, i.e., by using LaTeX commands between and (for instance, $\mathrm{e}^{tA}u_0$ gives $\mathrm{e}^{tA}u_0$).
- Please always "sign" your comment by writing ~~~~ (i.e., four times tilde) at the end of your contribution. This will be automatically converted to your user name with time and date.
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Remember that due to safety reasons, the server is off-line every day between 4:00 a.m. and 4:20 a.m. in local time (CEST).

Comments

Thesis on Crank-Nicolson scheme

Last week friday, my student Niels Besseling defended (with success) his Ph.D. thesis. He investigated the possible growth of the Crank-Nicolson scheme when the semigroup is not analytic, but is an arbitrary bounded semigroup on a Hilbert space. One of the main results is that the growth of the discretization is bounded by a constant times log(n). It is still unknown if this is the optimal estimate. On Banach spaces the optimal estimate is the square root of n. Furthermore, he gave a new proof of the fact that if A and its inverse generate a bounded semigroup, then the Crank-Nicolson scheme is uniformly bounded. This week's result in the lecture on analytic semigroups follows from this. The pdf-file of the thesis may be found at Stability analysis in continuous and discrete time (../../../external.html?link=http://doc.utwente.nl/79376/) .

HansZwart 12:50, 27 January 2012 (CET)

some comments

Dear IsemTeam,

the participants seem to be somewhat exhausted (and you as well, actually). So, finally I start the comments.

p.149, line 8 from below: of **bounded** analytic ...

p.151, in Prop.13.3 b): I'm not sure whether commutation of an unbounded operator with a bounded operator was defined. If B is an operator and C is a bounded (everywhere defined) operator, then it means that $BC \subseteq CB$.

Theorem 13.10: We think that you do not want the hypothesis that $|r(\infty)| < 1$.

Theorem 13.11: In previous occurrences, the order of the rational approximation was always specified. Is this order 1, here?

We had problems to understand the text following Figure 13.3. I think that I understand now that the estimates you indicate in the following are always for *z* lying on γ, and satisfying the additional restrictions. (Maybe you should make this clearer.) And I cannot follow the reasoning for the proof of the estimate on line 8 from below (on p.159). For me, from the hypothesis that r is an approximation of order 1, I would estimate

$$|r(z)| \leq |e^z| + C|z|^2$$

for *z* near 0, and this implies the estimate you indicate.

And an additional word concerning the proof. You prove the estimate only for $n > \frac{1}{\rho}$. Maybe you should loose a word why it is also true for $t \leq \frac{1}{\rho}$.

Typos and style

line 4 of the text: ... considered **as** multiplication ...

p.150, line above Definition 13.1: ... fixed for the remainder of this ...

In Definition 13.1: δ.

p.157, line 2: Three times *hA* instead of *A*.

Theorem 13.11: `i.e.', in my perception, means that the foregoing is expressed equivalently after `i.e'. This is not the case here. (D(A) should also be dense.)

p.159, line 1: Let $\frac{\pi}{2} - \delta < \cdots$

Best wishes, JurgenVoigt 21:00, 2 February 2012 (CET)

comments

Dear all, To join Jurgen and make this week's discussion more like a "discussion", I have the following additional remarks:

1)p.158, Sec. 13.4, first line: An "of" too much

2)p.156: 9th line from below: Should be "γ_2 is the part of γ that lies outside...",

last equation on the same page: first equality should rather be an $\leq$

3)p.153: To my taste, it should be stated that $|\eta| < \pi - \delta'$. Furthermore, I think you want μ instead of 1 in the third last line (in the denominator of the last term)

4) the definition of d on p.151 (and $F(\infty)$ elsewhere) is clear form the context, but to write the limit as $z \to \infty, z \in$ mirrored Σ_δ looks a bit strange to me since the set mirrored Σ_δ is in the left half plane. But okay, that's really a matter of taste.

5) To emphasize Jurgen's comment: Thm 13.10 should really not include $|r(\infty)| < 1$ in the assumptions (I guess this happened since you copied the paragraph from Thm 13.9. while texing. And then forgot to delete this item :))

Thanks for the lectures, Greetings from Enschede,FelixSchwenninger 09:21, 3 February 2012 (CET)

Some remarks

Dear ISem team,
I have the following remarks:
Prop. 13.3 b): Add "it holds that" before "if" to write a complete sentence.
Prop. 13.5, proof of a), at the the beginning: Maybe you could add a comma in "is holomorphic at 0, we have" to make the sentence easier to read
p. 153, first integral: It should be $\gamma_{\delta',a}$ instead of δ', a under the integral sign.
p. 153, second integral: I think you concluded the equality of the three terms using the equality of the first and the third expression by definition and the equalitiy of the first and the second expression by your argumentation above. So I suggest to put the expression $\Phi_A(G)$ to the left of the equality.
Prop. 13.7: where γ _is_ the positively
Prop. 13.8 c): $\beta \in \mathbb{N}$, not η
Thm. 13.9, proof, two lines before (13.3): I think the resolvent is $R(\frac{1}{h}, A)$, not $R(1,A)$.

Thm 13.10, proof, last line: The first f in the first expression should not be there.

Thm. 13.11: In my oppinion it sounds like you tell the reader that the property $|r(\infty)| < 1$ belongs to the definition of $A(\alpha)$-stabiliy. Maybe you could make it a bit clearer that this is not the case.

Thm. 13.12: I think, it should be $\frac{\pi}{2} - \delta < \cdots$ here too, cf. Juergen's comment to Thm. 13.11.

JohannesEilinghoff 16:07, 4 February 2012 (CET)