Talk:Lecture 12

From 15th ISeminar 2011/12 Operator Semigroups for Numerical Analysis

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How to contribute?

Here you can discuss the material of Lecture 12.

- **To make comments you have to log in.**
- You can add a new comment by clicking on the plus (+) sign on the top of the page.
- Type the title of your comment to the "Subject/headline" field.
- Add your contribution to the main field.
- You can write mathematical expressions in a convenient way, i.e., by using LaTeX commands between and (for instance, $\mathrm{e}^{tA}u_0$ gives $\mathrm{e}^{tA}u_0$).
- Please always "sign" your comment by writing ~~~~ (i.e., four times tilde) at the end of your contribution. This will be automatically converted to your user name with time and date.
- You can preview or save your comment by clicking on the buttons "Show preview" or "Save page", respectively.

Although all participants are allowed to edit the whole page, we kindly ask you not to do it and refrain from clicking on the "Edit" button. Please do follow the above steps.

Remember that due to safety reasons, the server is off-line every day between 4:00 a.m. and 4:20 a.m. in local time (CEST).

Comments

Proof of Theorem 12.12

Dear ISem Team, I think the definition of r_0 on p. 145 (and on p. 146 in the proof of Thm. 12.13) should be $r_0 := \sup\{|r(z)| :\ z \in -\overline{\Sigma}_\delta,\ |z| \geq h_0\}$.

By the way, how do you mirror the Σ in LaTeX? Greetings, ChristianSeifert 11:20, 23 January 2012 (CET)

Dear Christian,

absolutely! Thanks for pointing that out. To mirror essentially anything in LaTeX you can use \reflectbox contained in the package graphicx. This is, we admit, not the nicest way to produce a reflected symbol.

Cheers, IsemTeam 20:13, 23 January 2012 (CET)

comments

p.141, Remark 12.7: Should this be $|r(\infty)| = 1$? (See the 1-1-term in the Pade-tableau!) And can one understand why this holds, generally?

concerning Proposition 12.8: In the second line, one would want to have c>0.

p.142, line 1: ... on \Amgis_\alpha... (instead of -\Amgis). (Amgis baptised by M. Waurick.)

p.142, first sentence of Section 12.2: Let A be a **closed** linear ... (The resolvent set is only defined for closed operators.)

Misprints and style:

p.142, line 2: ... we conclude **that** ?? (I am not a native speaker, but I think that one can conclude contracts, agreements, but not equations.)

p.142, line 9 of Section 12.2: ... $a_2 z^2$ ...

p.143, line 3: Omit `are'. (It is somewhat unusual to have full sentences in the defining part of a set.)

p.143, line 10 ... having **all** zeros ...

p.143, line 5 from below: comma following 'However'.

Greetings, JurgenVoigt 17:17, 24 January 2012 (CET)

Some comments

p. 137, last displayed formula: Is there a special reason why you first write only 1 two times and then $\exp^{(p)}(0)$ (which is, of course, the same)?
Exa. 12.2: time step *h*_,_ we are led
Exa. 12.3: In the line above (12.4) the *k* should be bold.
Exa. 12.5: comma after the displayed formula
Prop. 12.8: The ε_0 from the second line never appears again. Probably you mean h_0.

Prop. 12.8, proof, p.142, first displayed line: The inequality $e^{-\frac{|z_n|}{n}} < |r(z_n)|$ is strict.
same proof: of a $c' > 0$ _as_ asserted
In the second line of the last displayed formula the factor *n* is missing.
In the middle term of this line the factor h_0 is missing in the exponent of the first e-function.

Prop. 12.11, proof: Delete the *f* in the second expression. Furthermore, the third expression should be $|(r(z))^n|$ after the sup. Additionally, the square at the end of the proof is twice.
Thm. 12.12, proof: Therefore_,_ with ... $C'' > 0$_,_ we ...
also there: Hence in _the_ case
Thm. 12.13, proof, middle: In the factor $|tm_k|^{p+1-\beta}$ the summand $-\beta$ in the exponent ist missing.
Greetings, JohannesEilinghoff 17:45, 24 January 2012 (CET)

A couple of typos

Dear ISEM Team, it seems there are a couple of minor typos on the p. 146:

- 'as before we choose h_0 s.t. $\sup\{|r(z)|, z \in -\Sigma_\delta, |z| \geq h_0\} < 1$ ' doesn't make sense since $|r(\infty)| = 1$ (one wants the theorem to be applicable to diagonal Pade approximants)
- the Cauchy-Schwarz inequality at the very end of the proof has to contain the term $\sup_{k \in N, m_k \neq 0} |r(hm_k)^n(-m_k)^{-\beta} - e^{tm_k}(-m_k)^{-\beta}|^2$ instead of what is written.

Maryna Kachanovska 19:08, 24 January 2012 (CET)

Dear Maryna, and dear ISemTeam,

some more comments on the proof of Theorem 12.13: The lines 3 and 4 of the proof should simply be discarded. And for the remaining proof one can simply take $h_0 = 1$.

On line 9 of the proof, the power *n* is missing at $r(hm_k)$..

And on line 10, *for all ... we obtain* should be discarded. (Alternatively, the $\sup$ on line 11 can be discarded.)

Greetings, JurgenVoigt 18:57, 25 January 2012 (CET)

more comments

Concerning the Pade tableau (p.139): According to my computations, at the place (l,k)=(2,1) the coefficient of z in the denominator should be $-\frac{2}{3}$. And at the place (l,k)=(3,1) the coefficient of z in the denominator should be $-\frac{3}{4}$. (In fact, the sum of the first order coefficient in the numerator and the negative first order coefficient in the denominator should always be 1.) Also, at the place (l,k)=(1,2), I think that the coeffitient of $\frac{z^2}{2!}$ should be $\frac{1}{3}$.

Incidentally, concerning the Radau II A method: The table given in Appendix B, Example B.1 6. yields the (corrected) formula at the place (l,k)=(2,1) in the Pade tableau (not the formula indicated on the bottom of p.138 (i.e., the formula at the place (l,k)=(3,2)).

Best wishes, JurgenVoigt 18:45, 25 January 2012 (CET)

Proposition 12.8, Theorem 12.12 and Theorem 12.13

Dear ISEM Team,

I have a question concerning Proposition 12.8:

By the assumption about the approximation order we only get

"There are constants $h_0 > 0$, $C' > 0$ s.t. $| r(z) - e^z | <= C' | z |^{p+1}$ for all $z \in \mathbb{C}$ with $|z| \leq h_0$"

instead of

"For all $\varepsilon_0 = h_0$ there is a constant $C' > 0$ s.t. $| r(z) - e^z | <= C' | z |^{p+1}$ for all $z \in \mathbb{C}$ with $|z| \leq h_0$".

So Proposition 12.8 just works for some small h_0.

If this is correct, I don't see how we can use this Proposition in the Proof of Theorem 12.12 and Theorem 12.13 with $h_0 > 0$ so large that $r_0 < 1$.

NilsKintscher 20:10, 28 January 2012 (CET)

Dear Nils,

as I understand, your problem is with the second line of the proof. Let $h_0 > 0$. By hypothesis, there exists $\delta > 0$ such that the inequality holds for all $|z| \leqslant \delta$. However, on the set $\{z \in \overline{\mathrm{Amgis}_\alpha}; \delta \leqslant |z| \leqslant h_0\}$ the functions r and e^z are bounded, and therefore, by increasing the constant C' suitably, one obtains the asserted bound for all $z \in \overline{\mathrm{Amgis}_\alpha}$ with $|z| \leqslant h_0$.

Best wishes, JurgenVoigt 09:38, 30 January 2012 (CET)