Talk:Lecture 1

From 15th ISeminar 2011/12 Operator Semigroups for Numerical Analysis

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- Type the title of your comment to the "Subject/headline" field.
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Comments

First comment

Lecture 1 is already uploaded.

PetraCsomos 16:49, 6 October 2011 (CEST)

Let us hope there will be lots of comments.AndrasBatkai 17:18, 6 October 2011 (CEST)

Thanks for a first lecture that struck me as easy to follow and well motivated. The only "error" I noticed on a first reading was that on the bottom of page 7, did you mean $\sup_{s\in\mathbb{R}}$ and not $\sup_{x\in\mathbb{R}}$? JamesKennedy 18:44, 6 October 2011 (CEST)

Dear James, thanks for pointing that out!

BalintFarkas 20:19, 6 October 2011 (CEST)

Mistake in the definition of the Dirichlet Laplace operator

I agree with James that this is a well-motivated lecture, as far as I've read up to now. Still, it seems to me that there is another mistake: On page 2, the domain D(A) is too large. It should contain only functions, whose second distributional derivative is in L^2. But as it is written down, it contains also functions whose derivative is locally given by the Cantor function. More precisely, take an antiderivative of the Cantor function and multiply by a cut-off function to realize Dirichlet boundary conditions. Then the resulting function is in D(A), but not in the domain of what usually is called the Dirichlet Laplace operator. A simple way to correct this is to assume in addition that $g'(t) = c + \int_0^t g''(s)ds$ for some constant c in the definition of D(A).

RobinNittka 20:27, 6 October 2011 (CEST)

Dear Robin, you are absolutely right, the domain as written now is indeed too large. That is a grave mistake, because with this domain the integration by parts in Proposition 1.1 fails. Of course, the definition you suggest works out. Thanks!

BalintFarkas 22:57, 6 October 2011 (CEST)

Dear virtual lecturers,

in consideration of the problem mentioned by Robin, maybe a little comment in the proof of Prop.1.1 would be helpful: Although the calculation is formally integration by parts (in the same way as for differentiable functions), the argument here has more to do with the characterization of absolute continuity noted by Robin above and Fubini's theorem.

Best regards, AlexanderUllmann 00:09, 7 October 2011 (CEST)

Sobolev space vs absolute continuity

Concerning the integration by parts formula used in Prop 1.1: It holds for H^1 functions, therefore can be used. Alternatively, for absolutely continuous functions the fundamental theorem of calculus holds, hence the proof of the integration by parts formula can be reproduced. To repeat what Robin and Alexander said, the absolute continuity (which can be expressed by $g'(t) = c + \int_0^s g''(s)ds$ for some constant c) of the derivative is needed. However, to see the situation in terms of Sobolev spaces (and with distributional derivatives) makes the whole discussion maybe easier; the functions are C^1 by Sobolev embedding anyway. Hope, this helps a bit everybody who followed the discussion.FelixSchwenninger 09:51, 10 October 2011 (CEST)

Thanks for the comments

We are pleased to see that a small exclamation mark(!) caused such a vigorous commenting which eventually turned into a discussion indeed.

IsemTeam 13:10, 10 October 2011 (CEST)

Why not $L^2[0,\pi]$?

As done in Page 2 of Lecture 1, usually, we take $L^2(0,\pi)$ instead of $L^2[0,\pi]$. However, on the other hand, each function in $D(A)$ is expected to take value at the endpoints. So, why not just take $L^2[0,\pi]$? JianHuaChen 11:53, 12 October 2011 (CEST)

Note that it is part of the definition of the L^p-spaces that functions are identified that coincide almost everywhere. Here, $\{0,\pi\}$ is a nullset of $[0,\pi]$ with respect to the Lebesgue measure and hence $L^2(0,\pi) = L^2[0,\pi]$ (up to subtleties). So you may safely take $L^2[0,\pi]$ if you feel more comfortable with that. An alternative answer could be that $D(A)$ (by convention) consists of those continuous functions on $(0,\pi)$ that can be continuously extended to the closure.

MathGod83 11:01, 13 October 2011 (CEST)

MG83, I find your answer is a bit dismissive. JHC, is raising (maybe a bit laterally) an important issue: "What is the meaning to $u(0)$,$u(\pi)$ in $L^2(0,\pi)$ appearing in (1.1+2)?" Understanding these "boundary values" (or traces if you will) is essential. In fact, how do we know that functions in $D(A)$ as defined here (see the discussion above) do have traces. The answer is either (a) you show that they are (absolutely) continuous (i.e., that each a.e.-equivalence class has a rep which is continuous), or (b) you assume that they are continuous. Since (b) seems to have been eschewed by the authors, I think (a) ought to be given (maybe as an exercise) with a good pointer to the literature (e.g., Avner Friedman's Foundation of Modern Analysis). OmarLakkis 06:58, 14 October 2011 (CEST)

?? on page 2

Taking into account the convergence assumption of the serie. For $f \in \; lin\{f_n : n \in \mathbb{N}\}$, I think we should write $f = \sum_{n=1}^{\infty} a_n f_n$, instead of $f = \sum_{n=1}^{N} a_n f_n$ the action of A is ...... AboubakariTraore 18:48, 12 October 2011 (CEST)

Dear Aboubakari Traore,

$lin\{f_n : n \in \mathbb{N}\}$ denotes the space of finite linear combinations of the elements f_n. It is introduced here, because for finite sums you don't have any problems with the interchange of summation an differentiation.

MoritzEgert 19:30, 12 October 2011 (CEST)

Eigenvalues of the Dirichlet/Neumann Laplacian

Dear virtual lecturers, I think there should be a remark about the fact that 0 is not an eigenvalue of the Dirichlet Laplacian (in contrast to the case of Neumann boundary conditions). For me this issue is not clear in the text - maybe I have missed something, although I have tried to read Section 1.1 as carefully as possible.AdrianViorel 09:22, 13 October 2011 (CEST)

Terminological question

I have got a historical/terminological question. The definition of semi-group as used in semigroup theory explicitly requires the existence of an identity element (if the evolution operators T(t) with concatenation are regarded as an algebraic structure). In abstract algebra however, such a structure is not only a semigroup, but even a monoid. While the terminology is not wrong, and the semigroups of this seminar are not just purely algebraic objects, I am still curious where is terminological situation comes from. I just guess they stem from two distinct terminological traditions. --MartinLicht 02:19, 14 October 2011 (CEST)

Dear Martin, in the classical books as Hille/Phillips, Yosida, Dunford/Schwarz, operator semigroups are indeed introduced as semigroups in the algebraic sense, and also topological one-parameter semigroups are semigroup-homomorphisms $((0,\infty),+) \to (L(X),\circ), t \mapsto T(t)$ (where the latter is endowed with the strong operator topology). Nevertheless, also weaker notions, as measurable semigroups are studied there (where you do not need any kind of continuity up to 0). Moreover, the algebraic difference between a monoid and a semigroup is in my view marginal, since every semigroup can be made to a monoid by simply adding one more element - indeed, this is also the way how it is done for one-parameter semigroups $((0,\infty),+) \to (L(X),\circ), t \mapsto T(t)$ by defining $T(0) := Id_X$. The more subtle question is of course if there is also any kind of continuity or at least measurability of this extended map. --AlexanderUllmann 09:08, 17 October 2011 (CEST)

A pathological example

The simplest case is $X = \mathbb{R}$. It is not difficult to show that then $T(t) = e^{ta}$ are the only strongly continous semigroups. But even here continuity is essential.

In fact, I was a bit surprised when I recently learned that without continuity there are other semigroups on $\mathbb{R}$. To see this, let H be a Hamel basis of $\mathbb{R}$. Then each t can be uniquely written as $t = \sum q_h h$ where h are from H and q_h are rationals which are 0 except for finitely many. Defining

$$T(t) = T(\sum q_h h) := \prod e^{q_h h a_h}$$

where a_h are choosen arbitrarily, we get a semigroup (which is in general nowhere continous). Andreas Geyer-Schulz 19:58, 17 October 2011 (CEST)

Let us mention that on $X = \mathbb{R}$ all Lebesgue measurable funcions $T : [0, \infty) \to \mathbb{R}$ having the semigroup property are of the form $T(t) = e^{ta}$ (this was independently proved by Banach and Sierpinski and published in 1920 in Fundamenta Mathematicae volume 1). In this sense, in the pathological example it is quite natural to use a Hamel basis.

IsemTeam 00:08, 18 October 2011 (CEST)

minor suggestion

Just a minor suggestion for better comprehensibility:

On the bottom of page 5, the letter t is first used to denote the boundary of an interval, but in the next line is used as an arbitrary $t \in \mathbb{R}$ to express that these functions are equal at every point of the real line. At first glance this might suggests that the functions are only equal on some point related to the boundary of the interval. Writing $\dot{u}(s) = v(s) = Au(s)$ instead would eliminate this opportunity for confusion.

FlorianMueller 14:54, 20 October 2011 (CEST)

Dear Florian,

you are probably right. Thank you.

IsemTeam 09:53, 31 October 2011 (CET)