Crank-Nicolson scheme for bounded semigroups

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In the lecture notes we have encountered the Crank-Nicolson scheme (or method) at several occasions. This scheme replaces the differential equation

$$\dot{x}(t) = Ax(t), \qquad t \geq 0, \quad x(0) = x_0 \tag{1}$$

by the difference equation

$$x_d(n+1) = (I + \frac{hA}{2})(I - \frac{hA}{2})^{-1} x_d(n), \qquad n \in \mathbb{N}, \quad x_d(0) = x_0. \tag{2}$$

In Theorem 13.12 it is shown if A generates a bounded analytic semigroup, then $\|A_d^n\|$ is uniformly bounded, where $A_d = (I + \frac{hA}{2})(I - \frac{hA}{2})^{-1}$.

In this project we want to investigate this property when A is just the infinitesimal generator of a bounded C_0-semigroup. Hence not necessarily analytic. It turns out that the estimate

$$\|A_d^n\| \leq M\sqrt{n}$$

is the best estimate possible for general Banach spaces, but for Hilbert spaces we can get uniform boundedness for several cases:

- A generates a contraction semigroup,
- A generates an analytic semigroup,
- A and A^{-1} generate a bounded semigroup.

The aim of this project is understand these results and to apply it to some p.d.e.'s. A possible extension is to look at the best estmates if A is a matrix. These estimates will depend on the size of the matrix.

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References

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