

## Perturbation theory of $C_0$ -semigroups (the Miyadera theorem)

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The objects of this project are the Miyadera perturbation theorem and applications. If  $T$  is a  $C_0$ -semigroups with generator  $A$ , and  $B$  is an operator then (along with suitable technical conditions) the condition that

$$\int_0^\alpha \|BT(t)x\| dt \leq \gamma \|x\|$$

for suitable  $\alpha > 0$ ,  $\gamma < 1$  and all  $x \in D(A)$  implies that  $A + B$  is a generator. One part of the project is to understand the proof of this theorem.

The application to ‘Schrödinger semigroups’ (alias heat equation with absorption) yields the relation between Miyadera perturbations and the ‘Kato class’ of potentials.

Another application of interest is the ‘substochastic perturbation’ of substochastic semigroups on  $L_1$ -spaces, a general version of Kolmogorov’s differential equations.

A third application could be the perturbation theory of delay equations, but I do not intend to include this topic in the project.

For the Miyadera perturbation theorem I refer to [4], [5], [7], [2; III.3.c], but I suggest to follow the presentation in [11; Section 3].

For the application to Schrödinger semigroups I refer to [8] (and possibly [10]).

For the application to substochastic semigroups on  $L_1$ -spaces I refer to [3], [9] (and to [6] for a generalisation).

A standard reference for the application to delay equations is [1].

The papers by Kato and Miyadera as well as my papers quoted below can be obtained under

<http://www.math.tu-dresden.de/~voigt/isem11/proj-mpt>.

## References

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