Some positivity preserving schemes for semilinear problems

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The aim of this project is to study the convergence and positivity properties of second-order exponential Runge-Kutta and Strang splitting methods applied to inhomogeneous and semilinear parabolic problems.

It is known that in many applications, such as e.g. population dynamics, mathematical finance, reaction kinetic..., the positivity is an important feature. Looking for numerical schemes that preserve positivity and study their convergence is not a trivial task. However, it was shown by Bolley and Crouzeix [1] that the order of an unconditionally positive Runge-Kutta method for an inhomogeneous linear parabolic problem can not exceed one.

As we will see in this project, the use of exponential integrators permit to preserve positivity and improve the convergence.

The project is divided into two parts. The first one deals with the second-order exponential Runge-Kutta method applied to the inhomogeneous Cauchy problem

$$u'(t) = Au(t) + f(t), \quad u(0) = u_0. \tag{1}$$

We propose to show that the second-order exponential Runge-Kutta method preserves positivity in (1).

In the second part we are interested in applying the Strang splitting to the semilinear problem

$$u'(t) = Au(t) + f(u(t)), \quad u(0) = u_0. \tag{2}$$

Here we study consistency and convergence of the Strang splitting applied to (2). Finally, we deduce that the Strang splitting preserves positivity.

Here A with domain $D(A)$ generates a positive C_0-semigroup on a Banach lattice X and f satisfies appropriate assumptions.

The first part is mainly contained in [4] and the second one in [2]. For the theory of positive semigroups we refer to [5] and for problems of type (1) and (2) we refer to [3].

References

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