Geometric theory of semilinear problems

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This project is concerned with the geometric theory of semilinear parabolic equations

$$u'(t) = Au(t) + g(u(t)) \tag{1}$$

and their numerical discretisations. Geometric theory is concerned with the qualitative behaviour of solutions, the geometry of the flow and stability questions. A good introduction into this field is the book by Dan Henry [2]. The simplest objects to be studied are asymptotically stable equilibria of (1). Such a study was partially carried out in our last ISEM lecture.

A hyperbolic equilibrium (saddle-point) is more involved as it possesses in its neighbourhood a stable and an unstable invariant manifold which generalise the concepts of stable and unstable subspaces for the linear problem. Numerical discretisations posses discrete counterparts thereof. The largest part of the project is concerned with the construction of these invariant sets.

Possible extensions cover periodic orbits (see [1] and [4]), and Hopf bifurcations (see [3]). The latter require the construction of an invariant centre manifold in which the bifurcation takes place.

References

[1] F. Alouges and A. Debussche: On the discretization of a partial differential equation in the neighborhood of a periodic orbit. Numer. Math. 65 (1993) 143-175

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[3] Ch. Lubich, A. Ostermann: Hopf bifurcation of reaction-diffusion and Navier-Stokes equations under discretization. Numer. Math. 81 (1998) 53-84

[4] Ch. Lubich, A. Ostermann: Runge-Kutta time discretization of reaction-diffusion and Navier-Stokes equations: Nonsmooth-data error estimates and applications to long-time behaviour. Applied Numerical Math. 22 (1996) 279-292