THE SEMIGROUP APPROACH TO STOCHASTIC PARTIAL DIFFERENTIAL EQUATIONS DRIVEN BY NOISE

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The stochastic wave equation driven by additive noise,

$$\begin{aligned} &\mathrm{d}\dot{u} - \Delta u \,\mathrm{d}t = f(u)\,\mathrm{d}t + \mathrm{d}W && \text{in } \mathcal{D}\times(0,\infty),\\ &u = 0 && \text{in } \partial\mathcal{D}\times(0,\infty),\\ &u(\cdot,0) = u_0, \quad \dot{u}(\cdot,0) = v_0 && \text{in } \mathcal{D}, \end{aligned}$$

can be given a rigorous formulation

$$X(t) = \mathrm{e}^{-tA}X_0 + \int_0^t \mathrm{e}^{-(t-s)A} B f(X_1(s))\,\mathrm{d}s + \int_0^t \mathrm{e}^{-(t-s)A} B\,\mathrm{d}W(s), \tag{1}$$

where

$$A = \begin{bmatrix} 0 & -I \\ \Lambda & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ I \end{bmatrix}, \quad X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} u \\ \dot{u} \end{bmatrix}, \quad X_0 = \begin{bmatrix} X_{0,1} \\ X_{0,2} \end{bmatrix} = \begin{bmatrix} u_0 \\ v_0 \end{bmatrix}.$$

The article [2] provides a so-called weak convergence analysis for finite element approximations of linear equations of this kind ($f = 0$).

The aim of the project is to extend the analysis in [2] for the linear wave equation to the semilinear equation (1). Such analysis was done earlier for the semilinear Schrödinger equation in [1] and it uses the fact that the operator family $\{\mathrm{e}^{-tA}\}$ is a group in order to re-write the equation to a form which is easier to analyze.

References

[1] A. de Bouard and A. Debussche, *Weak and strong order of convergence of a semidiscrete scheme for the stochastic nonlinear Schrödinger equation*, Appl. Math. Optim. **54** (2006), 369–399.

[2] M. Kovács, S. Larsson, and F. Lindgren, *Weak convergence of finite element approximations of linear stochastic evolution equations with additive noise*, BIT Numer. Math. (2011). [doi:10.1007/s10543-011-0344-2]

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