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Project: Exponential quadrature

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The aim of this project is to study the numerical approximation to solutions of linear abstract differential equations

$$u'(t) + Au(t) = f(t), \qquad u(t_0+) = u_0$$

on a Banach space X by exponential quadrature formulas.

To define such quadrature formulas we choose non-confluent collocation nodes $c_1, \dots, c_s$ and define approximations $u_n \approx u(t_n)$, where $t_n = t_0 + nh$, $n = 0, 1, \dots$ via

$$u_{n+1} = \mathrm{e}^{-hA} u_n + h \sum_{i=1}^{s} b_i(-hA) f(t_n + c_i h)$$

with weights

$$b_i(-hA) = \frac{1}{h} \int_0^h \mathrm{e}^{-(h-\tau)A} \ell_i(\tau)\ d\tau.$$

Here, ℓ_j is the Lagrange interpolation polynomial

$$\ell_j(\tau) = \prod_{m \neq j} \frac{\tau/h - c_m}{c_j - c_m}.$$

The project involves

- construction of exponential quadrature formulas
- convergence analysis in different Banach spaces (e.g. in L^p) and with different boundary conditions
- numerical experiments (using Matlab or any other programming language)

Reference:

M. Hochbruck, A. Ostermann: Exponential Runge-Kutta methods for parabolic problems, Appl. Numer. Math., vol. 53, no. 2-4, pp. 323-339 (2005)