THE STABILITY AND CONVERGENCE RESULTS OF BRENNER AND THOMÉE

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This project comes back to the promise of the virtual lecturers from Section 14.1 in the Isem lecture notes, that we will hear more on the stability and convergence theorems of Brenner and Thomée in the project phase. It invites you to dwelve into the proof of these two theorems.

The proof of both theorems is mainly based on the Hille-Phillips functional calculus. We saw in the course that functional calculi are a powerful tool to get stability and convergence results for rational approximation schemes and this was illustrated using the Dunford calculus for analytic semigroups in Section 13, which is a very natural choice. However, there are many interesting semigroups that are not analytic, such as shift semigroups or all semigroups that are actually groups.

That is where the Hille-Phillips calculus comes in. As was already pointed out in Section 14.1, the basic idea is to write a holomorphic function F as the Laplace transform of a bounded Borel measure μ on $[0,\infty)$, i.e.,

$$F(z) = \int_0^\infty \mathrm{e}^{sz}\mathrm{d}\mu(s) \quad (\mathrm{Re} z \leq 0)$$

and then to substitute the semigroup e^{sA} for the term e^{sz}

$$F(A) = \int_0^\infty \mathrm{e}^{sA}\mathrm{d}\mu(s)$$

and to hope for the best, i.e. that the resulting integral makes sense and that this procedure gives rise to a useful functional calculus.

So, for the project there will be two main tasks: First, to build up and explain the Hille-Phillips functional calculus and then to understand and present the proofs of Theorems 14.1 and 14.2 of the lecture notes.

The project will be based on the original article of Brenner and Thomée [1] for their results and on the book of Hille and Phillips [2], as well as the PhD thesis of Mihály Kovács [3] concerning the Hille-Phillips calculus.

References

[1] P. Brenner, V. Thomée: On rational approximations of semigroups, *SIAM Journal on Numerical Analysis* **16** (1979), no. 4, 683–694.

[2] E. Hille, R.S. Phillips: *Functional analysis and semigroups*, Colloquium Publications, American Mathematical Society (AMS), 1957.

[3] M. Kovács: *On qualitative properties and convergence of time-discretization methods for semigroups*, PhD thesis, Louisiana State University, 2004.