12th Austrian Numerical Analysis Day

April 28 – 29, 2016

Innsbruck, Austria

Welcome

We wish you a warm welcome to Innsbruck, and we are looking forward to an interesting *12th Austrian Numerical Analysis Day.*

The goal of this workshop is to inform about present/recent and future/planned research activities in the fields of numerical analysis and applied mathematics. Scientists from different groups at Austrian Universities and other Research Institutions are invited to present their research problems and discuss their results. Apart from strengthening already well established contacts this annual workshop should also provide an opportunity to start new collaborations.

The workshop will take place at the *Senatssitzungssaal, Innrain 52/1, Room 1050* in the main building of the University of Innsbruck.

We wish you a scientifically inspiring and enjoyable time in Innsbruck. If you have any questions, please do not hesitate to contact us.

Alexander Ostermann and Peter Kandolf

Scientific Program

Contributions

All communications will be given in plenary sessions. Each contribution is scheduled for 20 minutes including a brief discussion.

The conference language is English.

Equipment

The seminary room is equipped with a data projector. A Windows computer with Adobe Acrobat Reader will be provided as well as a presenter. Talks can be transferred to the conference computer through USB sticks or CD/DVD. **We highly recommend this option to keep the change of speakers as smooth as possible and provide everybody with the full 20 minutes.**

Conference Dinner

The conference dinner will be held on Thursday at 19:15 at the *Gasthof Weisses Rössl*[1] in the historic centre of town.

[1]Kiebachgasse 8, 6020 Innsbruck

Schedule

	Thursday, 28 April 2016
14.00 – 14.10	**Opening**
14.10 – 14.30	Lukas Einkemmer Overcoming Order Reduction in Diffusion-Reaction Splitting
14.30 – 14.50	Robert Altmann Constrained PDEs and their Formulation as Operator DAEs
14.50 – 15.10	Lothar Banz Regularization and *hp*-BEM for hemivariational inequalities modeling delamination problems
15.10 – 15.30	Matthias Hochsteger A high-level Python interface for Netgen/NGSolve
15.30 – 15.50	Alexander Rieder Convolution quadrature for the wave equation with a nonlinear boundary condition
15.50 – 16.20	**Coffee break**
16.20 – 16.40	Bernhard Stiftner A generalized midpoint scheme for the Landau-Lifshitz-Gilbert equation in computational micromagnetics
16.40 – 17.00	Othmar Koch Time-splitting FEM discretization of the Schrödinger–Poisson equation
17.00 – 17.20	Dante Kalise High-order, high-dimensional schemes for Hamilton-Jacobi-Bellman equations
17.20 – 17.30	**Break**
17.30 – 17.50	Michael Neunteufel Tangential-rotation and normal-normal-momentum continuous mixed finite elements for non-linear shell models
17.50 – 18.10	Katharina Rafetseder A new mixed approach for discretizing Kirchhoff-Love plates
18.10 – 18.30	David Cohen Numerical discretisations of stochastic wave equations by trigonometric integrators
19.15	**Dinner at the Gasthof Weisses Rössl**

	Friday, 29 April 2016
08.30 – 08.50	Ulrich Langer Space-time isogeometric analysis of parabolic initial boundary value problems
08.50 – 09.10	Gerhard Unger Numerical analysis of boundary element methods for Maxwell's eigenvalue problems

09.10 – 09.30	Stefan Takacs Robust approximation error and inverse estimates for B-splines and applications to isogeometric analysis
09.30 – 09.50	Peter Gangl A Locally Modified Finite Element Method for Interface Problems in Shape and Topology Optimization
09.50 – 10.10	Philip Lederer An exact divergence-free reconstruction operator for the Taylor-Hood element
10.10 – 10.40	**Coffee break**
10.40 – 11.00	Francesca Bonizzoni Padé approximation for the parametric Helmholtz equation
11.00 – 11.20	Christoph Hofer Continuous and discontinuous Galerkin Dual-Primal Isogeometric Tearing and Interconnecting methods for multipatch IgA equations
11.20 – 11.40	Scott Congreve *hp*-Version Trefftz Discontinuous Galerkin Method for the Homogeneous Helmholtz Equation
11.40 – 12.00	Jarle Sogn Robust preconditioners for optimality systems using Isogeometric Analysis
12.00 – 13.10	**Lunch**
13.10 – 13.30	Thomas Wick Numerical Methods for p-Laplace type problems
13.30 – 13.50	Duy Phan Riccati based feddback stabilization to trajectories for parabolic equations
13.50 – 14.10	Markus Wess A frequency dependent complex scaling for Helmholtz resonance problems
14.10 – 14.30	Christoph Wintersteiger Mapped Tent Pitching Method for Hyperbolic Conservation Laws
14.30 – 15.00	**Coffee break**
15.00 – 15.20	Gregor Milicic Fixed point schemes for systems of parabolic reaction-diffusion equations
15.20 – 15.40	Lena-Maria Pfurtscheller A Comparison of Stochastic Models for El Niño
15.40 – 16.00	Tobias Hell Spatial modeling of meteorological extremes
16.00 – 16.05	**Closing**

Abstracts of Talks

In alphabetical order

Constrained PDEs and their Formulation as Operator DAEs

Robert Altmann (University of Innsbruck, Austria)

Constrained PDEs appear in several applications such as in fluid dynamics (constrained by the incompressibility condition), flexible multibody dynamics (constrained through boundary conditions), or coupled systems in general. With the help of suitable Sobolev-Bochner spaces we may formulate these systems as DAEs in an abstract setting, also called abstract or operator DAEs. This then corresponds to the weak formulation of the underlying PDE systems. Within this talk, we discuss the well-posedness of initial conditions as well as their consistency.

Regularization and hp-BEM for hemivariational inequalities modeling delamination problems

Nina Ovcharova (Universität der Bundeswehr München)
Lothar Banz * (University of Salzburg, Austria)

In [1] we couple regularization techniques of nondifferentiable optimization with the hp-adaptive version of the boundary element method (hp-BEM) for the efficient numerical solution of linear elastic problems with nonmonotone contact boundary conditions. As a model example we treat the delamination of composite structures with a contaminated interface layer. The variational formulation of this problem leads to a hemivariational inequality with a nonsmooth functional defined on the contact boundary. This problem is first regularized and then, discretized by an hp-BEM. We give conditions for the uniqueness of the solution, prove convergence of the BEM Galerkin solution of the regularized problem in the energy norm, and obtain an a-priori error estimate for the regularized problem based on a novel Céa-Falk approximation lemma. Furthermore, we derive an a-posteriori error estimate based on an equivalent regularized mixed formulation thus enabling hp-adaptivity. Numerical experiments illustrate the behavior, strengths and weaknesses of the proposed approximation scheme.

References

[1] Ovcharova, N., Banz, L., 2016. Coupling regularization and adaptive hp-BEM for the solution of a hemivariational inequality modeling a delamination problem. preprint.

Padé approximation for the parametric Helmholtz equation

Francesca Bonizzoni * (Faculty of Mathematics, University of Vienna, Austria)
Fabio Nobile (MATHICSE - CSQI, Ecole Polytechnique Fédérale de Lausanne)
Ilaria Perugia (Faculty of Mathematics, University of Vienna)

Let $D \subset \mathbb{R}^d$ $(d = 1, 2, 3)$, and let $K := [k^2_{min}, k^2_{max}] \subset \mathbb{R}^+$ be the interval of frequencies we are interested in. We consider the parametric Helmholtz problem

$$-\Delta u - k^2 u = f \quad \text{in } D, \quad \text{with } k^2 \in K, \tag{1}$$

endowed with either Dirichlet or Neumann homogeneous boundary conditions on ∂D. Since the solution u is a function of the space variable $\mathbf{x} \in D$, as well as of the wavenumber k^2, we can introduce the *solution map* $\mathcal{S} : K \to V$ defined as $\mathcal{S}(k^2) = u(k^2, \cdot)$, where $V = H^1(D)$ or $V = H^1_0(D)$ depending on the considered boundary conditions.
Since the numerical evaluation of $\mathcal{S}(k^2)$ for all $k^2 \in K$ is computationally too expensive, we propose to approximate $\mathcal{S}$ on K starting from evaluations of $\mathcal{S}$ and its derivatives only at few frequencies. For a review on interpolatory model order reduction methods we refer e.g. to [1].
We extend the solution map to the complex plane $\mathbb{C}$, and we prove that the Helmholtz problem with complex wavenumber $z \in \mathbb{C}$ is well-posed provided that $z \notin \{\lambda_i\}$, $\{\lambda_i\}$ being the set of eigenvalues of the Laplacian with the considered boundary conditions. When $z = \lambda_i$, the solution map $\mathcal{S}$ presents a singularity (pole of order one). Namely, the solution map $\mathcal{S}$ is a meromorphic map. See [2].
We then approximate $\mathcal{S}$ with a *rational (Padé) approximation technique*, able to catch the poles of $\mathcal{S}$. The Padé approximant of $\mathcal{S}$, denoted as $\mathcal{S}_{[M/N]}$, is given by the ratio $\mathcal{S}_{[M/N]}(z) = \frac{\mathcal{P}_{[M/N]}(z)}{\mathcal{Q}_{[M/N]}(z)}$, where $\mathcal{Q}_{[M/N]} \in \mathbb{P}_N(\mathbb{C})$ is a polynomial of degree at most N, and $\mathcal{P}_{[M/N]} \in \mathbb{P}_M(\mathbb{C}; V)$, with $\mathbb{P}_M(\mathbb{C}; V) = \left\{ P(z) = \sum_{m=0}^{M} p_m(z) z^m, p_m \in V \right\}$.
We generalize the *Least Square Padé definition* presented in [3] for complex-valued multivariate functions, to Hilbert space-valued univariate maps. In particular, the Padé approximant $\mathcal{S}_{[M/N]}$ is computed by solving a minimization problem. Moreover, we prove a convergence theorem for the approximation error $\|\mathcal{S}(z) - \mathcal{S}_{[M/N]}(z)\|_V$, and we verify our a priori error upper bound with 2D numerical results. See [2].

References

[1] Hetmaniuk, U., Tezaur, R., Farhat, C., 2012. Review and assessment of interpolatory model order reduction methods for frequency response structural dynamics and acoustics problems. International Journal for Numerical Methods in Engineering 90 (13), 1636–1662.

[2] Bonizzoni, F., Nobile, F., Perugia, I. Convergence analysis of Padé approximations for Helmholtz frequency response problems. In preparation.

[3] Guillaume, P., Huard, A., Robin, V., 1998. Generalized Multivariate Padé Approximants. Journal of Approximation Theory 95 (2), 203 – 214.

Numerical discretisations of stochastic wave equations by trigonometric integrators

David Cohen * (University of Innsbruck and Umeå University, Austria and Sweden)
Stig Larsson (Chalmers/University of Gothenburg)
Magdalena Sigg (University of Basel)

Stochastic wave equations can be used as mathematical models to describe, for instance, the motion of a strand of DNA floating in a fluid or the motion of shock waves on the surface of the sun [2].

Analytic solutions to these stochastic partial differential equations (SPDEs) is, in general, not available: we must resort to numerical simulations in order to understand such problems!

Inspired by the construction of the trigonometric schemes for highly oscillatory deterministic problems and deterministic wave equations, see [1, Chapter XIII] and references therein, we propose a new numerical method for the time discretisation of linear stochastic wave equations [3].

We begin the presentation with a concise crash course on SPDEs. A fully discrete approximation of the linear stochastic wave equation driven by additive noise is then presented. A standard finite element method is used for the spatial discretisation and a stochastic trigonometric scheme for the temporal approximation. This explicit time integrator allows for error bounds independent of the space discretisation and thus does not have a step size restriction as in the often used Störmer-Verlet-leap-frog scheme. Furthermore, it enjoys a trace formula as does the exact solution of our problem. These favourable properties are next demonstrated with numerical experiments. Finally, we comment on recent results on the numerical analysis of semilinear stochastic wave equations driven by multiplicative noise [4, 5].

References

[1] E. Hairer, C. Lubich and G. Wanner, *Geometric Numerical Integration*, Springer Series in Computational Mathematics, 2006 (**31**).

[2] R.C. Dalang, *The stochastic wave equation*, A minicourse on stochastic partial differential equations, 2009 (**1962**), 39–71.

[3] D. Cohen, S. Larsson and M. Sigg, *A trigonometric method for the linear stochastic wave equation*, SIAM J. Numer. Anal., 2013 (**51(1)**), 204–222.

[4] D. Cohen and L. Quer-Sardanyons, *A fully discrete approximation of the one-dimensional stochastic wave equation*, IMA J NUMER ANAL, 2016 (**36(1)**), 400–420.

[5] R. Anton, D. Cohen, S. Larsson and X. Wang, *Full discretization of semilinear stochastic wave equations driven by multiplicative noise*, accepted for publication in SIAM J. Numer. Anal., 04.02.2016.

hp-Version Trefftz Discontinuous Galerkin Method for the Homogeneous Helmholtz Equation

Scott Congreve * (University of Vienna, Austria)
Paul Houston (University of Nottingham, UK)
Ilaria Perugia (University of Vienna)

We consider a Trefftz discontinuous Galerkin finite element (TDG) approximation of the solution to the homogeneous Helmholtz equation $-\nabla u - k^2 u = 0$. The TDG method uses (local) solutions to the Helmholtz equation as basis functions (such as the plane waves $\mathrm{e}^{ikd_l \cdot (x - x_K)}$, where d_l, $l = 1, \ldots, p_K$, are distinct propagation directions), rather than polynomial basis functions.

The general TDG formulation includes three different flux parameters, $\alpha > 0$, $\beta > 0$ and $0 < \delta < \frac{1}{2}$. Different functions for these flux parameters (constant and mesh-dependent functions) have been proposed in the literature. We study the effects of various selections of these parameters on the error in both mesh-dependent and L^2-norms, for both uniform and non-uniform meshes, with varying h (mesh size) and p (effective polynomial degree). We also compare the numerical results to a (polynomial) discontinuous Galerkin approximation, where the polynomial degree is the same as the effective polynomial order of the TDG approximation. Furthermore, we consider adaptive mesh refinement and adaptivity of the propagation directions of the plane waves basis functions based on *a posteriori* error analysis.

Overcoming Order Reduction in Diffusion-Reaction Splitting

Lukas Einkemmer * (University of Innsbruck, Austria)
Alexander Ostermann (University of Innsbruck)

For diffusion-reaction equations employing a splitting procedure is attractive as it reduces the computational demand and facilitates a parallel implementation. Moreover, it opens up the possibility to construct second-order integrators that preserve positivity independent of the time step size used. However, for boundary conditions that are neither periodic nor of homogeneous Dirichlet type order reduction limits its usefulness. In the situation described the Strang splitting procedure is no more accurate than Lie splitting.

In this talk, we introduce a modified Lie/Strang splitting procedure that, while retaining all the favorable properties of the original method, does not suffer from order reduction. That is, the modified Strang splitting is second order accurate in time. Furthermore, we will show how to efficiently compute the required correction term.

We demonstrate our results by presenting numerical simulations in one and two space dimensions with Dirichlet, Neumann, and mixed boundary conditions. In addition, a mathematically rigorous convergence analysis that explains the results observed in the numerical simulations is presented.

This talk is based on [1] and [2].

References

[1] L. Einkemmer, A. Ostermann, 2015, Overcoming order reduction in diffusion-reaction splitting. Part 1: Dirichlet boundary conditions. SIAM J. Sci. Comput., 37 (3), A1577–A1592.

[2] L. Einkemmer, A. Ostermann, 2016, Overcoming order reduction in diffusion-reaction splitting. Part 2: oblique boundary conditions. arXiv preprint, arXiv:1601.02288.

A Locally Modified Finite Element Method for Interface Problems in Shape and Topology Optimization

Peter Gangl * (Doctoral Program "Computational Mathematics", JKU Linz, Austria)
Ulrich Langer (Institue of Computational Mathematics, JKU Linz)

We consider the design optimization of electrical machines by means of sensitivity-based shape and topology optimization. The shape derivative of a domain-dependent functional provides information about its sensitivity with respect to a change of the shape of the underlying domain. On the other hand, the topological derivative gives information about the sensitivity of the functional with respect to the introduction of a hole or of an inclusion of a different material. We present numerical algorithms for the design optimization of an electric motor which rely on the shape derivative and the topological derivative.

In every iteration of the optimization procedure, the interface between different subdomains is updated. On the updated geometry, which is in general not resolved by the finite element discretization, the state and adjoint equations have to be solved. We present a numerical method that allows us to resolve the interface in every iteration exactly by only locally modifying the underlying triangular mesh. Moreover, the chosen mesh adaptation strategy ensures a maximum angle condition which yields optimal order of convergence independent of the location of the interface relative to the mesh.

Spatial modeling of meteorological extremes

Tobias Hell * (University of Innsbruck, Austria)
Harald Schellander (Central Institute of Meteorology and Geodynamics (ZAMG), Austria)

The maximum of consecutive measurements of meteorological parameters such as the yearly maximum of snow depth, of the amount of precipitation within an hour or of wind gust can by modeled by a *generalized extreme value* (GEV) distribution. The GEV distribution is determined by three parameters: The location $\mu \in \mathbb{R}$, the scale $\sigma > 0$ and the shape $\xi \in \mathbb{R}$.
Measured time series are only available at measurement stations and so are point estimates for the GEV parameters. Subsequent interpolation onto a grid may lead to poor results. Therefore, combining the estimation at the stations and the interpolation process to one act is preferable which we refer to as spatial modeling.
When spatially modeling meteorological extremes, at least two properties are desirable: Coherent spatial dependence between climate regions as well as reasonable estimates for so called return levels. Applied to Austrian snow depth data, this can be achieved by the approach of smooth spatial modeling with extremal coefficients presented in this talk.

A high-level Python interface for Netgen/NGSolve

Matthias Hochsteger * (TU Wien, Austria)
Joachim Schöberl (TU Wien, Austria)

We introduce the new Python interface for the Finite Element software Netgen/NGSolve. The software itself is written in modern C++, which is very efficient but time consuming for new users to get accustomed with. By providing an interface to a highly abstracted language, it's much easier to get started with NGSolve. In this talk we present the interface as well as techniques used to retain computational efficiency while providing a high-level abstraction layer.

Continuous and discontinuous Galerkin Dual-Primal Isogeometric Tearing and Interconnecting methods for multipatch IgA equations

Christoph Hofer * (Johannes Kepler University Linz, Austria)
Ulrich Langer (Johannes Kepler University Linz)

In this talk, we construct and investigate fast solvers for large-scale linear systems of algebraic equations arising from isogeometric analysis (IgA) of diffusion problems with heterogeneous diffusion coefficient on multipatch domains. In particular, we investigate the adaption of the Dual-Primal Finite Element Tearing and Interconnecting (FETI-DP) method to IgA, called Dual-Primal IsogEometric Tearing and Interconnecting (IETI-DP) method. The use of open knot vectors is very crucial since in this case we can still distinguish between basis functions corresponding to the boundary and to the interior of the patches (subdomains). We consider the cases where we have matching and non-matching meshes on the interfaces. In the latter case we use a discontinuous Galerkin (dG) method to couple the different patches. This requires a special extension of the IETI-DP method to the dG-IgA formulation. We use ideas from the finite element case in order to formulate the corresponding IETI-DP method, called dG-IETI-DP. We design the dG-IETI-DP method in such a way that it can be seen as a IETI-DP method on an extended discrete interface space. These methods are highly suited for parallelization. We present numerical results for complicated two and three dimensional domains. We observe a quasi-optimal behavior of the condition number κ of the preconditioned system with respect to the mesh-size h and the patch-size H. More precisely, this condition number κ behaves like $O((1+\log(H/h))^2)$, and is robust with respect to jumping diffusion coefficients and changing mesh-sizes across patch interfaces.

High-order, high-dimensional schemes for Hamilton-Jacobi-Bellman equations

Dante Kalise (Johann Radon Institute for Computational and Applied Mathematics, Austria)

Feedback control design plays a fundamental role in modern engineering. For an optimality-based formulation of the control problem, the Dynamic Programming Principle allows the characterization of the associated value function as the viscosity solution of a first-order, fully nonlinear Hamilton-Jacobi-Bellman equation. The equation is defined over the state-space of the cont rolled dynamical system and therefore, even control problems over low-dimensional dynamics lead to HJB equations of high complexity. In this talk, we present an approximation framework to compute (sub)optimal feedback controllers based on the solution of a Generalized HJB equation and a policy iteration algorithm. Problems arising from the feedback control of partial differential equations illustrate the effectiveness of our approach in a high-dimensional context.

References

[1] D. Kalise and K. Kunisch. *An spectral element method for Hamilton-Jacobi-Bellman equations*, in preparation.

[2] D. Kalise, A. Kröner and K. Kunisch. *Local minimization algorithms for dynamic programming equations*, to appear in SIAM Journal on Scientific Computing.

[3] O. Bokanowski, M. Falcone, R. Ferretti, L. Grüne, D. Kalise and H. Zidani. *Value iteration convergence of ϵ-monotone schemes for stationary Hamilton-Jacobi equations*, Discrete and Continuous Dynamical Systems - Series A **35**(9)(2015), 4041–4070.

[4] A. Alla, M. Falcone and D. Kalise. *An efficient policy iteration algorithm for the solution of dynamic programming equations*, SIAM Journal on Scientific Computing **37**(1)(2015), 181–200.

Time-splitting FEM discretization of the Schrödinger–Poisson equation

Winfried Auzinger (Technische Universität Wien, Austria)
Thomas Kassebacher (Universität Innsbruck, Austria)
Othmar Koch * (Universität Wien, Austria)
Mechthild Thalhammer (Universität Innsbruck, Austria)

We analyze the convergence of time-splitting methods for the Schrödinger–Poisson equation

$$\mathrm{i}\,\partial_t\,\psi(x,t) = -\tfrac{1}{2}\,\Delta\psi(x,t) + \Delta^{-1}(|\psi(x,t)|^2)\,\psi(x,t)\,, \quad (x,t) \in \Omega \times [0,T]\,,$$

on a truncated finite domain Ω with an underlying space discretization by conforming piecewise polynomial finite elements, where we impose homogeneous Dirichlet boundary conditions. The motivation for this approach is the possibility to treat the Poisson equation separately by dedicated solvers for the arising linear equations. The classical convergence orders in both the time and space discretization are established theoretically under natural assumptions on the regularity of the exact solution and illustrated by numerical experiments. Adaptive time-stepping relying on a defect-based error estimator is shown to reflect the solution behaviour.

References

[1] W. Auzinger, Th. Kassebacher, O. Koch, and M. Thalhammer. Convergence of a Strang splitting finite element discretization for the Schrödinger–Poisson equation. Submitted.

Space-time isogeometric analysis of parabolic initial boundary value problems

Ulrich Langer * (RICAM, Austria)
Stephen E. Moore (RICAM)
Martin Neumüller (Johannes Kepler University Linz)

We present and analyze a new stable space-time Isogeometric Analysis (IgA) method for the numerical solution of parabolic initial-boundary value problems in fixed and moving spatial computational domains. The discrete bilinear form is elliptic on the IgA space with respect to a discrete energy norm. This property together with a corresponding boundedness property, consistency and approximation results for the IgA spaces yields an a priori discretization error estimate with respect to the discrete norm. The theoretical results are confirmed by several numerical experiments with low- and high-order IgA spaces. Instead of solving a (space) system of linear algebraic equations at each time-step of an implicit time-stepping method, we solve one huge linear (space-time) system of linear algebraic equations. At the first glance, this seems to be a disadvantage. However, this disadvantage turns into a big advantage if parallel computers with many thousands of cores are used since now a complete parallelization in space and time is possible, and the curse of sequentiality of time can be overcome as our numerical studies presented in [1] show.
The authors would like to express their thanks to the Austrian Science Fund (FWF) for supporting our research project S117-03 within the National Research Network (NFN) S117 on "Geometry + Simulation".

References

[1] Langer, U., Moore, S., Neumüller, M., 2015. Space-time isogeometric analysis of parabolic evolution equations. RICAM Report 2015-19, also available at http://arxiv.org/abs/1509.02008.

An exact divergence-free reconstruction operator for the Taylor-Hood element

Philip Lederer * (TU Wien, Austria)
Alexander Linke (Weierstrass Institute for Applied Analysis and Stochastics)
Christian Merdon (Weierstrass Institute for Applied Analysis and Stochastics)
Joachim Schöberl (TU Wien)

In this talk we focus on a well-known issue of discretization techniques for the incompressible Navier Stokes equations. The numerical solution is only discrete divergence-free, which may have a major impact on quantitative and qualitative properties of the solution.
In recent years Alexander Linke and cooperators (see [1] and [2]) developed a methodology to reconstruct exactly divergence-free solutions from discrete divergence-free ones, and use this operator within the Navier Stokes solver.
In this work we extend this approach from discontinuous pressure elements to continuous pressure elements including the popular Taylor-Hood element. While for discontinuous pressures the reconstruction operator is given by element-wise local procedures, we have to extend the construction to vertex or element patches. The reconstruction leads to non conforming methods, where the consistency error is estimated in dual norms. Convergence of optimal order is proven.
The method is implemented in NGS-Py which is based on the finite element library Netgen/NGSolve. Several examples are presented.

References

[1] A. Linke, 2013 On the role of the Helmholtz decomposition in mixed methods for incompressible flows and a new variational crime. Computer Methods in Applied Mechanics and Engineering, Vol. 268, 782-800

[2] C. Brennecke, A. Linke, C. Merdon, J. Schöberl, 2015 Optimal and pressure-independent L2 velocity error estimates for a modified Crouzeix-Raviart Stokes element with BDM reconstructions. Journal of Computational Mathematics, Vol. 33(2), 191-208

[3] P. Lederer, 2016 Pressure Robust Discretizations for Navier Stokes Equations: Divergence-free Reconstruction for Taylor-Hood Elements and High Order Hybrid Discontinuous Galerkin Methods. Institute of Analysis and Scientific Computing, TU Wien.

Fixed point schemes for systems of parabolic reaction-diffusion equations

Gregor Milicic * (University of Salzburg, Austria)
Andreas Schröder (University of Salzburg)

In this talk, we discuss time stepping schemes for fully coupled systems of parabolic reaction-diffusion equations. The schemes are based on IMEX and, specifically, fixed point approaches which allow for the decoupling of the equations and, thus, for a significant reduction of the computational effort in each iteration step. Several numerical experiments confirm the applicability of the proposed techniques. In particular, we study an appropriate relaxation which leads to convergence of the fixed point schemes even for stiff problems where the IMEX schemes fail. The methods are applied to reaction-diffusion equations in biophysics, which model the signaling process of a yeast cell involving various, non-linearly dependent species.

Tangential-rotation and normal-normal-momentum continuous mixed finite elements for non-linear shell models

Michael Neunteufel * (Vienna University of Technology, Vienna, Austria)
Joachim Schöberl (Vienna University of Technology)

Many engineering applications involve thin elastic structures, which are typically treated by models of reduced dimensions, so called shell models.
We sketch the derivation of a 5-parameter shell model from geometric nonlinear 3D elasticity, and discuss proper continuity conditions of the involved fields: displacements shall be continuous, rotations tangential continuous, and bending moments normal-normal continuous.
We present a new finite element method, which is based on two existing formulations: The Hellan-Herrmann-Johnson method for Kirchhoff plates, and the tangential-displacement normal-normal-stress method for solid mechanics. Both are mixed methods using normal-normal continuous tensor-valued elements, and standard H1-elements or Nedelec elements, respectively.
We obtain a discrete constrained energy minimization problem, which is solved by Newton's method with line-search. We discuss the implementation in ngs-py, and present several numerical examples including piecewise smooth structures with junctions.

A Comparison of Stochastic Models for El Niño

Hermann Mena (University of Innsbruck)
Lena-Maria Pfurtscheller * (University of Innsbruck, Austria)
Chiara Piazzola (University of Innsbruck)

El Niño is a quasi-periodic climate pattern in the tropical Pacific ocean which has an impact on the whole world. Numerical models are an important tool in oceanographic research. In the recent years techniques specifically designed for El Niño have been proposed, see e.g. [1]. The models are usually described by means of stochastic partial differential equations or stochastic ordinary differential equations (SDEs). In this talk we consider SDE models with different type of noise. We perform numerical experiments in various scenarios and study the behavior of the solution. In addition, we briefly describe how to compute the covariance of the solution which is defined by a differential Lyapunov equation.

References

[1] Ewald, B.,Penland, C.,Temam R., 2003. Accurate Integration of Stochastic Climate Models with Application to El Niño. Monthly Weather Review 132 (1), 154–164.

Riccati based feddback stabilization to trajectories for parabolic equations

Duy Phan * (RICAM-ÖAW, Linz)
Sérgio S. Rodrigues (RICAM-ÖAW)

The problem which we address here is the *local* exponential stabilization to trajectories for parabolic systems, for $t \in (0, +\infty)$ in the form

$$\partial_t y - \nu \Delta y + F(y) + \nabla \cdot G(y) + f + \sum_{i=1}^{M} u_i \Phi_i = 0; \qquad y|_\Gamma = g;$$

or in the form

$$\partial_t y - \nu \Delta y + F(y) + \nabla \cdot G(y) + f = 0; \qquad y|_\Gamma = g + \sum_{i=1}^{M} u_i \Psi_i,$$

where F and G may be nonlinear functions and vector functions respectively, and $u : (0, +\infty) \longrightarrow \mathbb{R}^M$ is a control function. That is, given a positive constant $\lambda > 0$ and a solution $\hat{y}(t) = \hat{y}(t, \cdot)$ of the uncontrolled system (with $u = 0$), our goal is to find u such that the solution $y(t) := y(t, \cdot)$ of the system, supplemented with the initial condititon

$$y(0) := y(0, x) = y_0(x),$$

is defined on $[0, +\infty)$ and satisfies, for a suitable Banach space X,

$$|y(t) - \hat{y}(t)|_X^2 \le C\mathrm{e}^{-\lambda t} |y(0) - \hat{y}(0)|_X^2, \quad \text{provided} \quad |y(0) - \hat{y}(0)|_X < \epsilon.$$

We give conditions on the family of controllers $\{\Phi_i\}$, respectively $\{\Psi_i\}$, for the existence of a stabilizing control. Then we compute the feedback stabilizing controller by solving a suitable differential Riccati equation. The results of some numerical simulations are presented, both for internal and boundary controls, showing that the presence of the feedback controller can avoid instability observed in the case of free dynamics.

References

[1] D. Phan and S. S. Rodrigues, 2016. Stabilization to trajectories for parabolic equations. In preparation.

A new mixed approach for discretizing Kirchhoff-Love plates

Katharina Rafetseder * (Johannes Kepler University Linz, Austria)
Walter Zulehner (Johannes Kepler University Linz)

In this talk, we introduce a new mixed variational formulation of a Kirchhoff-Love plate. The plate is considered to be clamped, simply supported and free on different parts of the boundary. The new formulation satisfies Brezzi's conditions and is equivalent to the original primal variational problem. This nice properties come at the cost of a nonstandard Sobolev space. However, we provide a Helmholtz-like decomposition of this space, which allows us to derive an equivalent decoupled formulation.
Based on the Helmholtz-like decomposition, we can derive in a natural way families of finite elements for triangular and quadrilateral meshes and also isogeometric discretizations.
Finally, we show first numerical experiments.

Convolution quadrature for the wave equation with a nonlinear boundary condition

Lehel Banjai (Heriot-Watt University, Edinburgh, UK)
Alexander Rieder * (TU Wien, Austria)

In this talk, we consider a wave scattering problem with a nonlinear impedance boundary condition. We apply a discretization scheme using multistep convolution quadrature and the boundary element method. We present a stability and convergence analysis based on an equivalence principle to the multistep discretization of a related semigroup. We obtain unconditional weak convergence, and, assuming the regularity of the exact solution, we can show that the method realizes the full convergence rate.

Robust preconditioners for optimality systems using Isogeometric Analysis

Walter Zulehner (Johannes Kepler University)
Jarle Sogn * (Johannes Kepler University, Austria)

In this talk we consider optimization problems in function space with objective functionals of tracking type and elliptic partial differential equations (PDEs) as constraints, like inverse problems for elliptic PDEs or optimal control problems with elliptic state equations. Such problems typically involve an additional regularization/cost term depending on a regularization/cost parameter. In a recent paper [1] it has been shown that parameter-robust preconditioners for the associated optimality system are available, if the PDE-constraint is treated in strong form rather than in standard weak form. This strategy can be carried over to discretized optimality systems and requires approximation spaces of correspondingly smooth functions. While C^1 finite element spaces were used in [1], we will report on the advantages of Isogeometric Analysis for preconditioning discretized optimality systems.

References

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A generalized midpoint scheme for the Landau-Lifshitz-Gilbert equation in computational micromagnetics

Dirk Praetorius (TU Wien)
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Bernhard Stiftner * (TU Wien, Austria)

Micromagnetic phenomena on a ferromagnetic sample $\Omega \subset \mathbb{R}^3$ are described by the time-dependent Landau-Lifshitz-Gilbert equation (LLG). Besides a geometric non-linearity, the reliable and effective numeric integration faces the following problems: first, the continuous solution $\boldsymbol{m} : \Omega \to \mathbb{R}^3$ satisfies a non convex side-constraint $|\boldsymbol{m}| = 1$; second, each time-step requires the computation of the so-called stray field and thus leads to a coupling with an elliptic PDE in full space. The discretization in space employs lowest-order Courant finite elements, where the side-constraint is satisfied in all vertices of the underlying discretization. The approximate stray field is computed via a FEM-BEM approach to cope with the unbounded domain. In our talk, we discuss the extension of the midpoint scheme proposed in [1], where only the so-called exchange contribution (Laplace operator) is treated implicitly, while the lower-order terms (including the computationally expensive stray field) are treated explicitly in time. The resulting scheme is still unconditionally convergent to weak solutions of LLG and formally preserves second-order accuracy in time. Numerical experiments rely on an appropriate coupling of the Netgen/NGSolve package [2] with the BEM++ library [3].

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Robust approximation error and inverse estimates for B-splines and applications to isogeometric analysis

Stefan Takacs (RICAM, Austrian Academy of Sciences, Austria)

In this talk, we will discuss approximation error estimates for B-splines of polynomial degree p and maximal smoothness $p-1$, which are robust in the polynomial degree p. We will see that there are large subspaces of the spline space satisfying a corresponding inverse estimate. One example is the space of splines whose odd derivatives vanish on the boundary. However, the inverse estimate does not extend to the whole spline space, i.e., there is a small subspace of outlier splines. For many numerical methods, it is important to have both, a robust approximation error estimate and a robust inverse estimate. We will discuss possibilities how a precise characterization of the outliers can be used for the construction of fast linear solvers for problems in isogeometric analysis.

Numerical analysis of boundary element methods for Maxwell's eigenvalue problems

Gerhard Unger (TU Graz, Austria)

We consider a Galerkin approximation of boundary integral formulations of Maxwell's eigenvalue problem in bounded and unbounded domains. An analysis of the boundary integral formulations and their numerical approximations is given in the framework of eigenvalue problems for holomorphic Fredholm operator-valued functions. We show that Galerkin approximations yield a so-called regular convergent sequence for the underlying operator-valued function of the eigenvalue problem. For that we use a recent general result of M. Halla [1]. This allows us to apply general results of the numerical analysis of holomorphic eigenvalue problems which guarantee the convergence of the eigenvalues as well as of the eigenspaces. Numerical examples confirm the theoretical results.

References

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A FREQUENCY DEPENDENT COMPLEX SCALING FOR HELMHOLTZ RESONANCE PROBLEMS

Lothar Nannen (TU Wien)
Markus Wess * (TU Wien, Austria)

Using complex scaling (often referred to as perfectly matched layer method) is a popular approach to apply finite element methods to scattering or resonance problems in open systems. In order to optimize the complex scaling for scattering problems it is common to use damping parameters scaled by the given frequency. When considering resonance problems the use of constant damping parameters leads to linear eigenvalue problems. In contrast we study the effects of frequency dependent damping parameters resulting in non-linear eigenvalue problems. In particular we focus on the behavior of the essential spectrum and the spurious eigenvalues generated by discretization.

Numerical Methods for p-Laplace type problems

Ioannis Toulopoulos (RICAM Linz, Austria)
Thomas Wick * (RICAM Linz, Austria)

In this presentation, we consider numerical methods for solving p-Laplacian model problems [1]. For space discretization we use continuous Galerkin finite element methods. Assuming sufficient regularity for the exact solution, we derive higher order error estimates. Our second goal is a very detailed comparison of three different solution algorithms: first, solving the primal variable with Newton-type algorithms and residual-based or error-oriented globalization. Second, we address the saddle point problem by applying augmented Lagrangian techniques in form of a splitting algorithm (sALG) and a monolithic scheme (mALG). All proposed algorithms are compared with respect to computational cost and convergence rates in three different examples. In all examples, the p-power and the regularization parameter are varied.
This talk is based on a joint work with Ioannis Toulopoulos. We gratefully acknowledge the financial support of this research work by the Austrian Science Fund (FWF) under the grant NFN S117-03.

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Mapped Tent Pitching Method for Hyperbolic Conservation Laws

Jay Gopalakrishnan (Portland State University, USA)
Joachim Schöberl (TU Wien, Austria)
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Tent pitching algorithms construct space-time meshes by vertically erecting canopies over vertex patches. The main advantage is the ability to advance in time by different amounts at different spacial locations. These tent pitched meshes are usually combined with a space-time discretization, which leads to a rather large local problem on each tent. This talk considers a novel discretization technique, that exploits the structure of tent pitched meshes to reduce the local problem size. The reduction is obtained by transforming the tents to a reference domain with a space-time tensor product structure, which then allows to discretize space and time independently. These Mapped Tent Pitching (MTP) schemes can be applied to both, linear and non-linear systems. For linear systems a fully implicit MTP scheme is presented in [1] and this talk will focus on non-linear systems (see [1, 2]). Numerical results for the Euler equations in 2+1 dimensions and the linear wave equation in 3+1 dimensions will be shown.

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