

$\forall C \subset X$  convex, closed  $\forall x \in X \setminus C \exists f \in X^*$ :

Functional Analysis Group

$f(x) = a, f(C) < a$

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SEMINARVORTRAG

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# Rich families occurring across Maths or at least across Analysis, Topology, Set Theory ...

**Abstract.** Let  $P$  be a set and let  $\prec$  be a *partial order* on it. Assume moreover that  $P$  is *(up)-directed by  $\prec$* , i.e., for every  $t_1, t_2 \in P$  there is  $t_3 \in P$  such that  $t_1 \prec t_3$  and  $t_2 \prec t_3$ . A subset  $R \subset P$  is called *rich* if it is (i) *cofinal* (i.e. for every  $t \in P$  there is  $r \in R$  such that  $t \prec r$ ) and (ii)  *$\sigma$ -complete* (i.e. whenever  $r_1 \prec r_2 \prec \dots$  is an increasing sequence in  $R$ , then  $\sup_{i \in \mathbb{N}} r_i$  belongs again to  $R$ ). **Fact.** The intersection of countably many rich families is again rich.

**Important example:** Let  $X$  be a (rather non-separable) Banach space. Let  $\mathcal{S}(X)$  denote the family of all separable closed subspaces of  $X$  and endow it by the partial order ' $\subset$ '.

**Theorem.** Given *any* function  $f : X \rightarrow \mathbb{R}$ , there is a rich family  $\mathcal{R}$  in the poset  $(\mathcal{S}(X), \subset)$  such that for every  $Y \in \mathcal{R}$  and every  $x \in Y$  the function  $f$  is continuous and Fréchet differentiable and ... at  $x$  if (and only if) so is the restriction  $f|_Y$  at  $x$ .

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Gäste sind herzlich willkommen!

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