# Kolloquium

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***Evolution Equations beyond Banach Spaces***

Many phenomena appearing in nature can be modeled by abstract Cauchy problems of the form

$$\text{(ACP)} \quad \begin{cases} \frac{d}{dt}u(t) = Au(t) \ \text{ for } \ t \geqslant 0, \\ u(0) = u_0 \end{cases}$$

where $u\colon [0,\infty) \to X$ is a vector-valued function and $A\colon D(A) \to X$ with $D(A) \subseteq X$ is a linear operator.

For example, the heat equation $\frac{\partial}{\partial t}u(x,t) = \frac{\partial^2}{\partial x^2}u(x,t)$ describes the distribution of temperature in a given area $\Omega \subseteq \mathbb{R}$ over time. It can be written in the form above with $A = \frac{d^2}{dx^2}$ and a space $X$ of real valued functions over $\Omega$ if one identifies $u(\cdot,t)\colon \Omega \to \mathbb{R}$ with $u(t) \in X$.

If $X$ is a Banach space then there is a rich and well developed theory to solve (ACP) and to explore qualitative and quantitative properties of the solutions via so-called $C_0$-semigroups. If $X$ is not Banach then the picture is not so clear since already basic results from the classical theory can fail. From another perspective exactly these failures exhibit interesting effects and new possibilities beyond the Banach space world. In the talk we discuss phenomena and results of this kind.

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| **uniformly exponentially stable** | $\Longrightarrow$ <br> $\Longleftarrow$ | **strongly exponentially stable** | $\Longrightarrow$ <br> $\Longleftarrow$ | **super polynomially stable** | $\Longrightarrow$ <br> $\Longleftarrow$ | **uniformly stable** | $\Longrightarrow$ <br> $\Longleftarrow$ | **strongly stable** |

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