Kolloquium

Institut für Mathematik

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Briot-Bouquet differential equations

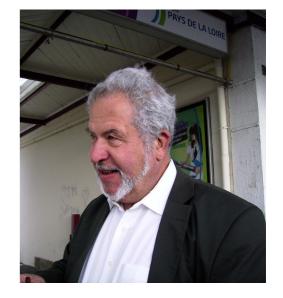A Briot-Bouquet differential equation is a complex differential equation

$$zw'(z) = az + bw(z) + \sum_{\alpha+\beta \geq 2} a_{\alpha\beta} z^\alpha w(z)^\beta,$$

where $w(0) = 0$ and the power series on the right-hand side is given. Cauchy's theorem on existence and uniqueness cannot be applied directly.

We discuss the important special case where $b = n$ is a natural number. Here a formal solution $w(z)$ exists if, and only if, a certain polynomial $P(a, b, a_{\alpha\beta} : \alpha + \beta \geq n)$ vanishes. If so, then the equation is called solvable of type n, and all solutions take the shape

$$w_t(z) = c_1 z + \ldots + c_{n-1} z^{n-1} + tz^n + \sum_{\nu \geq n+1} Q_\nu(t) z^\nu$$

for polynomials $Q_\nu(t)$. Coefficients c_i are uniquely determined; $w_t(z)$ is convergent if the given right-hand side is convergent.

In the talk, we will discuss regularization; hence applicability of classical existence theorems; analytic dependence of solutions on z and t; and characterization of families of analytic functions arising as solutions of Briot-Bouquet equations of type n.

Do·28·Mai

16:15·HSB 8