Kolloquium

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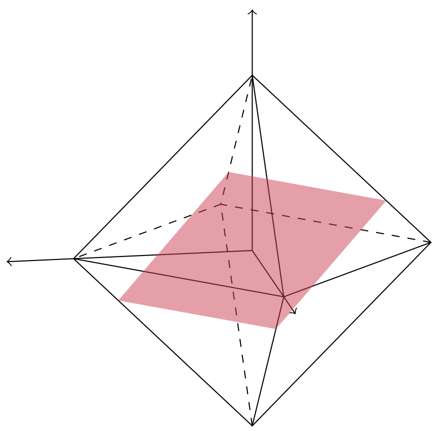
Slicing inequalities for measures of convex bodies



We consider the following problem. Does there exist an absolute constant C such that for every $n \in \mathbb{N}$, every integer $1 \leq k < n$, every origin-symmetric convex body L in $\mathbb{R}^n$, and every measure μ with non-negative even continuous density in $\mathbb{R}^n$,

$$\mu(L) \leq C^k \max_{H \in \mathrm{Gr}_{n-k}} \mu(L \cap H)|L|^{k/n} \tag{1}$$

where Gr_{n-k} is the Grassmanian of $(n-k)$-dimensional subspaces of $\mathbb{R}^n$, and $|L|$ stands for volume? This question is an extension to arbitrary measures (in place of volume) and to sections of arbitrary codimension k of the hyperplane conjecture of Bourgain, a major open problem in convex geometry.

We show that (1) holds for arbitrary origin-symmetric convex bodies, all k and all μ with $C \sim \sqrt{n}$, and with an absolute constant C for some special classes of bodies. We also prove that for every $\lambda \in (0,1)$ there exists a constant $C = C(\lambda)$ such that inequality (1) holds for every $n \in \mathbb{N}$, every origin-symmetric convex body L in $\mathbb{R}^n$, every measure μ with continuous density and the codimension of sections $k \geq \lambda n$. The proofs are based on a stability result for generalized intersection bodies and on estimates of the outer volume ratio distance from an arbitrary convex body to the classes of generalized intersection bodies.

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