Wider Research Context

Noncommutative semialgebraic geometry studies specific sets of matrices of any size. *Operator algebra* studies linear operators, not individually, but in terms of their interplay. Many interesting concepts can be studied from both perspectives simultaneously, for example operator systems. The goal of this project is twofold. First, we develop theory at the intersection of the two areas. Second, we apply the results to problems in convex algebraic geometry and quantum information theory.

Objectives

We examine abstract operator systems with a finite-dimensional realization, also known as free spectrahedra. We develop new criteria for having such a realization, and apply them to large classes of operator systems.
We define and study generalizations of operator systems, involving families of non-commutative cones beyond positive matrices.
We apply the results to problems from convex algebraic geometry, answering questions on linear matrix inequality representations of sets. Generalized versions of abstract operator systems will answer the same questions, but for more general conic representations.
Generalized operator systems are also a particularly promising tool to approach important open problems from quantum information, such as the quest for NPT bound entanglement, and the PPT2 conjecture.

Methods

Our methods include theorems by Arveson, Choi, Effros, Ruan, and Stinespring from operator algebra, which for generalized operator systems will have to be established first. We also use methods from convex and semialgebraic geometry to study (free) spectrahedra, such as hyperbolic polynomials, determinantal representations, sums of squares, and matrix convexity theory. Non-standard models of real and complex numbers have recently been used successfully to solve problems related to our proposed project. We fully generalize these techniques to the noncommutative setup, and use them to tackle the proposed problems. Quantum information theory provides also provides specific methods that we will use, such as purifications of states, dilation techniques and results on undecidability.

Innovation

Combining semialgebraic geometry and operator algebra is a novel approach, that has recently led to first significant results. Its full potential is by far not exhausted, and the two different communities are not well aware of the others' methods and results. Besides connecting the two areas better, we develop new theory at their intersection, with influence on the development of both. The results also have applications in the highly active areas of convex algebraic geometry and quantum information theory, whose future development and impact can hardly be underestimated.

Primary researchers involved

The primary researcher involved in this project is Tim Netzer, full professor at the University of Innsbruck. He applies for funding of two additional research positions.