Abstract of FWF Project

Free Semialgebraic Geometry and Convexity

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Classical semialgebraic geometry examines subsets of euclidean space defined by polynomial inequalities. This algebraic structure of sets allows for strong methods and results. The projection theorem for example states that the projection of a semialgebraic set is again semialgebraic. From this it also follows that the convex hull of a semialgebraic set is again semialgebraic. This is of particular importance in optimization, where the geometry can often be simplified significantly by passing to the convex hull, whereas the optimal value of the problem is unchanged. Finding easy algebraic descriptions of the convex hull is then of great interest for practical purposes.

The project addresses similar question, however in a free, i.e. non-commutative context. Semialgebraic sets here consist of matrices of all sizes simultaneously, i.e. they are dimension-free. The free theory of semialgebraic sets is a recent development, triggered by applications in linear systems engineering, quantum physics, optimization and group theory. Many of the main building-blocks of the commutative theory have not been transferred successfully to the free context however. This is one of the main goals of the project. The most important questions concern the projection theorem in the free context, the correct notion of a free semialgebraic set, and how free convex hulls of semialgebraic sets can be described as simple as possible. The methods applied to solve these questions stem from operator algebra, group theory and functional analysis over real closed fields. The results are of great importance for the future developments of the field of free semialgebraic geometry. They also provide a sound basis for many of the algorithmic applications via semidefinite optimization, that have become very popular recently. Any result also opens the way for more of these applications to emerge.