ABSTRACT

THE CONE OF COMPLETELY POSITIVE SEMIDEFINITE MATRICES

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The cone of completely positive semidefinite matrices is a recent discovery that admits numerous applications, for example in quantum non-local games, quantum graph theory and quantum behaviors. The cone is a generalization of the cone of completely positive matrices, which has been studied since the middle of the last century. Completely positive semidefinite matrices have however not been studied much so far, and understanding this cone better is the main goal of my proposed dissertation project.

A symmetric matrix $M \in \mathrm{Sym}_s(\mathbb{R})$ ist completely positive semidefinite, if it admits a representation

$$M = (\mathrm{tr}(P_i P_j))_{i,j=1,\ldots,s}$$

where $P_1, \ldots, P_s$ are positive semidefinite matrices of an arbitrary large size d. Going through larger and larger sizes of d is a procedure to produce all completely positive semidefinite matrices. Certifying that a matrix is *not* completely positive semidefinite is however much more complicated; no suitable method seems to exist so far. In particular, since it was recently shown that the cone of completely positive semidefinite matrices is not closed, there is no bound on d depending just on s. This rises many wide open questions. For example, is the cone of positive semidefinite matrices semialgebraic? Is complete positive semidefiniteness a decidable problem at least?

The goal of my disseration is to solve these problems, among others. I want to find a good description of the cone of completely positive semidefinite matrices and its dual. I will also try to find good approximations of the cone by easier ones, for example spectrahedral cones. The results are of foundational interest in pure mathematics, but will also have significant impact on the mentioned applications, in particular their computational aspects.