Intermediate Scale for Plasma Sheath

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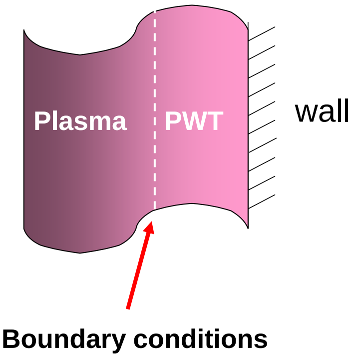
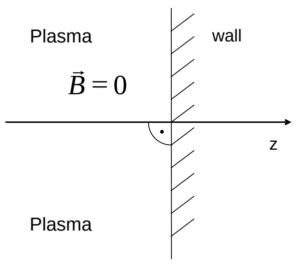
Plasma wall transition (PWT)
is a narrow plasma layer in front of the wall



- Estimation of particle and heat fluxes to the wall
- Boundary conditions for the numerical codes
- Diagnostics (plasma probe theory)

Geometry



$\hat{n}_i = n_i / n_s,$
$\hat{n}_e = n_e / n_s$
$\varphi = -e\Phi / kT_{es}$
$\tau_S = \dfrac{T_{iS}}{T_{eS}^{\ *} + T_{iS}}$

Basic Equations

$$\frac{d}{dz}(\hat{n}_i u) = \frac{\sigma}{L}$$

$$u\frac{du}{dz} - \frac{d\varphi}{dz} = -\frac{1}{L}\left(\nu + \frac{\sigma}{\hat{n}_i}\right)u - \tau_s\left(\frac{1}{\hat{n}_i}\frac{d\hat{n}_i}{dz} - \frac{1}{\hat{n}_e}\frac{d\hat{n}_e}{dz}\right)$$

$$\frac{d^2\varphi}{dz^2} = \frac{1}{\lambda^2{}_{DS}}(\hat{n}_i - \hat{n}_e)$$

$$\hat{n}_e(\varphi) = 1 - \varphi + c\varphi^2$$

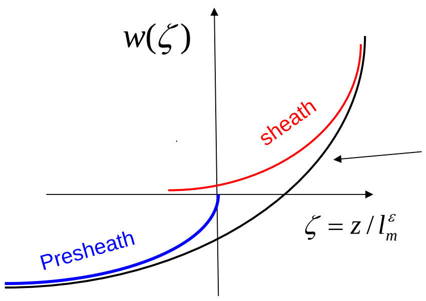
λ_{DS} Debye length
$u = v_i / c_s$ ion velocity
σ ionization rate
L mean free path
ν collision frequency

Motivation: Usually unmagnetized PWT consists of two scales *L* and λ_{DS}

$$L >> \lambda_{DS}$$

which results in different solutions:

- Presheath solution: quasineutral, collision dominated region with the scale L
- Debey sheath: nonneutral, collisionless region with the scale λ_{DS}



Our goal is to introduce some intermediate scale l_m^{ε}

$$L >> l_m^{\varepsilon} >> \lambda_{DS}$$

which allow to find a unique solution including the both: sheath and presheath solutions

$$\varepsilon = \frac{\lambda_{DS}}{L} << 1$$

- Integrated continue-equation

$$u(z)=u(0)\frac{\left(1+\frac{\sigma_s z\alpha(\varepsilon)}{L\hat{n}_i(0)u(0)}\right)}{\left(1+\frac{\Delta}{\hat{n}_i(0)}\right)}$$

$$\alpha(\varepsilon)=1-\frac{\kappa n_e(0)}{\nu_i}\left(1+C_1\varepsilon^{q_1}\right)$$

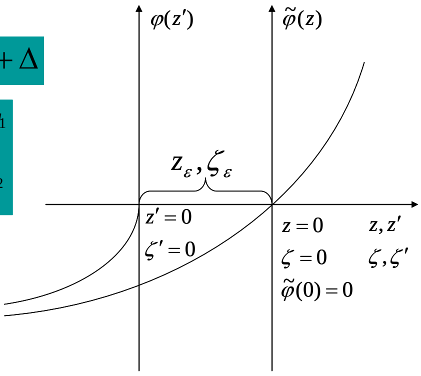
κ... Recombination rate

ν_i··· Collision frequency

$$\hat{n}_i(z)=\hat{n}_i(0)+\Delta$$

$$\hat{n}_i(0)=1+C_1\varepsilon^{q_1}$$

$$u(0)=1+C_2\varepsilon^{q_2}$$



- Integrated equation of motion

$$\frac{1}{2}u^2(0)\frac{\left(1+\frac{\sigma_s z\alpha(\varepsilon)}{L\hat{n}_i(0)u(0)}\right)^2}{\left(1+\frac{\Delta}{\hat{n}_i(0)}\right)^2}-(1-\tau_s)\varphi(z)+\frac{1}{L}\left(\nu+\frac{\sigma_s\alpha(\varepsilon)}{\hat{n}_i(0)}\right)u(0)z-\tau_s\left(c-\frac{1}{2}\right)\varphi^2(z)-\frac{1}{2}u^2(0)-\tau_s\ln\hat{n}_i(0)+\tau_s\ln\hat{n}_i=0$$

$$\Delta=\frac{1}{\left(\frac{\tau_s}{\hat{n}_i(0)}-\frac{u^2(0)}{\hat{n}_i(0)}\right)}\left\{(1-\tau_s)\varphi(z)-\frac{1}{L}\left(\nu+\frac{2\sigma_s\alpha(\varepsilon)}{\hat{n}_i(0)}\right)u(0)z+\tau_s\left(c-\frac{1}{2}\right)\varphi^2(z)\right\}-\frac{1}{2}\frac{\left(\frac{3u^2(0)}{n_i{}^2(0)}-\frac{\tau_s}{n_i{}^2(0)}\right)}{\left(\frac{\tau_s}{n_i(0)}-\frac{u^2(0)}{n_i(0)}\right)^4}(1-\tau_s)^2\varphi^2(z)$$

$$z'=z+\frac{(\hat{n}_i(0)-1)-\frac{W^2}{4V}}{U}$$

$$\zeta'=\frac{z'}{l_m^\varepsilon}$$

- $\lambda^2{}_{Ds}\sqrt{\frac{l_m^\varepsilon}{VU}}\frac{1}{\left(l_m^\varepsilon\right)^3}\frac{d^2w}{d\zeta'^2}=w^2(\zeta')+\zeta'$ $\quad$ $\lambda^2{}_{DS}\sqrt{\frac{l_m^\varepsilon}{VU}}\frac{1}{\left(l_m^\varepsilon\right)^3}\overset{!}{=}1$ $\quad$ $\Rightarrow l_m^\varepsilon=\lambda^{4/5}{}_{DS}\left(\frac{1}{VU}\right)^{1/5}$

Painleve´-I-Equation

$$U=-\frac{1}{\frac{\tau_s}{\hat{n}_i(0)}-\frac{u^2(0)}{\hat{n}_i(0)}}\frac{1}{L}\left(\nu+\frac{2\sigma_s\alpha(\varepsilon)}{\hat{n}_i(0)}\right)u(0)$$

$$V=\frac{\tau_s\left(c-\frac{1}{2}\right)}{\frac{\tau_s}{\hat{n}_i(0)}-\frac{u^2(0)}{\hat{n}_i(0)}}-\frac{1}{2}\frac{\frac{3u^2(0)}{\hat{n}^2{}_i(0)}-\frac{\tau_s}{\hat{n}^2{}_i(0)}}{\left(\frac{\tau_s}{\hat{n}_i(0)}-\frac{u^2(0)}{\hat{n}_i(0)}\right)^4}(1-\tau_s)^2-c$$

$$W=\frac{1-\tau_s}{\frac{\tau_s}{\hat{n}_i(0)}-\frac{u^2(0)}{\hat{n}_i(0)}}+1$$

$$\tau_s=\frac{\gamma_i T_{is}}{T_{es}{}^*+\gamma_i T_{is}}$$

$$\gamma_i=1$$

- Ansatz: $\hat{n}_i(0) = 1 + C_1\varepsilon^{q_1}$ $u(0) = 1 + C_2\varepsilon^{q_2}$ $q_{1,2}, C_{1,2} > 0$

Expansion of l_m^{ε}

$$\Rightarrow l_m^{\varepsilon} = l_m^{Riem} + l_m^{Riem}.T(\varepsilon) \qquad T(\varepsilon) = \varepsilon^{q_1} A + \varepsilon^{q_2} B$$

$$A = \frac{\tau_s(c-1/2)}{c(1-\tau_s) - 1/2(3-\tau_s) - (1-\tau_s)(\tau_s - 1/2)\tau_s}\left[-1/2C_1 - \ldots\ldots\right]$$

$$B = \frac{\tau_s(c-1/2)}{c(1-\tau_s) - 1/2(3-\tau_s) - (1-\tau_s)(\tau_s - 1/2)\tau_s}\left[-\frac{C_2}{1-\tau_s} - \ldots.\right]$$

$$l_m^{Riem} = \varepsilon^{4/5} L \left(\frac{1-\tau_s}{(3/2-c)^{1/2}(\nu + 2\bar{\sigma}_s)^{1/2}}\right)^{2/5} \qquad \bar{\sigma}_s = \sigma_s\left(1 - \frac{\kappa n_e(0)}{\nu_i}\right)$$

$$l_m^{\varepsilon} = l_m^{Riem} + l_m^{Riem}.\underbrace{\left[\varepsilon^{q_1} A + \varepsilon^{q_2} B\right]}_{\text{new}} \xrightarrow{\varepsilon \to 0} l_m^{Riem}$$

$q_{1,2}, C_{1,2} \rightarrow$?

$$C_1 = 0.7152 \frac{2^{1/5}(\nu + 2\bar{\sigma}_S)^{4/5}}{(1-\tau_S)^{3/5}(3-2c)^{1/5}},$$

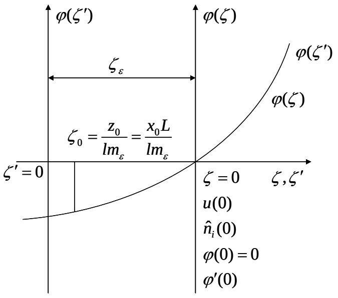
$$C_2 = \frac{1}{2}C_1(1-\tau_S), \quad q_1 = q_2 = 4/5$$

New boundary conditions

$$\hat{n}_i(0) = 1 + C_1\varepsilon^{4/5}, \quad u(0) = 1 + C_2\varepsilon^{4/5}, \quad \phi'(0) = \sqrt{\frac{Ul_m^{\varepsilon}}{V}} w'(\zeta_{\varepsilon}),$$

$$\zeta_{\varepsilon} = \frac{C_1(1-\tau_s)^{3/5}(3-2c)^{1/5}}{2^{1/5}(\nu + 2\bar{\sigma}_s)^{4/5}} - \frac{\varepsilon^{4/5}}{2^{6/5}} \frac{\left(C_1(1-\tau_s) - 2C_2\right)^2}{(3-2c)^{4/5}(\nu + 2\bar{\sigma}_s)^{4/5}(1-\tau_s)^{2/5}}$$

Boundary conditions at an arbitrary point $\zeta = \zeta_0$



The boundary conditions $u(0), \varphi(0), \varphi'(0), \hat{n}_i(0)$ or $\frac{\rho(0)}{n_e(0)}$ at $\zeta_0 = 0$ can be transformed to boundary conditions at an arbitrary point $\zeta_0 = \frac{z_0}{l_m^\varepsilon}$ with $0 \le \zeta_0 \le 1$

Summary:

- The precise intermediate scale for finite ε is derived.
- Corresponding new boundary conditions are formulated.
- These conditions can be applied to any arbitrary point $\zeta_0 = \frac{z_0}{l_m^\varepsilon}$.