Plasma-wall transition in the presence of an oblique magnetic field

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Plasma: ionized gas containing various kinds of neutral and charged particles.

- Plasma properties are roughly characterized by temperature $T_{i,e}$ and number density of charged $n_{i,e}$ and neutral particles n_o, and is influenced by applied and self-induced fields.

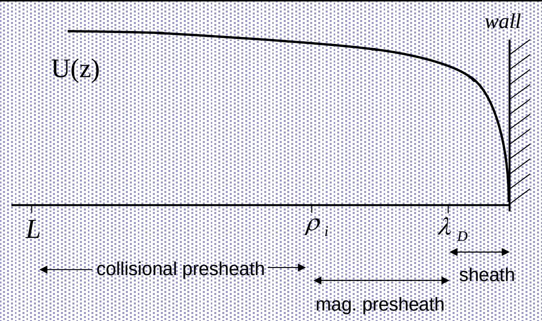
- Apart from its interior composition, a plasma is influenced by the *boundary conditions*.

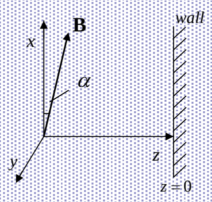
- In most of the terrestrial situations, plasmas are bounded by material walls. Examples: Plasma confining machines, laboratory plasma devices, plasma processing of materials etc.

- The boundary region of plasmas involves plentiful and complex physical behaviours.

- In tokamaks, an oblique magnetic field reducesthe risk of melting and sublimation.

Structure of Magnetized Plasma-Wall Transition





- Collisional plasma
- Electric field along z-axis
- Magnetic field at small angle with x-axis
- Plane absorbing wall at position z=0
- Electrons in Boltzmann equilibrium

Equations

Ion continuity eq.

$$\frac{d}{dz}(n_i v_z) = n_e \nu_i - \kappa n_e n_i \qquad (1)$$

Ion momentum balance

$$v_z \frac{dv_x}{dz} = \omega_z v_y - (\nu_c + \nu_i n_e / n_i - \kappa\, n_e) v_x \qquad (2)$$

$$v_z \frac{dv_y}{dz} = \omega_x v_z - \omega_z v_x - (\nu_c + \nu_i n_e / n_i - \kappa\, n_e) v_y \qquad (3)$$

$$v_z \frac{dv_z}{dz} = -\frac{Ze}{m_i}\frac{d\varphi}{dz} - \omega_x v_y - \frac{\gamma k T_i}{m_i n_i}\frac{dn_i}{dz} - (\nu_c + \nu_i n_e / n_i - \kappa\, n_e) v_z \qquad (4)$$

Ion polytropic law

$$\nabla p_i = \gamma k T_i \nabla n_i \qquad (5)$$

Electron Boltzmann distribution

$$\frac{dn_e}{dz} = \frac{e n_e}{k T_e}\frac{d\varphi}{dz} \qquad (6)$$

Poisson's eq.

$$Z n_i - n_e = \frac{\varepsilon_o}{e}\frac{d^2\varphi}{dz^2} \qquad (7)$$

Length Scales

A three-scale problem, including

1. The Debye length $\lambda_D = (\varepsilon_o kT_e / n_s e^2)^{1/2}$
2. The ion mean free path $\lambda_c = c_s / \nu_{ci}$
3. The ion gyro-radius $\rho_i = c_s / \omega_i$

$$\lambda_D >> \rho_i >> \lambda_c$$

Aim: To find the approximate solution of all regions near transition points and relate them to find the intermediate solutions for 'smooth and realistic' transition

Normalization

We normalize our equations at sheath edge.

$$v_{ref} = c_s, \quad T_{ref} = T_e + \gamma T_i, \quad \varphi_{ref} = -\frac{kT_e}{e}, \quad n_{ref} = n_s,$$

$$p_{ref} = n_{ref} kT_{ref}, \quad z_{ref} = \lambda_D, \rho_i, \lambda_c. \tag{8}$$

- **Asymptotic Analysis**

$$\varepsilon_{Dm} = \lambda_D / \rho_i \to 0 \quad \text{and} \quad \varepsilon_{mc} = \rho_i / \lambda_c \to 0$$

Asymptotic magnetic presheath solution

Due to the asymptotic limit $\varepsilon_{Dm} \to 0$, the magnetic presheath is quasi-neutral.

$$Zn_i^* = n_e^*$$

x-momentum

$$v_z^* \frac{dv_x^*}{d\zeta} = \sin \alpha v_y^*$$

y-momentum

$$v_z^* \frac{dv_y^*}{d\zeta} = \cos \alpha v_z^* - \sin \alpha v_x^*$$

z-momentum

$$\left(v_z^* - \frac{1}{v_z^*} \right) \frac{dv_z^*}{d\zeta} = -\cos \alpha v_y^*$$

All three momentum equations combine to give:

$$v_z^* (v_z^{*2} - 1) v_z^{*\prime\prime} + (v_z^{*2} + 1) v_z^{*\prime 2} + v_z^{*2} - C v_z^* + \sin^2 \alpha = 0, \tag{12}$$

where C is constant of integration which is to be determined later.

We now investigate the behaviour of this equation near the sheath edge for which we choose $\zeta = 0$.

$$v_z^*(\zeta) = 1 + w(\zeta),$$

and we assume that in leading order

$$w(\zeta) = a(-\zeta)^m, \qquad a < 0 \quad \text{and} \quad m > 0$$

$$a^4 m(2m-1)(-\zeta)^{4m-2} + a^3 m(5m-3)(-\zeta)^{3m-2} + 2a^2 m(2m-1)(-\zeta)^{2m-2} + a^2(-\zeta)^{2m}$$
$$+(2-C)a(-\zeta)^m + 1 + \sin^2\alpha - C = 0. \qquad (13)$$

Case 1: $1+\sin^2\alpha - C > 0$

we get $v_z^* = 1 - \sqrt{\dfrac{-(1+\sin^2\alpha - C)}{2}}\,\zeta.$ physically not acceptable

Case 2: $1+\sin^2\alpha - C < 0$

we get $v_z^* = 1 - \sqrt{\dfrac{C-1-\sin^2\alpha}{2}}\,\zeta.$ acceptable

Case 3: $1+\sin^2\alpha - C = 0$

we get $v_z^* = 1 - \dfrac{\cos^2\alpha}{12}\zeta^2.$ acceptable

$$C \geq 1 + \sin^2\alpha. \qquad (14)$$

We perform a similiar analysis at $\zeta \to -\infty$. For that we assume:

$$v_z^* = b(-\zeta)^s. \qquad b > 0$$

Eq. (12) gives:

$$s(2s-1)b^4(-\zeta)^{4s-2} + sb^2(-\zeta)^{2s-2} + b^2(-\zeta)^{2s} - Cb(-\zeta)^s + \sin^2\alpha = 0 \qquad (15)$$

the powers $s > 0$ are excluded due to non-physical results and for $s = 0$ we get

$$b^2 - Cb + \sin^2\alpha = 0$$

which is a quadratic ewuation in b, whose solution is:

$$b = \frac{C \pm \sqrt{C^2 - 4\sin^2\alpha}}{2}$$

and for $C > 1 + \sin^2\alpha$ and $C^2 = 4\sin^2\alpha$,we get

$$v_z^* = \sin\alpha \qquad (16)$$

which is called Bohm-Chodura criterion for the velocity at the magnetic presheath entrance.

Next step: analysis of the two intermediate regions between

- collisional presehath and magnetic presheath
- magnetic presheath and Debye sheath

Thank you!