

# The Economic Value of Predicting Bond Risk Premia: Can Anything Beat the Expectations Hypothesis? \*

Lucio Sarno<sup>†</sup>   Paul Schneider<sup>‡</sup>   Christian Wagner<sup>§</sup>

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## Abstract

This paper studies whether the evident statistical predictability of bond risk premia translates into economic gains for bond investors. We show that affine term structure models (ATSMs) estimated by jointly fitting yields and bond excess returns capture this predictive information otherwise hidden to standard ATSM estimations. The model's excess return predictions are unbiased, produce regression  $R^2$ s beyond those reported in the literature, exhibit high forecast accuracy, and allow to generate positive bond portfolio excess returns in- and out-of-sample. Nevertheless, these models cannot beat the expectations hypothesis (EH) out-of-sample: the forecasts do not add economic value compared to using the average historical excess return as an EH-consistent estimate of constant risk premia. We show that in general statistical significance does not necessarily translate into economic significance because EH deviations mainly matter at short horizons and standard predictability metrics are not compatible with common measures of economic value. Overall, the EH remains the benchmark for investment decisions and should be considered an economic prior in models of bond risk premia.

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<sup>†</sup>Cass Business School and Centre for Economic Policy Research (CEPR), London. Corresponding author. Faculty of Finance, Cass Business School, City University, London EC1Y 8TZ, UK. [lucio.sarno@city.ac.uk](mailto:lucio.sarno@city.ac.uk).

<sup>‡</sup>Institute of Finance, University of Lugano, Via Buffi 13, CH-6900 Lugano. [paul.schneider@usi.ch](mailto:paul.schneider@usi.ch).

<sup>§</sup>Department of Finance, Copenhagen Business School, DK-2000 Frederiksberg, Denmark. [cwa.fi@cbs.dk](mailto:cwa.fi@cbs.dk).

# 1 Introduction

Empirical research documents that the expectations hypothesis (EH) of the term structure of interest rates is rejected by the data and argues, almost unequivocally, that deviations of the EH reflect time-varying risk premia.<sup>1</sup> Fama (1984), Fama and Bliss (1987), and Campbell and Shiller (1991) are among the first to provide such evidence, while more recent studies that document the violation of the EH include Bekaert et al. (1997), Bekaert and Hodrick (2001), and Sarno et al. (2007). This evidence is strengthened by work showing that bond risk premia are predictable; see e.g. Cochrane and Piazzesi (2005). In this paper, we evaluate the relevance of EH deviations by studying whether bond investors benefit from conditioning on information about time-varying risk premia.

We estimate risk premia using affine term structure models (ATSMs). Based on the pioneering work of Duffie and Kan (1996) and Dai and Singleton (2000), ATSMs receive a particular focus in the finance literature on dynamic term structure models because of their richness, tractability, and ability to produce reasonable risk premium dynamics. Interestingly, research on the EH and on ATSMs has, to a large extent, evolved along separate paths.<sup>2</sup> Only a few papers attempt to bridge this gap and, for example, the results of Backus et al. (2001) and Dai and Singleton (2002) support the notion that the failure of the EH is due to the invalid assumption of constant risk premia. Recent research, however, argues that the evident predictability of bond risk premia cannot be captured by ATSMs because the necessary predictive information is not spanned by the cross-section of yields (see e.g. Duffee, 2011; Joslin et al., 2010). By contrast, in this paper we show that such ATSMs do capture the predictability of bond excess returns when employing an extended estimation procedure that jointly fits yields and past risk premia to the data. This finding suggests that ATSMs represent a suitable vehicle for evaluating the economic consequences of EH deviations for bond investors.

Our paper contributes to the literature by evaluating whether ATSM forecasts are statistically more accurate and economically more valuable than EH-consistent forecasts or whether

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<sup>1</sup>The EH is the postulate that the long-term interest rate is determined by the current short-term rate and the market expectation of the short-term rate over the maturity of the long-term rate, plus a constant risk premium. The case where the risk premium is zero is termed the “pure EH.” Under the EH, therefore, pure discount bonds are perfect substitutes and bond excess returns are not predictable. The EH is originally credited to Fisher (1896, 1930), and further refined and popularized by Keynes (1930), Lutz (1940) and Hicks (1953).

<sup>2</sup>While empirical EH research often argues that the theory’s failure is due to time-varying risk premia, these papers put little effort into modeling risk premia, focusing instead on formal statistical tests of the EH. Similarly, research on ATSMs is usually motivated by the empirical rejection of the EH, but does not establish a direct link to the EH.

presuming that the EH holds is a suitable first-order approximation for bond investment decisions. We conduct an empirical evaluation of the EH that is in many respects more comprehensive than evaluations in previous research. First, using ATSMs, we consistently model the whole term structure and not only a subset of yields or excess returns, as e.g. in [Fama and Bliss \(1987\)](#), [Campbell and Shiller \(1991\)](#), [Bekaert and Hodrick \(2001\)](#), [Cochrane and Piazzesi \(2005\)](#). Second, the extended estimation proposed in this paper accounts for predictive information in and beyond the term structure, thereby producing a stronger challenge to the EH. Through this extension, we allow estimates of the state variables and the parameters to not only reflect information embedded in the term structure but, beyond that, any (unspanned) information that conveys predictive ability for bond excess returns. Recent research suggests that such additional information that adds to statistical predictability may originate from forward rates, macroeconomic factors, technical indicators, option markets, the market variance risk premium, or a ‘hidden factor’ (see, e.g., [Cochrane and Piazzesi, 2005](#); [Ludvigson and Ng, 2009](#); [Goh et al., 2012](#); [Almeida et al., 2011](#); [Mueller et al., 2011](#); [Duffee, 2011](#)). Third, while related research generally either focuses on a particular segment of the term structure (short end or long end) or analyzes a single prediction horizon only, we analyze the term structure of bond risk premia for prediction horizons ranging from 1 month to 5 years.<sup>3</sup> Fourth, while many other papers focus on statistical evidence in-sample, e.g. [Fama and Bliss \(1987\)](#), [Campbell and Shiller \(1991\)](#), [Bekaert and Hodrick \(2001\)](#), [Cochrane and Piazzesi \(2005\)](#), we measure both the statistical accuracy as well as the economic value added by conditional risk premium predictions, and we complement the in-sample results with an out-of-sample analysis. Finally, unlike the aforementioned papers, with the exception of [Bekaert and Hodrick \(2001\)](#), we expand our analysis beyond the US bond market and show that our findings apply uniformly to Switzerland, Germany, the UK, and Japan. Our paper is thus related to, but more general than [Thornton and Valente \(2012\)](#), who specifically investigate the economic value that can be generated in US bond markets when using one-year out-of-sample forecasts based on the single factor of [Cochrane and Piazzesi \(2005\)](#).

We use data for five countries to evaluate 25 combinations of prediction horizons and bond maturities, with bond maturities ranging from one month to ten years. The patterns of statistical predictability and economic value results are very similar across countries. We find that the extended estimation strategy increases predictive ability and adds economic value over standard estimations which, in line with e.g. [Duffee \(2011\)](#), cannot account for the pre-

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<sup>3</sup>The usual horizon in related papers is one year, e.g. [Fama and Bliss \(1987\)](#), [Cochrane and Piazzesi \(2005\)](#).

dictability of bond risk premia. Conditional risk premia from the extended estimation are generally unbiased, thereby explaining deviations from the EH, and entail high explanatory power for bond risk premia beyond results reported in related work. For instance, the average  $R^2$  of regressing realized on model expected excess returns across maturities and across countries is about 26% at the one-month prediction horizon and about 79% at the one-year horizon. These findings suggest that our estimation strategy is flexible enough to capture long- and short-term predictive information that emerges from different sources. As a result, the model allows investors to forecast bond risk premia with high accuracy and to earn positive bond portfolio excess returns in-sample and out-of-sample.

Compared to the standard procedure, the forecast errors from the extended procedure are substantially smaller and bond investors would be willing to pay an annual premium in the range of 2% to 4.8% to switch from the standard to the extended estimation. To evaluate the model against the EH, we use the average historical bond excess return as a consistent estimate for the EH-postulate of constant risk premia. The extended estimation beats the EH in terms of statistical forecast accuracy, however, the model's comparably higher predictive ability does not lead to superior portfolio performance out-of-sample: while the model forecasts are more accurate than the EH for 61% of the horizon/maturity combinations across countries, bond portfolio investors using the model instead of presuming that the EH holds earn higher portfolio returns in only 26% of combinations but suffer economic losses in more than 50% of combinations. These results suggest that there is a wedge between the statistical and economic relevance of EH deviations. Overall, we find that the EH presumption of constant risk premia still provides a useful benchmark to investors for out-of-sample purposes, and we view the finding that bond investors generally cannot benefit from using conditional risk premia relative to using the historical average as the bond market analogue to the result of [Goyal and Welch \(2008\)](#) for stock markets. All of our results are robust across countries, yield data sources, and ATSM specifications.

One may argue that our findings could be specific to the use of ATSMs. We therefore provide a general discussion on why – for any forecast model – conclusions based on metrics of forecast accuracy may deviate from those reached using economic value measures. On the one hand, EH deviations may be statistically significant but too small to be meaningfully exploited by bond investors. On the other hand, common predictive ability measures evaluate loss functions that are in many respects unrelated to the economic success of bond investments. As a consequence it cannot be taken for granted that even models with high forecast accuracy

allow for economically meaningful bond investment returns. We illustrate the validity of these general arguments using the results of our model estimations, but these arguments are equally valid for the mounting number of papers on statistical predictability of bond excess returns.

Finally, to acknowledge the usefulness of the EH as a benchmark, we show how ATSM estimations can be augmented to use the EH as an economic anchor for model-implied bond risk premia. We impose the EH as a prior in the estimation procedure to limit (excessive) variability of risk premia and we find that doing so increases that economic value added by model forecasts. While these results further support the role of the EH, they also leave room for future research on the optimal balance between economic restrictions to prevent overfitting and keeping sufficient flexibility to model the dynamics of bond risk premia.

## 2 Empirical Model and Estimation

Consider a long-term bond with  $T$  years maturity and a short-term bond with  $\tau$  years maturity. We denote by  $p_t^T$  the time- $t$  price of a  $T$ -year zero coupon bond with a certain payoff of 1 at maturity. The corresponding (effective) yield is given by

$$y_t^T = -\log[p_t^T]. \quad (1)$$

Analogously, we use the notation  $p_t^\tau$  and  $y_t^\tau$  for the price and the yield of the short-term bond with  $\tau \leq T$ . The prices (or equivalently yields) of the short- and long-term bonds imply the time- $t$  forward rate effective for  $T - \tau$  periods beginning at  $t + \tau$

$$f_{t,\tau}^{T-\tau} = \log[p_t^\tau / p_t^T]. \quad (2)$$

The return of buying a  $T$ -year bond at time  $t$  and selling it at time  $t + \tau$  ( $h_{t+\tau}^T$ ) is given by

$$h_{t+\tau}^T = \log[p_{t+\tau}^{T-\tau} / p_t^T], \quad (3)$$

and the corresponding bond excess return ( $rx_{t+\tau}^T$ ) is thus

$$rx_{t+\tau}^T \equiv f_{t,\tau}^{T-\tau} - y_{t+\tau}^{T-\tau}. \quad (4)$$

The EH presumes that the forward rate is equal to the expected yield (under the physical

probability measure) plus a constant risk premium. To accommodate potentially time-varying risk premia, we now turn to the specification of an affine term structure model (ATSM). Model-implied conditional expectations of bond excess returns are affine in the state variables and they contain a time-invariant as well as a time-varying component. Subsequently, we describe our Bayesian approach for the estimation of the model, where we use (i) a standard estimation procedure to fit model-implied to observed yields and (ii) an extended estimation that additionally requires model risk premia to match past bond excess returns.

## 2.1 Affine Term Structure Model and Bond Risk Premia

Based on the findings of [Litterman and Scheinkman \(1991\)](#), it has become well-established practice to employ term structure models with three factors. Accordingly, we use an ATSM specification with three latent factors. Our main model is the  $A_0(3)$  purely Gaussian three factor model of [Joslin et al. \(2011\)](#) but all our findings are robust to changing the ATSM specification to account for a larger number of factors and/or stochastic volatility (see [Section 5.3](#)).

### 2.1.1 Affine Term Structure Model

For our empirical analysis, we use a continuous-time affine term structure model for an economy that is driven by the latent state variables  $X$  living on a canonical state space  $\mathcal{D} = \mathbb{R}_+^m \times \mathbb{R}^n$ ,  $m, n \geq 0$ ,  $d = m + n \geq 1$ . Under a given probability measure  $\mathbb{M}$  the evolution of  $X$  solves the stochastic differential equation

$$dX_t = (b^{\mathbb{M}} - \beta^{\mathbb{M}} X_t)dt + \sigma(X_t)dW_t^{\mathbb{M}}, \quad (5)$$

where  $\sigma(x)\sigma(x)^\top = a + \alpha x$ ,  $a$  is a  $d \times d$  matrix, and  $\alpha$  is a  $d \times d \times d$  cube. Throughout we assume boundary non-attainment conditions for  $X_{i,t}$ ,  $1 \leq i \leq m$  in order to ensure existence of transition densities ([Filipović et al., 2013](#)) and to use generalized affine market prices of risk from [Cheridito et al. \(2007\)](#) in addition to the admissibility conditions from [Duffie et al. \(2003\)](#). This means that  $2b_i^{\mathbb{M}} > \alpha_{i,ii}$ ,  $1 \leq i \leq m$ . In what follows we will make use of two specific probability measures:  $\mathbb{Q}$ , the pricing measure, and  $\mathbb{P}$ , the time-series measure.

We impose a lower-triangular form of the mean-reversion matrix  $\beta^{\mathbb{M}}$  for  $\mathbb{M} \in \{\mathbb{P}, \mathbb{Q}\}$ . Furthermore, we restrict its diagonal to strictly positive values. This ensures a stationary system and existence of unconditional moments. The remaining parameterization (in particular

the diffusion function) is modeled in its most flexible form according to the [Dai and Singleton \(2000\)](#) specification.

We model the instantaneous short rate to be affine in  $X$ ,  $r(t) \equiv \delta_0 + \delta_X^\top X_t$ , which implies that bond prices  $p_t^T$  at time  $t$  for a maturity  $T$  are exponentially affine in the state variables  $X$

$$p_t^T = \mathbb{E}_t^{\mathbb{Q}} \left[ e^{-\int_t^{t+T} r(u) du} \right] = e^{\phi(T) + \psi(T)^\top X_t}, \quad (6)$$

where  $\phi$  and  $\psi$  solve the differential equations

$$\dot{\psi} = -\delta_X - \beta^{\mathbb{Q}\top} \psi + \frac{1}{2} \psi^\top \alpha \psi, \quad \psi(0) = 0, \quad (7)$$

$$\dot{\phi} = -\delta_0 + b^{\mathbb{Q}\top} \psi + \frac{1}{2} \psi^\top \alpha \psi, \quad \phi(0) = 0. \quad (8)$$

We collect the set of parameters governing the evolution of  $X$  by defining  $\theta^{\mathbb{P}} \equiv \{b^{\mathbb{P}}, \beta^{\mathbb{P}}, a, \alpha\}$ ,  $\theta^{\mathbb{Q}} \equiv \{b^{\mathbb{Q}}, \beta^{\mathbb{Q}}, a, \alpha, \delta_0, \delta_X\}$ , and  $\theta^{\mathbb{Q}\mathbb{P}} \equiv \theta^{\mathbb{Q}} \cup \theta^{\mathbb{P}}$ . The coefficients  $\psi$  and  $\phi$  are functions of time and the parameters, but we will suppress this dependence if the context permits for lighter notation.

### 2.1.2 Bond Risk Premia: Conditional Expectations of Bond Excess Returns

We combine Eqs. (1) and (6) to express the yield from  $t$  to  $t + T$  as

$$y_t^T = -\log[p_t^T] = -(\phi(T) + \psi(T)^\top X_t) \quad (9)$$

and use the no-arbitrage condition  $f_{t,\tau}^{T-\tau} = y_t^T - y_t^\tau$  to compute the forward rate  $f_{t,\tau}^{T-\tau}$ :

$$f_{t,\tau}^{T-\tau} = \phi(\tau) - \phi(T) + (\psi(\tau) - \psi(T))^\top X_t. \quad (10)$$

Equipped with these relations, we calculate expected yields and expected excess returns (risk premia). To appreciate the structure of the risk premium induced through the affine state variables we note that for affine models

$$\mathbb{E}_t^{\mathbb{P}} [X_{t+\tau}] = A(\tau) + B(\tau)X_t, \quad (11)$$

where  $B(\tau) = e^{-\beta^{\mathbb{P}}\tau}$  and  $A(\tau) = b^{\mathbb{P}} \int_0^\tau B(u)du$ ; see [Fisher and Gilles \(1996a,b\)](#). We may then write

$$\mathbb{E}_t^{\mathbb{P}} [y_{t+\tau}^{T-\tau}] = -(\phi(T-\tau) + \psi(T-\tau)^\top (A(\tau) + B(\tau)X_t)). \quad (12)$$

Putting together Eqs. (10) and (12), we can express

$$\mathbb{E}_t^{\mathbb{P}} [rx_{t,\tau}^T] = (\psi(\tau) - \psi(T))^\top X_t + \psi(T-\tau)^\top (A(\tau) + B(\tau)X_t).$$

Collecting coefficients, making explicit the dependence of  $\phi$  and  $\psi$  on the parameters, and introducing

$$\gamma^{\tau,T}(\theta_{\mathbb{Q}}) \equiv \psi(\tau, \theta_{\mathbb{Q}})^\top - \psi(T, \theta_{\mathbb{Q}})^\top + \psi(T-\tau, \theta_{\mathbb{Q}})^\top B(\tau, \theta_{\mathbb{P}}) \quad (13)$$

$$\eta^{\tau,T}(\theta_{\mathbb{Q}}) \equiv \psi(T-\tau, \theta_{\mathbb{Q}})^\top A(\tau, \theta_{\mathbb{P}}) \quad (14)$$

the time- $t$  risk premium turns is affine in  $\eta$  and  $\gamma$

$$\mathbb{E}_t^{\mathbb{P}} [rx_{t,\tau}^T] = \eta^{\tau,T} + \gamma^{\tau,T} X_t. \quad (15)$$

The risk premium in Eq. (15) depends on  $\tau$ ,  $T$ , and on  $t$  (through  $X$ ). It comprises a constant component as well as a time-varying component that is driven by the evolution of  $X_t$ . By adding and subtracting the unconditional expectation of  $X$  we can rewrite the conditional expectation in Eq. (15) as

$$\mathbb{E}_t^{\mathbb{P}} [rx_{t,\tau}^T] = \eta^{\tau,T} + \gamma^{\tau,T} \mathbb{E}^{\mathbb{P}} [X] - \gamma^{\tau,T} (\mathbb{E}^{\mathbb{P}} [X] - X_t). \quad (16)$$

This relation interprets the time-variation in risk premia as deviations of  $X_t$  from its unconditional expectation. The first two terms only depend on  $\tau$  and  $T$  and are thus time-invariant, consistent with the EH notion of a constant risk premium. Empirically, the question whether the EH holds can be assessed by analyzing whether the last term, which should be just noise under the EH, induces predictability of bond excess returns. Note that when estimating the model, the sum of the first two terms will correspond to the average excess return observed in the data and the last term will average to zero. In that sense, the time-invariant part determines for a given horizon the shape of the (average) term structure of risk premia. Building on these insights from Eqs. (15) and (16), we estimate the EH-postulated constant risk



premia using historical sample averages of bond excess returns. To estimate ATSM-implied conditional risk premia that additionally capture the time-varying component, we employ the estimation methodology described in the next subsection.

## 2.2 Model Estimation

For our empirical analysis, we follow two estimation strategies. The first is standard likelihood-based inference where the filtering equation requires model-implied yields to match the observed term structure. In the second, extended estimation, we additionally require that model-implied bond excess returns match past realized excess returns. We choose Bayesian methodology over moment-based, or maximum likelihood procedures to naturally accommodate the notion of an investor updating her beliefs about the model's predictability and to include past failures and successes into the parameter and state variable estimates. Without changing the structure of the model, this explicitly accounts for the time-series properties of EH deviations in addition to the cross-sectional properties of yields. With this novel approach we account for information that is not embedded in the term structure of interest rates but adds to predictive ability for bond excess returns.

### 2.2.1 Standard Estimation Procedure

Our data set comprises zero yields with 24 maturities (expressed in years)  $T_1, \dots, T_{24}$ , covering 1, 2, 3, 4, 6, 7, 9, 12, 13, 15, 18, 24, 25, 27, 30, 36, 48, 60, 61, 63, 66, 72, 84, 120 months; for details about the data, see Section 3. We estimate our model using filtering equations

$$\frac{y_t^{T_i}}{T_i} = -\frac{\phi(T_i, \theta^{\mathbb{Q}}) + \psi(T_i, \theta^{\mathbb{Q}})^\top X_t}{T_i} + \varepsilon_t^{T_i}, \quad (17)$$

where  $\varepsilon_t^{T_i}, i = 1, \dots, 24$  are assumed i.i.d normally distributed with mean zero and  $\mathbb{V}[\varepsilon_t^{T_i}] = e^{-2(a_0 + a_1 T_i + a_2 T_i^2)}$ . We use these equations for filtering and smoothing the latent state variables  $X$  and define  $\theta^\epsilon \equiv \{a_0, a_1, a_2\}$  and finally  $\theta \equiv \theta^{\mathbb{Q}\mathbb{P}} \cup \theta^\epsilon$ .

In a Bayesian setting, for a discretely observed data sample at times  $t_1, \dots, t_N$  the joint log posterior  $\ell$  of the latent states with the parameters for a window  $[t_m, t_n], t_1 \leq t_m < t_n \leq t_N$  is

$$\ell_m^n(\theta, X) = \sum_{k=m}^n \left\{ \log p(X_{t_k} | X_{t_{k-1}}, \theta^{\mathbb{P}}) + \sum_{i=1}^{24} \log p(\varepsilon_{t_k}^{T_i} | \theta^\epsilon) + \log \pi(\theta) \right\}, \quad (18)$$

with

$$\pi(\theta_i) \propto \begin{cases} \mathbb{1}_{\{\theta_i \text{ admissible}\}} & \theta_i \in \mathbb{R} \\ \frac{\mathbb{1}_{\{\theta_i \text{ admissible}\}}}{\theta_i} & \theta_i \in \mathbb{R}_+ \end{cases}. \quad (19)$$

The first term on the right hand side of Eq. (18) contains the transition densities, the second reflects yield pricing errors, and the third the prior distribution of the parameters. Draws  $\theta, X$  from the complicated distribution in Eq. (18) are obtained by sampling in turn from  $X \mid \theta$  and  $\theta \mid X$ .

### 2.2.2 Extended Estimation Procedure

Bond investors pay close attention to bond excess returns and evaluate past forecast errors to account for this information in their predictions and portfolio choices. To reflect this behavior we propose an extended estimation which matches model risk premia with past realized excess returns using Eq. (15). We therefore additionally consider the set of all possible (34) forecast equations given the available yield maturities

$$f_{t,\tau_i}^{T_i,j-\tau_i} - y_{t+\tau_i}^{T_i,j-\tau_i} = \eta^{\tau_i,T_i,j}(\theta^{\mathbb{Q}^{\mathbb{P}}}) + \gamma^{\tau_i,T_i,j}(\theta^{\mathbb{Q}^{\mathbb{P}}})X_t + \epsilon_{t+\tau_i}^{\tau_i,T_i,j}. \quad (20)$$

The forecast errors  $\epsilon_{t+\tau_i}^{\tau_i,T_i,j}$  are assumed i.i.d normal with mean zero and variance  $\mathbb{V} \left[ \epsilon_{t+\tau_i}^{\tau_i,T_i,j} \right] = e^{-2(C(a_0+a_1T_{i,j}+a_2T_{i,j}^2)+(b_0+b_1\tau_i+b_2\tau_i^2))}$ . We now define  $\theta^{\epsilon\epsilon} \equiv \{a_0, a_1, a_2, b_0, b_1, b_2, C\}$  and finally  $\theta \equiv \theta^{\mathbb{Q}^{\mathbb{P}}} \cup \theta^{\epsilon\epsilon}$ , and use Eq. (20) in addition to Eq. (17) for filtering and smoothing the latent state variables  $X$ . The joint, augmented log posterior  $\tilde{\ell}$  of the latent states with the parameters is now<sup>4</sup>

$$\begin{aligned} \tilde{\ell}_m^n(\theta, X) &= \sum_{k=m}^n \left\{ \log p(X_{t_k} \mid X_{t_{k-1}}, \theta^{\mathbb{P}}) + \sum_{i=1}^{24} \log p(\epsilon_{t_k}^{T_i} \mid \theta^{\epsilon\epsilon}) \right. \\ &+ \left. \sum_{1 \leq i \leq 5, 1 \leq j \leq J_i} \log p(\epsilon_{t_k}^{\tau_i,T_i,j} \mid \theta^{\epsilon\epsilon}) \mathbb{1}_{\{t_k+\tau_i \leq t_n\}} \right\} + \log \pi(\theta), \end{aligned} \quad (21)$$

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<sup>4</sup>The augmented likelihood contains both a filtering (second term in the first line) and a forecasting (first term second line) component. We keep the filtering component in this likelihood, since it is necessary for out-of-sample forecasting. At time  $t_i$  the bond investor can learn about the realizations of the latent state variables only from the time  $t_i$  term structure of interest rates, but not from past forecast errors. However, she needs the current state variables to form her conditional expectations, i.e. to make her forecast.

with  $\pi(\theta_i)$  as in Eq. (19). The first term in the second line of Eq. (21) reflects the excess return forecast errors  $\varepsilon$ , which affect estimates of  $\theta$  and  $X$ .<sup>5,6</sup> We stress here that in the out-of-sample bond investment decision to be made at time  $t_i$ , the investor first samples from the joint distribution of the parameters and latent states through the augmented likelihood Eq. (21) using only forecast error information available *prior* to time  $t_i$ . For each draw of  $\theta$  and  $X$  from this joint distribution she then makes an out-of-sample forecast, records it, and with enough draws (we use 100,000) chooses the sample mean of all recorded forecasts as the forecast to be used in her investment decision.

### 3 Data and Yield Pricing Errors

We obtain monthly interest rate data for Switzerland, Germany, the UK, Japan, and the US from Datastream. The data set comprises money market (Libor) rates with maturities of 1 through 11 months, and swap rates with maturities of 1 to 10 years. We bootstrap riskless zero-coupon yields from these money market and swap rates; [Feldhütter and Lando \(2008\)](#) show that swap rates are the best parsimonious proxy for riskless rates.<sup>7</sup> Given the availability of data, our sample period starts in April 1987 for Germany, the UK and the US, January 1988 for Switzerland, and September 1989 for Japan. For the United States, we also report results using the yield data set of [Sarno et al. \(2007\)](#); this data set covers the period from 1952 to 2003 and the authors also show that it is virtually identical to that of [Campbell and Shiller \(1991\)](#) over the respective period from 1952 to 1987. Our results using these data are thus directly comparable to the large EH literature on the US bond market.

Table 1 summarizes the  $A_0(3)$  models' yield pricing accuracy when using the standard estimation and the extended estimation procedure that also matches risk premia. The fact that the latter has to match 34 risk premia in addition to the 24 yields has, not surprisingly, an

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<sup>5</sup>Augmenting the likelihood function with forecast errors, any information in bond excess returns is absorbed by the latent state variables and the parameters regardless of the drivers. For example, if the data were truly Markovian, these forecasting equations would be irrelevant and would affect neither parameter nor state variable estimates. Note also that the approach chosen is very different from [Cieslak and Povala \(2011\)](#). In their latent variable exercise expected excess returns are treated as observables. In our extended estimation the information from past realized forecast errors is allowed to affect state variable and parameter estimates. This admits a *learning effect*, but we do not consider learning to be built into the conditional expectations directly, a computationally intensive approach taken by [Barberis \(2000\)](#).

<sup>6</sup>Note from Eq. (20) that the procedure of matching risk premia also incorporates information from forward rates, which [Cochrane and Piazzesi \(2005\)](#) find to be an important source of predictability.

<sup>7</sup>To obtain non full-year maturities greater than one year, we use the [Svensson \(1994\)](#) model. This model represents an extension to the approach by [Nelson and Siegel \(1987\)](#) and is used by many central banks to estimate yield curves, for instance the Federal Reserve Board, as described in [Gürkaynak et al. \(2007\)](#). Results are virtually identical when using other interpolation schemes.

impact on yield pricing errors. The standard model has in most cases lower root mean squared pricing errors (RMSEs) ranging from 6 to 24 basis points across countries and maturities as compared to 10 to 24 basis points for the extended estimation. Differences in RMSEs of the extended compared to the standard estimation are largest at the short end of the term structure (bond maturities less than one year) but become very small as the maturity increases, with the exception of Switzerland. Notwithstanding this trade-off in yield pricing accuracy, the extended estimation does a good job in fitting yields across countries; e.g. the pricing errors are smaller than the comparable numbers reported by [Tang and Xia \(2007\)](#) for their best model across various countries.<sup>8</sup> These statistics suggest that both estimation strategies match the term structure of yields satisfactorily. Moreover, for the long US data set we do not find yield pricing errors to be different for the two estimation procedures. The patterns are very similar for standard deviations of pricing errors.

## 4 Forecasting Bond Excess Returns and Economic Value

We now evaluate the statistical accuracy and economic value of bond excess return forecasts generated by ATSMs estimated with the extended procedure, both in- and out-of-sample. Extended estimation forecasts are unbiased predictors for realized excess returns with high regression  $R^2$ s. The forecasts are statistically more accurate compared to using standard estimation forecasts and compared to using the historical average bond excess return as EH-consistent forecast of constant risk premia. Investors are willing to pay a sizable premium to switch from the standard to the extended estimation; however, out-of-sample investors do not benefit compared to using the EH forecast.

### 4.1 Bond Risk Premium Regressions

Table 2 presents results for regressing realized excess returns on model risk premia for 25 combinations of horizons and maturities, for the standard estimation and the extended estimation, respectively.<sup>9</sup> We assess the significance of the slope coefficients  $b$  by calculating standard errors adjusted for autocorrelation and heteroskedasticity based on [Newey and West \(1987\)](#)

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<sup>8</sup>[Tang and Xia \(2007\)](#) report mean absolute pricing errors for maturities of 6 months to 10 years ranging from 17 to 34 basis points for Germany, 26 to 32 for the UK, 18 to 24 for Japan, and 22 to 36 for the US.

<sup>9</sup>As mentioned above, we included 34 combinations in our estimation, which corresponds to all possible combinations that can be formed from the yield maturities we use. The subset of 25 combinations that we report contains the most commonly examined horizon and maturity combinations. The results for the other 9 combinations are qualitatively identical to those reported in the paper, but are not reported to conserve space.

and [Andrews \(1991\)](#); we report statistical significance for the null hypothesis that  $b = 0$  and also for  $b = 1$ .

For the standard estimation, the results in Panel A reveal that most slope estimates are positive and many are close to one. However, most of them are neither different from zero (with rejection of the null  $b = 0$  indicated by the asterisks) nor from one (where rejection of the null  $b = 1$  is indicated by the diamonds), with the exception of the long US data set. Furthermore, there is large cross-country variation in the model's explanatory power, with  $R^2$ s being largest for Japan and the long US data set. Across countries and longer-term bond maturities, the average one-month and one-year prediction horizon  $R^2$ s are 3% and 8%, respectively.

The results for the extended estimation in Panel B show that model risk premia are generally unbiased predictors of realized excess returns. Almost all estimates are different from zero at a high level of significance and in most cases we cannot reject the null hypothesis that  $b = 1$ . The results also show that model-implied risk premia have high explanatory power for realized excess returns. In general, the explanatory power increases with horizon, at least up to a horizon of one year. At the one-month prediction horizon the average  $R^2$  across countries and maturities is around 26%, and at the one-year horizon it is around 79%.

Overall, we find that the extended model estimation dominates the standard estimation in terms of explanatory power for realized risk premia. These extended estimation results are consistent with previous research documenting that bond excess returns are predictable at shorter and longer horizons (see e.g. [Cochrane and Piazzesi, 2005](#); [Ludvigson and Ng, 2009](#); [Mueller et al., 2011](#)) and that this predictability is to a large extent not spanned by the term structure of bond yields and thus not captured in standard ATSM estimations (see e.g. [Duffee, 2011](#)). The finding that model expectations are unbiased is in line with research showing that accounting for risk premia from ATSMs can explain why coefficients of classical EH regressions suggest a rejection of the EH (see e.g. [Dai and Singleton, 2002](#)).<sup>10</sup> In what follows, we take a closer look at the relative forecast accuracy of the two estimation strategies and compute measures for the economic value that accrues to investors using the extended instead of the standard estimation procedure and relative to EH-consistent constant risk premium forecasts.

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<sup>10</sup>Note that this is true for ATSMs independent of the specification and estimation procedure. Our results show that regression coefficients based on standard estimation-implied risk premia are neither different from zero nor from one, whereas extended estimation-implied risk premia reflect unbiased and significant predictors.

## 4.2 Statistical Accuracy of Bond Excess Return Forecasts

To evaluate the accuracy of extended estimation forecasts against the standard estimation and the EH constant risk premium benchmarks, we report values for a  $R2$ -metric defined similarly as in [Campbell and Thompson \(2008\)](#):

$$R2 \equiv 1 - MSE^m / MSE^b, \quad (22)$$

where  $MSE^k = 1/(N - \tau + 1) \sum_{t=1}^{N-\tau} (rx_{t+\tau}^T - \mathbb{E}_t^{\mathbb{P},k} [rx_{t+\tau}^T])^2$  denotes the mean squared forecast error of the model ( $k = m$ ) and the benchmark ( $k = b$ ), respectively.  $R2$  takes positive values when forecasts from model  $m$  are more accurate than those from benchmark model  $b$  and negative values when the opposite is the case.<sup>11</sup> To judge the significance of  $R2$ -statistics, we estimate confidence intervals as the 5%- and 95%-percentiles using a block bootstrap procedure.

The results in [Table 3](#) reveal that the extended estimation produces more accurate forecasts than the benchmarks, both in- and out-of-sample. Panel A shows that the in-sample  $R2$  estimates of the extended versus the standard estimation are positive in 146 of 150 horizon/maturity combinations with estimates being significant in most cases (as indicated by the \*). Only a single forecast, the 5-year forecast of the 1-month excess return in Japan, is more accurate when using the standard estimation procedure (as indicated by the °). The extended estimation forecasts are also more accurate than constant risk premium forecasts with 135 of 150  $R2$  estimates being positive and 114 of these  $R2$  values being statistically significant. Out-of-sample (Panel B), the extended estimation beats the benchmarks as well, with  $R2$  estimates versus the standard estimation being positive (and significant) in 124 (68) of 150 horizon/maturity combinations whereas only 26 (9) are negative. Using the EH benchmark,  $R2$ s are positive in 91 (30) combinations while  $R2$  are negative in 59 (27) cases.

The results show that the extended estimation picks up information relevant for predicting bond risk premia that is hidden to affine models that are estimated by only fitting yields. Accounting for the information in forward rates and past bond excess returns substantially improves the model's forecast accuracy in- and out-of-sample. Moreover, the model's forecasts are more accurate than constant risk premium forecasts in 90% of horizon/maturity combinations in-sample and in 61% out-of-sample, suggesting that the model beats the EH from a statistical perspective.

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<sup>11</sup>Note that many common measures of predictive ability are based on squared loss functions (e.g. [Diebold and Mariano, 1995](#)) and therefore lead to the same conclusions that we reach in this paper using  $R2$ .

### 4.3 Economic Value of Bond Excess Return Forecasts

We now investigate whether the superior predictive ability of the extended estimation compared to benchmark forecasts translates into economic benefits for bond investors. We evaluate optimal bond portfolios within the quadratic utility framework of [West et al. \(1993\)](#).<sup>12</sup> For an investment horizon  $\tau$ , the investor chooses to allocate his wealth between bonds with maturities  $\tau$  and  $T > \tau$ . Since the maturity of the shorter-term bond exactly matches the investment horizon, the  $\tau$ -bond represents the risk-free asset. The longer-term  $T$ -bond, with remaining maturity  $T - \tau$  at the end of the horizon, represents the risky asset. Let  $\mu_{t+\tau}^{T,k}$  denote the  $N \times 1$  vector of conditional expectations of risky asset returns generated by model  $k$  and denote the associated covariance matrix by  $\Sigma_{t+\tau}$ . For a given target volatility  $\sigma^*$ , we maximize the portfolio excess return to obtain the  $N \times 1$  vector of mean-variance optimal portfolio weights ( $w_t^k$ ) for the risky asset

$$w_t^k = \frac{\sigma^*}{C_t} \Sigma_{t+\tau}^{-1} \mu_{t+\tau}^{T,k},$$

where  $C_t = \mu_{t+\tau}^{T,k\top} \Sigma_{t+\tau}^{-1} \mu_{t+\tau}^{T,k}$ . The weights of the riskless asset are given by  $\mathbf{1} - w_t^k$ , where  $\mathbf{1}$  is a  $N \times 1$  vector of ones and the resulting gross portfolio return from  $t$  to  $t + \tau$  is given by  $R_{t+\tau}^k = 1 + y_t^\tau + w_t^k \cdot r x_{t+\tau}^T$ .

To measure the economic value generated by model  $m$  over model  $b$ , we compute the performance measure  $\Theta$  proposed by [Goetzmann et al. \(2007\)](#).  $\Theta$  quantifies the risk-adjusted premium return that the portfolio based on forecasts from model  $m$  earns in excess of the benchmark portfolio and is calculated as

$$\Theta = \frac{12}{(1 - \rho)\tau} \ln \left( \frac{1}{N - \tau + 1} \sum_{t=1}^{N-\tau} [(1 + R_n^m)/(1 + R_n^b)]^{1-\rho} \right), \quad (23)$$

where  $\rho$  denotes the coefficient of relative risk aversion. In contrast to the commonly reported Sharpe ratio,  $\Theta$  alleviates concerns related to non-normality and related to underestimation of dynamic strategy performance. Furthermore, compared to the performance fee of [Fleming et al. \(2001\)](#) it does not assume a specific utility function. We repeated the empirical analysis using the performance fee and find qualitatively identical and quantitatively very similar results as we do for  $\Theta$  (not reported to conserve space). Throughout the empirical analysis we set the target volatility of the portfolio investor to  $\sigma^* = 2\%$  *p.a.* and the coefficient of relative

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<sup>12</sup>For US bond markets, this approach is used by, e.g., [Della Corte et al. \(2008\)](#) and [Thornton and Valente \(2012\)](#).

risk aversion to  $\rho = 3$ . All our results are robust to choosing different values of  $\sigma^*$ , varying  $\rho$  between 2 and 6, as well as short sale constraints.

We report portfolio excess returns of investors using forecasts from the extended estimation and performance measures relative to the standard estimation and EH forecasts in Table 4. The results show that bond excess returns increase with the maturity of the longer-term bond and decrease with prediction horizon. In-sample (Panel A), portfolio excess returns are positive for all horizon/maturity combinations. Relative to the standard estimation,  $\Theta$  values are positive and suggest a superior performance in 127 out of 150 horizon/maturity combinations, four  $\Theta$  values are negative, and the remaining 19 estimates are zero. Premium returns in excess of portfolios based on EH forecasts are also positive in 114 of the 150 horizon/maturity combinations, while negative only for 11 of the combinations, suggesting a rejection of the EH also in terms of economic significance.  $\Theta$  estimates increase with the maturity of the longer-term bond but decrease with prediction horizon, suggesting that, in particular, EH deviations over longer horizons are of limited relevance in economic terms.

In the out-of-sample analysis (Panel B), we find that portfolios allocated based on extended estimation forecasts deliver positive excess returns in 140 of 150 horizon/maturity combinations.  $\Theta$  estimates relative to the standard estimation are positive in 123 of the 150 combinations. With the exception of Switzerland, we find that the economic value added by the extended over the standard estimation tends to increase with bond maturity and to decrease with forecast horizon. For Switzerland, the standard estimation short-term forecasts deliver higher portfolio returns for 3 out of 5 bond maturities but long-horizon investors earn a risk-adjusted premium of 2% *p.a.* to 3% *p.a.* when they switch to the extended estimation strategy. For all other countries, bond investors earn highest premium returns for short-horizon long-term bond portfolios, ranging from approximately 2% *p.a.* to around 4.8% *p.a.* Relative to the EH, however, the  $\Theta$  estimates are positive in only 39 of 150 cases, negative in 79 cases, and zero in 32 cases. Thus, in contrast to the statistical predictability results above, these findings do not suggest that using extended estimation instead of constant risk premium forecasts adds economic value for bond investors.

Overall, these results suggest that the information hidden to affine models estimated with the standard procedure but captured through the extended procedure results in economic gains for bond investors. Relative to the EH, however, bond investors only earn premium returns in-sample whereas the EH cannot be outperformed - in an economic sense - out-of-sample.



## 4.4 Can anything beat the EH?

Our results show that extending ATSM estimations beyond fitting yields to additionally match past excess returns captures information otherwise unspanned or hidden to standard ATSM estimations (Duffee, 2011). The extension leads to a substantial improvement in forecast accuracy for bond excess returns that translates into economic gains for portfolio investors who would be willing to pay fees in the range of 2% *p.a.* to 4.8% *p.a.* to switch from the standard to the extended estimation.

More generally, model risk premia generate unbiased bond excess return predictions that entail high explanatory power for EH deviations; the average  $R^2$ s of 79% for one-year excess returns is beyond that of forward rate-based forecasts (Cochrane and Piazzesi, 2005). To evaluate the EH postulate of constant risk premia, we use the historical average bond excess return as a benchmark predictor. While the model beats EH forecasts in terms of statistical accuracy in- and out-of-sample, investors do not gain economic value from model forecasts out-of-sample. The finding that bond investors generally cannot benefit from using conditional risk premia as compared to using the historical average can be viewed as the bond market analogue to the result of Goyal and Welch (2008) for stock markets.

## 5 Discussion of Results, Extensions, and Robustness Checks

We first show why statistical and economic criteria may lead to apparently conflicting conclusions about the validity of the EH. Subsequently, we discuss the potential benefits of augmenting ATSM estimations by imposing EH priors. Finally, we summarize various robustness checks.

### 5.1 Statistical Accuracy versus Economic Value

While there are many papers on predictability of bond risk premia that are concerned with statistical forecast accuracy, it is important to note that statistical accuracy *per se* does not imply economic value for bond investors. Our results above indeed suggest that the EH is rejected from a statistical but not from an economic value perspective: 61% of model forecasts are more accurate than the EH but only 26% of forecasts add economic value. We present general, model-free arguments as to why there may be a gap between statistical and economic

significance and evaluate our model results along these lines. These arguments are also useful when interpreting the results of other papers that study the predictability of bond risk premia using various forecasting approaches.

### 5.1.1 Economic Relevance of EH Deviations

One reason for apparently conflicting results is that departures from the EH might be statistically significant but too small to be exploited by bond investors. In other words, failure to generate economic value may not imply that a model fails to capture EH deviations accurately but rather that the EH holds in an economic sense. Since there is no “natural” upper bound for economic value measures (similar to a regression  $R^2$  capped by one or forecast errors floored by zero), we compare the economic performance of model forecasts to the performance of the same strategy under perfect foresight. If perfect foresight returns of the strategy are high but the model evaluated only captures a (small) fraction of these excess returns, EH deviations are not exploited because the model fails. If the model captures a large fraction of perfect foresight returns but returns are nevertheless economically small, this suggests that “true” EH deviations are indeed economically irrelevant.<sup>13, 14</sup>

To get a feeling for the economic relevance of EH deviations, we plot average excess returns of buy-and-hold and perfect foresight portfolio investors in Figure 1. Buy-and-hold excess returns capturing constant risk premia exhibit very similar patterns across countries in that returns increase with maturity and decrease with forecast horizon. The patterns are very similar for perfect foresight investors but with average excess returns on a higher level. It is more valuable for investors to accurately predict short-horizon as compared to long-horizon bond excess returns. For instance, in the long US data, investors buying and holding the long-term bond ( $T - \tau = 60$  months) over horizons of  $\tau = 1, 12,$  and  $60$  months earn average excess returns of 3.12%, 1.25%, and 0.40% *p.a.*. The perfect foresight excess returns for the same horizon/maturity combinations are 14.41%, 4.83%, and 3.05% *p.a.* This shows that EH

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<sup>13</sup>For a simple investment strategy that just goes long when the expected excess return is positive and short when the expected excess return is negative, the returns of the strategy based on model forecasts relative to using perfect foresight forecasts would be bounded by minus one and plus one. For optimal portfolio investors, model-based returns could exceed those of perfect-foresight-based portfolios; however, this would imply a less than optimal risk-return-trade-off. Using relative risk-adjusted returns, i.e. the performance measure  $\Theta$  of model forecasts over constant risk premium forecasts relative to the  $\Theta$  of perfect foresight over constant risk premium forecasts, leads to qualitatively the same conclusions that we report for returns below in Figure 2.

<sup>14</sup>Note that even models that perfectly capture risk premia may not generate an economic performance equal to that based on perfect foresight because departures from the EH may not be exclusively driven by (predictable) risk premia. Similarly, in the presence of noise or other determinants of EH deviations, it would not be possible to achieve an  $R^2$  of 1 with perfect risk premium predictions in regressions of realized excess returns.

deviations are economically less important for increasing  $\tau$  and, as a consequence, having a less than perfect forecast model for short horizons may add more economic value than a perfect forecast model for longer horizons.

In Figure 2, we plot the excess returns of portfolios allocated using forecasts based on constant risk premia (in green), the standard estimation (in red), and the extended estimation (in black) relative to perfect foresight portfolio returns. The graphs show that EH deviations are not as important economically as statistical results might suggest: EH-consistent constant risk premium forecasts capture a large fraction (increasing with horizon) of perfect foresight returns. The extended estimation forecasts capture a larger fraction of perfect foresight returns than standard estimation forecasts. The fraction of perfect foresight returns captured by extended estimation forecasts generally increases with horizons, similar to the regression  $R^2$ s in Table 2 and  $R^2$ -statistics in Table 3. In contrast, the economic value measures reported in Table 4 decrease with horizon, consistent with comparably lower statistical accuracy at short horizons adding higher economic value than more accurate forecasts for longer horizons at which EH deviations are not relevant in economic terms. In other words, statistical accuracy cannot lead to economic value when EH deviations are too small to be exploited by investors.<sup>15</sup>

### 5.1.2 Information in Economic Value versus Statistical Accuracy Measures

Conflicting conclusions based on metrics of statistical accuracy and economic value may also result from the construction of the measures used. Common measures of predictive ability are based on loss functions involving squared or absolute forecast errors and – by definition – ignore the sign of forecast errors. Getting the sign right, however, is of utmost importance for investors since the sign of the forecast determines whether to take a long or a short position. To illustrate this point, consider two competing forecasts of excess returns being -1% and 8% and suppose the realization is 2%. Standard measures of predictive ability consider the forecast of -1% more accurate because the absolute error is just half of that of the 8% forecast. In terms of investment performance, however, the first forecast would have resulted in a loss while the second would have resulted in a positive performance.

As a measure of directional accuracy, we compute hit ratios measuring the fraction of correctly signed forecasts. Table 5 reports the hit ratios of the extended estimation relative to

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<sup>15</sup>Consider, for instance, the one-year out-of-sample forecasts in Japan: extended estimation forecasts have high accuracy ( $R^2$  from 0.11 to 0.70), capture more than 90% of perfect foresight returns, but their the economic value is close to zero (-2 to +7 basis points *p.a.*) because EH deviations are very small and constant risk premia already capture around 80% of perfect foresight performance.

the hit ratios of constant risk premium forecasts, with asterisks (circles) indicating that model hit ratios are significantly higher (lower) than those of constant risk premium forecasts. While the model generally has high directional accuracy in-sample, model hit ratios exceed constant risk premium hit ratios only in 49 of the 150 horizon/maturity combinations out-of-sample. This contrasts with the  $R2$  results in Table 3 where 91 of 150 combinations suggested that the model has higher predictive ability compared to constant risk premium forecasts. Thus, our finding that the economic value analysis is more in favor of the EH than the statistical accuracy results can partly be explained by forecasts having small squared/absolute errors but nonetheless pointing in the wrong direction.<sup>16</sup>

To further gauge the relation between statistical versus economic significance, we plot constant risk premium forecast errors (black circles) and model forecast errors (red crosses) against realized excess returns for the long US data set in Figure 3. The shaded areas represent areas where forecasts have the wrong sign and hence forecast errors that lead to bond portfolio losses. Differences in hit ratios are thus reflected in different numbers of observations falling in the shaded economic loss areas. While standard predictive ability measures are only concerned with the distribution of forecast errors across the x-axis in absolute terms, the economic value accruing to investors depends on the forecast errors' joint distribution with realizations (on the y-axis): the magnitude of signed forecast errors as well as their dispersion, kurtosis, and skewness matter. The graphs reveal that the distribution across the x-axis is relatively similar for model and constant risk premium forecasts for most of the 25 horizon/maturity combinations but also that the model forecast errors exhibit a larger dispersion across the y-axis. These patterns explain why statistical predictability of bond excess returns does not (necessarily) map into economic gains for bond investors.<sup>17</sup>

## 5.2 Extension: Imposing EH Priors on Affine Models

Given the empirical result that bond investors using (extended estimation) ATSM instead of EH forecasts suffer economic losses in 60% of horizon maturity/combinations, it is natural to ask whether imposing EH restrictions on ATSMs may improve performance. Despite the fact that model-implied risk premia can be decomposed into a constant and a time-varying

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<sup>16</sup>See, e.g., the 3-month out-of-sample forecasts in the long US data set for bonds with  $T - \tau$  up to two years: while the  $R2$ s of the model estimates are positive (Table 3), the model hit ratios are lower than those of constant risk premium forecasts, leading to negative estimates of economic value (Table 4).

<sup>17</sup>Examples that illustrate this point include the 1- and 3-month forecasts of 5-year bond risk premia where  $R2$ s are positive, model hit ratios are equal and slightly higher than constant risk premium hit ratios but portfolio returns are nevertheless lower and economic value added is negative.

component, see Eq. (15), it can be shown that it is not possible to impose parametric EH restrictions on ATSMs in general, but only in special cases that are very restrictive and hinder realistic modeling.<sup>18</sup> In a Bayesian setting, however, the EH can be imposed in a “soft way”, through a prior distribution rather than a hard parameter restriction. Consider

$$\log \pi(\theta^{\text{QP}}) \propto -\frac{1}{2}\gamma^{\tau,T}(\theta_{\text{QP}})\Xi\gamma^{\tau,T}(\theta_{\text{QP}})^{\top}, \quad (24)$$

with  $\Xi$  positive semi-definite. Depending on the specification of  $\Xi$ , this prior arbitrarily reduces or amplifies, for a given sample path of the latent state variables, the time-variability of risk premia across maturities. More specifically, the prior imposes a penalty on  $\gamma$ , the coefficient that controls the time-variation of expected excess returns in Eq. (15), with the penalty increasing in the determinant of  $\Xi$ . Intuitively, the prior should prevent overfitting (Duffee, 2010) and alleviate related concerns on parameter uncertainty (Feldhütter et al., 2012): while the prior does not directly restrict the variance of expected risk premia (which also depends on the variance of the state variables), it penalizes all parameters constellations that make expected risk premia excessively time-varying. This approach can be viewed as a soft version of excluding economically unrealistic model outputs, in the spirit of Duffee (2010).

In a preliminary exercise, we repeat the out-of-sample analysis for the long US data set using the extended estimation procedure with a “weak” and a “strong” EH prior and present results in Table 6.<sup>19</sup> Imposing the weak EH prior (Panel A) substantially augments predictive accuracy for horizons of one year or longer and leads to higher hit ratios in 24 of 25 horizon/maturity combinations compared to the estimation without prior; in nine cases these hit ratios exceed those using constant risk premium forecasts. Furthermore, there is an increase in bond portfolio excess returns and in economic value across horizons and maturities. While for the estimation without prior all  $\Theta$  estimates are negative (reported above in Table 4), we find positive estimates for ten horizon/maturity combinations with the weak prior. Nevertheless, the economic value added by conditioning on an affine model instead of constant risk premium forecasts is on average very small.

For the strong EH prior, we also find that predictive accuracy improves for horizons of one

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<sup>18</sup>First, the pure EH can be imposed when the term structure is flat and mean reversion in the state variables is infinitely fast. Infinite speed of mean reversion is in strong contrast to the empirical observation that interest rates are very persistent. Second, the EH can be imposed on a subset of yields by numerically finding the roots of  $\gamma$  and  $\eta$  from Eqs. (13) and (14), but not the entire yield curve. Proofs are omitted from the paper but available on request.

<sup>19</sup>We define the “weak” and “strong” prior with  $\Xi$  identity matrix times 10,000 and times 1e12, respectively.

year and beyond. The  $R^2$ -statistics are comparably small in absolute value, suggesting that model and constant risk premium forecasts errors are of similar magnitude. The hit ratios are very similar to those using constant risk premia, being equal in eleven cases and slightly higher or lower in seven cases each. Bond portfolio returns and economic value increase with the imposition of the strong EH prior, and the  $\Theta$  estimates averaged across horizons and maturities are slightly positive (albeit often close to zero). Finding that the predictive accuracy and hit ratios of strong EH prior and constant risk premium forecasts are very similar and that the economic value is close to zero illustrates how one can approach the aforementioned “soft” imposition of the EH on ATSMs in our Bayesian setup.<sup>20</sup>

Overall, we find that imposing EH priors increases the economic value generated by ATSM forecasts. We leave the precise specification of the prior that balances EH-imposed penalties versus variability in risk premia for future research.

### 5.3 Robustness Checks

We now summarize various robustness checks that support our findings. The results are not reported to save space but available from the authors on request.

#### 5.3.1 Alternative Yield Data

We use bond prices (from Datastream) directly to estimate zero yields using the approaches of Nelson and Siegel (1987) and Svensson (1994), as well as the smoothed and unsmoothed versions of the Fama and Bliss (1987) method. The results are qualitatively identical to those reported above. We also use term structure data provided by central banks (for countries where data is available) and reach the same conclusions. Thus, our findings do not depend on the mechanism used to estimate the zero curve in general and, more specifically, our conclusions are not affected by credit risk issues that have become relevant in Libor and swap markets during the recent crisis. The latter argument is also supported by the results reported above for the US yield data set from 1952 to 2003 since this data does not involve Libor or swap rates and the sample ends well before the recent crisis.

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<sup>20</sup>Given such a soft imposition of the EH, one might also consider testing the EH in a generalized version of the bivariate VAR framework of Bekaert and Hodrick (2001) using ATSMs to jointly model not only a pair of yields but the full term structure (under no-arbitrage).

### 5.3.2 Alternative ATSM Specifications

We verify that our conclusions are robust to changes in the ATSM specification and repeat the empirical analysis using a larger model with four factors ( $A_0(4)$  model) and a stochastic volatility model ( $A_1(3)$  model). In general, changing the specification can have an impact on yield pricing errors and/or forecast accuracy. We find that changing the specification may improve or deteriorate particular results but the overall picture does not change and our conclusions remain the same.

### 5.3.3 Forecasting Bond Excess Returns with Forward Rates

Previous research documents (in-sample) predictability of bond excess returns using lagged forward rates; see e.g. [Fama and Bliss \(1987\)](#) and [Cochrane and Piazzesi \(2005, CP\)](#). While we do not impose the CP-factor in the model structure, the extended estimation procedure that matches model risk premia to the data incorporates forward rates that the CP-factor is based upon and it additionally accounts for past forecast errors. In line with previous research, we find that forward rates contain information for in-sample predictions of bond excess returns but do not generate economic value out-of-sample ([Thornton and Valente, 2012](#)). Forecasts based on the extended ATSM estimation proposed in this paper have larger predictive ability and add more economic value than the CP-factor forecasts in- and out-of-sample, thus, posing a stronger challenge to the EH and thereby providing more general findings.

## 6 Conclusion

In this paper, we offer new insights on the validity of the expectations hypothesis (EH) by studying the economic benefits that accrue to bond investors who exploit predictable deviations from the EH. We estimate conditional bond risk premia using affine term structure models (ATSMs) by employing a novel estimation strategy that jointly fits the term structure of model yields to the observed yield curve and additionally matches model risk premia with bond excess returns observed in the past. This extended procedure allows investors to capture predictive information beyond the cross section of yields and to update beliefs about the model's predictive ability based on its past performance. We use the model to generate forecasts of bond excess returns and, based on these forecasts, we determine optimal bond portfolios. To evaluate the model against the EH, we compare the model's forecast accuracy

and corresponding portfolio performance to EH-consistent forecasts and accordingly allocated benchmark portfolios, where we use averages of historical bond excess returns to consistently estimate constant risk premia as postulated by the EH.

We find that, for 25 combinations of horizons and maturities ranging from one month to ten years, bond risk premia have very similar properties across countries, leading to uniform conclusions for the US, Switzerland, Germany, the UK, and Japan. We show that the extended estimation captures predictive information otherwise hidden to standard ATSM estimations, thereby providing investors with forecasts that are statistically more accurate and economically more valuable; out-of-sample, investors would be willing to pay an annual premium in the range of 2% to 4.8% to switch from the standard to the extended estimation procedure. More generally, regressing realized on model-implied excess returns reveals that extended estimation forecasts are unbiased and have high explanatory power with  $R^2$ s of about 26% at the one-month prediction horizon and about 79% at the one-year horizon. From a statistical perspective, the model beats the EH forecasts of constant risk premia as judged by standard metrics of predictive accuracy in- and out-of-sample. Furthermore, portfolios allocated based on the extended estimation forecasts earn positive excess returns; however, out-of-sample these portfolios perform worse than the corresponding EH benchmark portfolios. In other words, investors cannot beat the historical average, which suggests that our findings can be viewed as a bond market analogue to [Goyal and Welch \(2008\)](#).

At first sight, our results may appear to offer conflicting conclusions for statistical and economic assessments of the EH. We show that this finding is not rooted in the use of ATSMs but potentially applies to any approach for modeling and predicting bond risk premia. We demonstrate that there is a wedge between the statistical and economic relevance of EH deviations for two reasons. First, departures from the EH can be statistically significant but too small to be exploited by investors, in particular over longer horizons. Second, metrics of statistical accuracy evaluate loss functions that are in many respects unrelated to the economic success of bond investments. As such, even models with high regression  $R^2$ s or measures of predictive ability *per se* cannot guarantee to provide bond investors with economic gains relative to presuming that the EH holds.

Overall, our results suggest that the EH presumption of constant risk premia, while being statistically rejected by the data, still provides a good first approximation to the out-of-sample behavior of bond excess returns for the purpose of asset allocation in fixed income markets, especially so over long forecast horizons. This finding is in line with the EH, despite being cen-



turies old, having remained a benchmark for a number of practical purposes in many financial firms and policy institutions, e.g. for extracting information about future inflation, interest rates, and economic activity. At the same time, ATSMs are well established in the literature because of their virtues in modeling interest rates. Taken together, these insights suggest to examine models of bond risk premia that grant flexibility in their specification but account for the EH as an anchor. As a first step in this direction, we consider ATSMs on which we impose the EH through priors in the estimation in order to limit (excessive) variability of risk premia. The results of this preliminary exercise suggest that imposing EH priors improves the performance of bond portfolios and we leave it to future research to further explore modeling and estimation approaches that balance flexibility and economically reasonable restrictions.

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**Table 1: Yield Pricing Errors.**

Yield Maturity		All $T$	$1m \leq T < 12m$	$12m \leq T < 60m$	$60m \leq T \leq 120m$
Number of Yields		24	7	10	7
Switzerland					
RMSE	standard estimation	6.80	7.54	6.77	6.03
	extended estimation	11.92	12.49	11.21	12.30
Sd	standard estimation	6.80	7.54	6.77	6.03
	extended estimation	11.92	12.49	11.21	12.29
Germany					
RMSE	standard estimation	7.88	8.64	7.07	8.17
	extended estimation	19.74	33.34	10.02	8.96
Sd	standard estimation	7.88	8.65	7.06	8.11
	extended estimation	19.74	33.34	10.01	8.96
United Kingdom					
RMSE	standard estimation	11.02	15.95	8.94	6.89
	extended estimation	23.48	39.72	12.05	10.27
Sd	standard estimation	11.01	15.94	8.93	6.88
	extended estimation	23.46	39.61	12.03	10.22
Japan					
RMSE	standard estimation	5.84	5.63	5.86	6.02
	extended estimation	10.46	15.94	7.12	6.98
Sd	standard estimation	5.84	5.63	5.85	6.03
	extended estimation	10.45	15.90	7.12	6.98
United States					
RMSE	standard estimation	10.16	13.18	9.28	7.53
	extended estimation	15.85	25.36	10.65	7.48
Sd	standard estimation	10.15	13.17	9.28	7.41
	extended estimation	15.85	25.31	10.62	7.48
United States (long data set)					
RMSE	standard estimation	23.62	41.41	9.41	8.40
	extended estimation	23.61	41.22	9.88	8.55
Sd	standard estimation	23.61	41.41	9.37	8.32
	extended estimation	23.61	41.11	9.83	8.54

*Notes:* The Table summarizes root mean squared yield pricing errors and standard deviations of yield pricing errors for the standard estimation (the estimation procedure only fitting yields) and the extended estimation (the estimation procedure fitting yields and matching model risk premia to bond excess returns observed in the past) of the  $A_0(3)$  model. We estimate the models using monthly data from September 1989 for Japan, January 1988 for Switzerland, and April 1987 for Germany, UK, and US. The sample period ends in March 2011 for all countries. For the US we also report results for a longer data set covering the period from January 1952 to December 2003.

Table 2: Time-Varying Risk Premium Regressions.

Panel A: Standard Estimation Procedure							Panel B: Extended Estimation Procedure								
$T - \tau$			1m	3m	12m	24m	60m	$T - \tau$			1m	3m	12m	24m	60m
Switzerland							Switzerland								
$\tau$	1m	b	0.44	0.74*	0.51	1.33***	0.85***	$\tau$	1m	b	0.98***	0.90***	0.96***	1.21***	1.05***
		$R^2$	0.01	0.02	0.01	0.07	0.04			$R^2$	0.25	0.26	0.29	0.31	0.23
	3m	b	0.54	0.52	0.54	0.85*	0.64**		3m	b	0.85***	0.87***	1.00***	1.16***	1.07***
		$R^2$	0.03	0.03	0.03	0.06	0.06			$R^2$	0.54	0.57	0.61	0.60	0.52
	12m	b	-0.05	0.28	0.28	0.46	0.45		12m	b	0.98***	1.00***	1.02***	1.11***	1.04***
		$R^2$	0.00	0.01	0.01	0.04	0.06			$R^2$	0.73	0.88	0.84	0.87	0.82
	24m	b	0.86	0.35	0.10	0.08	0.14 $\circ$		24m	b	1.14***	1.02***	0.94***	0.94***	0.89***
		$R^2$	0.07	0.02	0.00	0.00	0.01			$R^2$	0.42	0.65	0.62	0.61	0.62
	60m	b	-0.04 $\circ$	-0.33 $\circ$	-0.41 $\circ$	-0.30 $\circ$	-0.20 $\circ$		60m	b	0.59	0.70***	0.71***	0.61**	0.59 $\circ$
		$R^2$	0.00	0.07	0.14	0.09	0.06			$R^2$	0.06	0.27	0.35	0.29	0.37
Germany							Germany								
$\tau$	1m	b	0.37 $\circ$	0.50	-0.27 $\circ$	0.12	0.51	$\tau$	1m	b	0.91***	0.79***	0.87***	1.16***	1.14***
		$R^2$	0.01	0.02	0.00	0.00	0.01			$R^2$	0.22	0.20	0.22	0.27	0.28
	3m	b	0.48	0.44	0.15	0.38	0.61		3m	b	0.80***	0.83***	0.99***	1.15***	1.04***
		$R^2$	0.03	0.02	0.00	0.01	0.02			$R^2$	0.33	0.40	0.51	0.55	0.49
	12m	b	-0.65 $\circ$	-0.06	0.20	0.40	0.61		12m	b	1.02***	1.07***	1.14***	1.20***	1.02***
		$R^2$	0.06	0.00	0.01	0.02	0.06			$R^2$	0.60	0.84	0.83	0.84	0.77
	24m	b	0.10	0.05	0.07	0.19	0.41		24m	b	1.28***	1.20***	1.11***	1.06***	0.90***
		$R^2$	0.00	0.00	0.00	0.01	0.06			$R^2$	0.30	0.56	0.58	0.56	0.53
	60m	b	0.44	0.01 $\circ$	-0.11 $\circ$	0.01 $\circ$	0.18 $\circ$		60m	b	0.96	0.83**	0.60***	0.52***	0.46***
		$R^2$	0.04	0.00	0.01	0.00	0.05			$R^2$	0.07	0.18	0.17	0.16	0.18
United Kingdom							United Kingdom								
$\tau$	1m	b	1.16**	0.90**	1.30***	1.56***	1.26**	$\tau$	1m	b	0.93***	0.86***	1.08***	1.19***	0.91***
		$R^2$	0.02	0.02	0.04	0.06	0.03			$R^2$	0.10	0.20	0.23	0.28	0.18
	3m	b	0.73*	0.80*	0.78*	0.87*	0.76		3m	b	0.91***	0.92***	1.01***	1.04***	0.90***
		$R^2$	0.03	0.04	0.04	0.05	0.03			$R^2$	0.44	0.46	0.47	0.51	0.38
	12m	b	1.00**	0.64	0.31	0.26	0.37		12m	b	1.25***	1.13***	1.12***	1.10***	0.97***
		$R^2$	0.08	0.05	0.01	0.01	0.02			$R^2$	0.62	0.85	0.86	0.86	0.78
	24m	b	0.77	0.32	0.00 $\circ$	0.13 $\circ$	0.30 $\circ$		24m	b	1.48***	1.23***	1.05***	0.98***	0.84***
		$R^2$	0.05	0.01	0.00	0.00	0.02			$R^2$	0.53	0.66	0.57	0.56	0.53
	60m	b	0.14 $\circ$	0.03 $\circ$	0.04 $\circ$	0.25 $\circ$	0.48***		60m	b	0.24 $\circ$	0.44 $\circ$	0.47	0.48	0.46***
		$R^2$	0.00	0.00	0.00	0.04	0.20			$R^2$	0.01	0.12	0.17	0.23	0.37
Japan							Japan								
$\tau$	1m	b	1.18***	1.18**	0.39	0.33	0.96**	$\tau$	1m	b	0.60***	0.77***	1.04***	1.13***	1.34***
		$R^2$	0.09	0.06	0.01	0.01	0.05			$R^2$	0.25	0.41	0.50	0.37	0.34
	3m	b	1.26*	0.86	0.76	0.81	1.05**		3m	b	0.72***	0.83***	1.07***	1.22***	1.30***
		$R^2$	0.12	0.05	0.07	0.09	0.13			$R^2$	0.54	0.69	0.82	0.75	0.64
	12m	b	0.11	0.50	0.81	0.85	0.83*		12m	b	1.24***	1.11***	1.14***	1.14***	1.08***
		$R^2$	0.00	0.06	0.22	0.29	0.33			$R^2$	0.61	0.78	0.81	0.80	0.70
	24m	b	-0.03 $\circ$	0.41 $\circ$	0.69	0.75*	0.76***		24m	b	0.08 $\circ$	0.71***	1.07***	1.07***	0.97***
		$R^2$	0.00	0.13	0.30	0.38	0.48			$R^2$	0.00	0.51	0.87	0.87	0.74
	60m	b	0.89***	0.65	0.53	0.62	0.62		60m	b	0.26 $\circ$	0.88***	1.11***	1.13***	1.08***
		$R^2$	0.16	0.27	0.18	0.25	0.28			$R^2$	0.02	0.60	0.93	0.92	0.82
United States							United States								
$\tau$	1m	b	0.15 $\circ$	0.36 $\circ$	1.13*	-0.40	0.77*	$\tau$	1m	b	0.73***	0.74***	1.00***	1.11***	0.98***
		$R^2$	0.00	0.00	0.02	0.00	0.01			$R^2$	0.16	0.21	0.29	0.26	0.21
	3m	b	0.33 $\circ$	0.31	0.84	0.13	0.55		3m	b	0.83***	0.86***	1.02***	1.09***	0.97***
		$R^2$	0.01	0.01	0.02	0.00	0.01			$R^2$	0.46	0.52	0.59	0.56	0.45
	12m	b	0.88	0.85	1.42	0.95	0.72		12m	b	1.38***	1.22***	1.20***	1.16***	0.96***
		$R^2$	0.03	0.04	0.10	0.05	0.05			$R^2$	0.65	0.85	0.86	0.86	0.74
	24m	b	-1.97	0.11	1.19	0.92	0.70		24m	b	1.20***	1.27***	1.25***	1.14***	0.91***
		$R^2$	0.12	0.00	0.12	0.09	0.10			$R^2$	0.21	0.55	0.61	0.60	0.58
	60m	b	0.16	0.43	0.40	0.36	0.33 $\circ$		60m	b	0.12	0.36	0.31 $\circ$	0.27 $\circ$	0.30 $\circ$
		$R^2$	0.00	0.05	0.06	0.06	0.11			$R^2$	0.00	0.03	0.03	0.03	0.06
United States (long data set)							United States (long data set)								
$\tau$	1m	b	0.57***	0.63**	0.96***	0.72***	0.92***	$\tau$	1m	b	0.59***	0.68***	0.82***	0.82***	0.88***
		$R^2$	0.09	0.04	0.03	0.02	0.04			$R^2$	0.08	0.12	0.14	0.12	0.11
	3m	b	0.70***	0.93**	1.00**	0.80*	0.87***		3m	b	0.75***	0.78***	0.94***	0.95***	0.99***
		$R^2$	0.10	0.07	0.03	0.02	0.04			$R^2$	0.28	0.29	0.36	0.35	0.32
	12m	b	1.08**	1.22**	1.37**	1.25**	1.15**		12m	b	0.91***	0.98***	1.12***	1.14***	1.18***
		$R^2$	0.14	0.14	0.15	0.15	0.19			$R^2$	0.74	0.85	0.89	0.88	0.79
	24m	b	0.79***	0.99***	1.04**	0.98**	0.93**		24m	b	0.83***	0.94***	1.06***	1.07***	1.07***
		$R^2$	0.09	0.13	0.14	0.15	0.21			$R^2$	0.45	0.62	0.66	0.68	0.66
	60m	b	1.18***	1.04***	0.98***	0.91***	0.84**		60m	b	0.97***	0.86***	0.88***	0.88***	0.84***
		$R^2$	0.21	0.28	0.27	0.27	0.29			$R^2$	0.21	0.30	0.34	0.37	0.40

Notes: The Table presents results for regressing realized bond excess returns on risk premia implied by the  $A_0(3)$  model using the standard estimation procedure that only fits yields (Panel A) and using the extended estimation procedure that fits yields and matches model risk premia to bond excess returns observed in the past (Panel B). For each country, we report estimates for 25 horizon ( $\tau$ ) and maturity ( $T - \tau$ ) combinations. The horizons are indicated in the rows, the maturities in the columns.  $b$  is the estimate of the slope coefficient. \*\*\*, \*\*, \* and  $\circ$ ,  $\circ$ ,  $\circ$  indicate that the estimate differs from zero and/or one respectively at the 99%, 95%, 90% level. Significance is assessed using standard errors with HAC adjustment based on [Newey and West \(1987\)](#) where the optimal truncation lag is chosen following [Andrews \(1991\)](#) using a quadratic spectral kernel (standard errors not reported for space reasons).  $R^2$  denotes the in-sample coefficient of determination. We estimate the models using monthly data from September 1989 for Japan, January 1988 for Switzerland, and April 1987 for Germany, UK, and US. The sample period ends in March 2011 for all countries. For the US we also report results for a longer data set covering the period from January 1952 to December 2003.

Table 3: Statistical Accuracy of Bond Excess Return Forecasts.

	Panel A: In-Sample										Panel B: Out-of-Sample									
	Extended vs. Standard Estimation					Extended Estimation vs. EH					Extended vs. Standard Estimation					Extended Estimation vs. EH				
	1m	3m	12m	24m	60m	1m	3m	12m	24m	60m	1m	3m	12m	24m	60m	1m	3m	12m	24m	60m
<i>T</i> - $\tau$																				
Switzerland																				
$\tau$	0.23*	0.25*	0.30*	0.21*	0.18*	0.23*	0.26*	0.29*	0.25*	0.22*	-0.04	0.02	0	-0.02	-0.22*	-0.38*	-0.24*	-0.10*	-0.26*	-0.31*
1m	0.53*	0.57*	0.62*	0.55*	0.50*	0.52*	0.55*	0.61*	0.57*	0.51*	0.16*	0.1	0.01	-0.05	-0.24*	-0.29*	-0.24	-0.25	-0.34*	-0.41*
3m	0.77*	0.89*	0.86*	0.87*	0.84*	0.73*	0.88*	0.84*	0.86*	0.82*	0.35	0.46*	0.42*	0.35	0.22	0.02	0.05	0.07	0.08	-0.01
12m	0.31	0.65*	0.69*	0.70*	0.73*	0.31	0.63*	0.62*	0.61*	0.61*	0.54*	0.67*	0.60*	0.57*	0.54*	0.02	0.15*	0.20*	0.13	0.02
24m	0.22	0.63*	0.73*	0.71*	0.77*	0.03	0.21	0.29	0.17	0.18	0.67*	0.84*	0.85*	0.84*	0.87*	-0.01	0.15	0.31*	0.26	0.36
60m																				
Germany																				
$\tau$	0.23	0.18	0.25*	0.26*	0.27*	0.19	0.18	0.21*	0.20*	0.24*	0.13	0.18*	-0.11	0.19*	0.11	0.04	0.26	0.05	0.14	0.07
1m	0.32	0.38	0.54*	0.54*	0.48*	0.31	0.37	0.51*	0.52*	0.47*	0.34*	0.19	0	0.19	0.06	0.31	0.26	0.04	0.15	0.04
3m	0.69*	0.85*	0.83*	0.82*	0.76*	0.59*	0.82*	0.81*	0.81*	0.77*	0.18	0.28	0.06	0.12	0.11	0.15	0.2	-0.05	0.02	0.03
12m	0.33*	0.63*	0.64*	0.61*	0.55*	0.18	0.54*	0.56*	0.55*	0.52*	0.42*	0.53*	0.21	0.23	0.26	0.33*	0.33*	-0.09	-0.08	-0.03
24m	0.13	0.51*	0.63*	0.60*	0.55*	-0.08	0.15	0.09	0.02	-0.06	0.54*	0.37	-0.41	-0.15	0.36	-0.12	-0.72*	-1.25*	-1.07*	-0.71*
60m																				
United Kingdom																				
$\tau$	0.10*	0.19*	0.21*	0.25*	0.15*	0.09*	0.20*	0.22*	0.27*	0.17*	0.04	0.30*	0.25	0.30*	0.18	0.07	0.29	0.26	0.27	0.08
1m	0.44*	0.44*	0.47*	0.50*	0.36*	0.43*	0.45*	0.47*	0.51*	0.37*	0.44*	0.34	0.23	0.21	0.01	0.41	0.32	0.17	0.11	-0.23
3m	0.61*	0.84*	0.87*	0.80*	0.80*	0.59*	0.83*	0.85*	0.78*	0.78*	0.41*	0.60*	0.52*	0.46*	0.27	0.40*	0.52*	0.33*	0.18*	-0.25
12m	0.54*	0.72*	0.70*	0.68*	0.60*	0.47*	0.63*	0.57*	0.51*	0.51*	0.42*	0.62*	0.54*	0.52*	0.29*	0.62*	0.39*	0.2	0.18	0.07
24m	0.55*	0.73*	0.71*	0.69*	0.61*	-0.15	-0.07	-0.05	-0.14	-0.14	0.17	0.53*	0.55*	0.59*	0.62*	-0.31	-0.25	-0.14	-0.07	-0.13
60m																				
Japan																				
$\tau$	0.07	0.34*	0.50*	0.33*	0.27*	0.11	0.37*	0.49*	0.23	0.30*	0.25*	0.40*	0.25	0.50*	0.29*	-0.37*	-0.09	0.02	-0.71*	-0.02
1m	0.40*	0.64*	0.80*	0.68*	0.52*	0.45*	0.66*	0.81*	0.70*	0.58*	0.41*	0.49*	0.39*	0.64*	0.36*	-0.24	-0.04	0.04	0.19	0.16
3m	0.63*	0.77*	0.72*	0.70*	0.56*	0.59*	0.77*	0.77*	0.78*	0.70*	0.23	0.54*	-0.91*	-0.17	0.64*	0.11	0.46*	0.57*	0.66*	0.70*
12m	-0.03	0.4	0.81*	0.80*	0.54*	-1.16*	0.23	0.84*	0.86*	0.73*	0.48*	0.76*	-1.22*	-0.36	0.65*	-2.44*	0.57*	0.71*	0.77*	0.82*
24m	-0.61*	0.62*	0.93*	0.93*	0.85*	-0.41*	0.57*	0.86*	0.89*	0.80*	0.39*	0.63*	-3.97*	-2.46*	0.24	-0.5	0.80*	0.70*	0.78*	0.88*
60m																				
United States																				
$\tau$	0.15	0.19*	0.27*	0.25*	0.20*	0.13	0.18*	0.29*	0.21*	0.21*	0.21	0.18	0.28*	0.27*	0.05	0.11	0.1	0.25*	0.16*	0.00
1m	0.46*	0.51*	0.58*	0.55*	0.44*	0.44*	0.50*	0.59*	0.54*	0.45*	0.26	0.17	0.16	0.21*	-0.05	0.16	0.1	0.12	0.15	-0.07
3m	0.58*	0.82*	0.82*	0.87*	0.73*	0.73*	0.82*	0.83*	0.84*	0.74*	0.48*	0.42*	0.15	0.24	0.17	0.46*	0.37*	0.11	0.19	0.15
12m	0.25*	0.55*	0.56*	0.58*	0.55*	0.1	0.52*	0.58*	0.58*	0.57*	0.40*	0.57*	0.40*	0.42*	0.34*	0.19*	0.40*	0.19*	0.20*	0.06
24m	-0.07	0.06	0.11	0.05	0.05	-0.14	-0.08	-0.16	-0.2	-0.27	0.2	0.43*	0.44*	0.39*	0.11	-1.08*	-1.49*	-1.35*	-2.13*	-6.09*
60m																				
United States (long data set)																				
$\tau$	-0.12	0.04	0.11*	0.11*	0.08*	-0.08	0.06	0.13*	0.12*	0.11*	-0.06	0.02	0.01	-0.01	0.03	-0.08	0	-0.01	-0.02	-0.00
1m	0.12	0.21*	0.34*	0.34*	0.30*	0.18	0.26*	0.35*	0.35*	0.31*	0.13	0.16	0.15	0.13	0.14*	0.04	0.07	0.04	0.02	0.04
3m	0.69*	0.83*	0.87*	0.85*	0.73*	0.72*	0.84*	0.88*	0.87*	0.77*	0.05	0.12	0.19*	0.21*	0.20*	-0.18	-0.09	0	0.02	0.05
12m	0.38*	0.56*	0.61*	0.62*	0.60*	0.43*	0.61*	0.66*	0.67*	0.65*	-0.18	-0.16	-0.03	0	0.07	-0.26*	-0.29*	-0.18	-0.16	-0.10
24m	0.07	0.1	0.15	0.2	0.27*	0.16*	0.29*	0.34*	0.36*	0.38*	-0.38*	-0.60*	-0.42*	-0.26	0.01	-0.52*	-0.78*	-0.65*	-0.50*	-0.22*
60m																				

Notes: The Table presents  $R2$  statistics measuring the predictive ability of the ATSM estimated with the extended procedure relative to the standard estimation and EH-consistent constant risk premium benchmarks. The  $R2$  statistic is defined as  $R2 \equiv 1 - MSE^m/MSE^b$  where  $MSE^m$  and  $MSE^b$  denote the mean squared errors of the model and benchmark forecasts, respectively. Panels A and B report in-sample and out-of-sample results for 25 horizon ( $\tau$ ) and maturity ( $T - \tau$ ) combinations, respectively. The horizons are indicated in the rows, the maturities in the columns. \* ( $^\circ$ ) identifies  $R2$ s that are significantly positive (negative) as judged by confidence intervals obtained as a block bootstrap procedure. For the in-sample analysis, we estimate the models using monthly data from September 1989 for Japan, January 1988 for Switzerland, and April 1987 for Germany, UK, and US. The sample period ends in March 2011 for all countries. For the US we also report results for a longer data set covering the period from January 1952 to December 2003. In the out-of-sample analysis, we estimate for each country data set the first model based on the first ten years of data available. Subsequently, we generate return forecasts every month  $t$  by re-estimating the model using only information available up to time  $t$ .

Table 4: Economic Value of Bond Excess Return Forecasts.

		Panel A: In-Sample										Panel B: Out-of-Sample																			
		Portfolio Excess Returns					Extended Estimation vs. EH Forecast					Portfolio Excess Returns					Extended Estimation vs. EH Forecast														
$T - \tau$		1m		3m		12m		24m		60m		1m		3m		12m		24m		60m		1m		3m		12m		24m		60m	
Switzerland																															
$\tau$	1m	68	122	245	489	633	21	48	148	81	116	23	69	157	153	267	-34	-17	-16	91	-64	-10	4	33	-68	-306	-101	-98	-109	-429	-626
	3m	61	104	229	367	526	25	47	109	128	220	34	71	139	181	282	11	23	26	2	-26	25	41	36	-65	-167	-30	-17	-90	-259	-331
	12m	39	80	158	225	371	22	40	84	111	184	20	40	79	114	177	21	45	78	114	132	31	52	74	88	42	-1	-4	-1	1	-51
	24m	37	69	124	175	290	2	12	32	44	77	5	16	35	47	72	49	67	83	117	191	50	68	82	113	159	0	0	0	0	4
	60m	27	82	185	272	466	0	0	0	0	0	0	0	0	0	0	28	73	156	235	405	14	44	88	118	171	0	0	0	0	0
Germany																															
$\tau$	1m	57	95	193	441	699	25	37	166	185	333	-1	27	84	53	200	29	111	140	560	663	32	75	27	146	177	-76	-2	72	-25	-7
	3m	38	73	193	331	516	13	35	144	187	255	8	36	90	118	228	55	76	137	299	381	51	50	59	91	84	0	22	22	14	44
	12m	34	76	164	233	384	35	61	123	163	200	14	32	73	110	169	18	52	79	118	197	14	25	27	37	55	-2	-3	-8	-5	-9
	24m	32	62	112	164	282	9	30	72	95	118	1	10	30	43	72	52	71	80	113	185	12	16	23	31	53	-1	-2	-2	-2	-2
	60m	29	69	162	252	419	1	5	8	10	26	0	0	0	0	-4	35	68	139	219	375	0	0	0	0	0	0	0	0	0	0
United Kingdom																															
$\tau$	1m	44	81	197	363	508	28	43	93	199	231	20	48	88	161	226	31	83	225	358	410	-9	22	127	190	189	-14	48	74	101	70
	3m	40	72	177	298	442	28	44	139	228	301	24	54	105	165	239	44	69	153	222	262	14	27	103	135	224	31	67	42	60	50
	12m	31	69	152	229	362	22	54	126	185	245	14	40	82	111	176	42	63	108	142	208	21	41	85	131	234	15	18	24	22	48
	24m	26	51	108	154	245	17	38	93	106	135	9	18	30	37	64	20	38	70	93	113	11	24	51	99	170	2	3	-6	-10	-13
	60m	27	65	127	184	262	25	59	104	127	107	0	0	1	-7	-55	8	25	46	36	17	-5	5	13	3	135	-10	-19	-45	-95	-180
Japan																															
$\tau$	1m	81	159	358	699	768	8	27	142	136	184	18	66	129	76	208	23	76	142	519	388	7	28	89	475	424	-76	-53	-28	-64	-42
	3m	65	116	263	433	559	1	25	89	115	149	17	38	79	96	187	32	62	94	227	214	5	26	77	178	190	-35	-24	-11	-4	-3
	12m	43	88	157	240	437	9	13	18	27	63	4	9	16	24	60	23	40	38	72	195	8	26	16	28	98	-1	-2	0	0	7
	24m	73	136	160	241	429	0	0	0	0	6	0	0	0	0	2	66	97	55	96	235	32	41	16	25	74	0	0	0	0	0
	60m	41	90	149	220	382	0	0	0	0	0	0	0	0	0	0	38	66	75	118	228	2	0	0	0	3	0	0	0	0	0
United States																															
$\tau$	1m	69	144	329	496	659	1	42	67	44	215	-2	38	67	44	206	76	130	307	629	569	20	50	99	113	195	-18	-2	57	-29	5
	3m	65	119	276	385	513	11	37	80	98	190	12	37	80	98	190	78	122	229	362	408	36	60	89	120	231	8	17	36	19	62
	12m	49	95	192	270	413	7	16	45	72	115	7	16	45	72	115	53	93	151	223	346	7	16	38	58	134	7	11	21	34	66
	24m	38	77	133	189	308	-1	-1	2	2	2	-1	-1	2	2	2	76	100	131	192	282	7	11	23	41	74	0	0	2	6	-2
	60m	36	82	167	250	441	0	0	0	0	0	0	0	0	0	0	43	83	164	238	254	5	12	26	51	10	0	0	0	-5	-181
United States (long data set)																															
$\tau$	1m	84	120	201	262	394	-19	1	102	121	140	-33	-18	67	138	181	105	102	79	76	204	-15	-26	35	28	227	-29	-57	-64	-46	-133
	3m	63	93	176	245	366	-2	12	104	157	216	-8	7	85	149	226	53	72	85	107	229	-25	-2	94	161	351	-31	-31	-9	-4	-23
	12m	44	76	148	208	303	12	34	84	110	154	9	28	83	130	201	14	19	36	62	123	-13	-5	64	125	220	-25	-30	-36	-47	-87
	24m	24	47	91	128	198	9	22	41	52	88	5	15	42	65	112	-1	0	6	15	74	-16	-20	-4	10	79	-17	-28	-49	-76	-106
	60m	13	23	46	62	93	1	3	6	10	11	-1	1	14	24	50	-6	-26	-27	11	80	-18	-31	-20	20	111	-19	-41	-70	-54	-35

Notes: The Table presents results on the economic value generated by ATSM forecasts using the extended estimation procedure. We report results for the in-sample and out-of-sample analysis in Panels A and B for 25 horizon ( $\tau$ ) and maturity ( $T - \tau$ ) combinations, respectively. The horizons are indicated in the rows, the maturities in the columns. In each Panel, we first present the excess returns of optimal bond portfolios using extended estimation forecasts. For an investment horizon  $\tau$ , an investor with target volatility of  $\sigma^* = 2\%$  and relative risk aversion  $\rho = 3$  optimally allocates his wealth between the risk-free bond with maturity  $\tau$  and the risky bond with maturity  $T > \tau$ . In the remaining columns, we report the economic value added by the extended compared to the standard estimation and compared to the EH-consistent constant risk premium forecasts using the performance measure of [Goetzmann et al. \(2007\)](#). All performance measures are annualized and reported in basis points. For the in-sample analysis, we estimate the models using monthly data from September 1989 for Japan, January 1988 for Switzerland, and April 1987 for Germany, UK, and US. The sample period ends in March 2011 for all countries. For the US we also report results for a longer data set covering the period from January 1952 to December 2003. In the out-of-sample analysis, we estimate for each country data set the first model based on the first ten years of data available. Subsequently, we generate return forecasts every month  $t$  by re-estimating the model using only information available up to time  $t$ .



**Table 5: Directional Accuracy of Bond Excess Return Forecasts.**

$T - \tau$	In-Sample					Out-of-Sample					
	1m	3m	12m	24m	60m	1m	3m	12m	24m	60m	
<b>Switzerland</b>											
$\tau$	1m	0.99	1.07	1.15	1.03	1.12*	0.45 <sup>oo</sup>	0.66 <sup>oo</sup>	1.04	0.72 <sup>oo</sup>	0.63 <sup>oo</sup>
	3m	1.12	1.18*	1.27**	1.25**	1.17*	0.63 <sup>oo</sup>	0.84	0.81 <sup>o</sup>	0.72 <sup>o</sup>	0.71 <sup>oo</sup>
	12m	1.31*	1.33**	1.30**	1.30**	1.32**	0.98	1.00	1.00	1.03	0.93
	24m	1.04	1.09	1.17	1.17	1.16	1.00	1.00	1.00	1.00	1.01
	60m	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
<b>Germany</b>											
$\tau$	1m	0.91	1.02	1.13	1.01	1.08	0.69 <sup>oo</sup>	1.00	1.13	1.01	1.03
	3m	1.00	1.18*	1.24*	1.21*	1.20*	0.99	1.18	1.07	1.06	1.05
	12m	1.23*	1.32**	1.31**	1.36**	1.30**	1.05	0.97	0.96	0.99	0.99
	24m	1.03	1.12	1.14	1.16	1.20*	0.98	0.98	1.00	0.99	0.99
	60m	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
<b>United Kingdom</b>											
$\tau$	1m	0.97	1.14*	1.20*	1.14*	1.14*	1.03	1.41**	1.21*	1.21*	1.08
	3m	1.32**	1.45**	1.36**	1.28**	1.20*	1.33**	1.15	1.21	1.24*	1.11
	12m	1.38*	1.42**	1.39**	1.31**	1.29**	1.39*	1.27	1.12	1.06	1.10
	24m	1.19*	1.20*	1.21*	1.19*	1.26*	1.11	1.07	1.03	1.08	1.05
	60m	0.99	1.01	1.01	0.99	0.92	0.77 <sup>o</sup>	0.87	0.90	0.80	0.64 <sup>o</sup>
<b>Japan</b>											
$\tau$	1m	0.88	1.00	1.05	0.95	1.03	0.52 <sup>oo</sup>	0.64 <sup>oo</sup>	0.94	0.88 <sup>oo</sup>	0.90
	3m	0.94	1.03	1.04	1.04	1.08	0.59 <sup>oo</sup>	0.68 <sup>oo</sup>	0.91	0.97	1.00
	12m	1.04	1.03	1.02	1.07	1.12	0.89	0.92	0.99	1.00	1.02
	24m	1.00	1.00	1.00	0.99	1.01	1.00	1.00	1.00	1.00	1.00
	60m	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
<b>United States (87-11)</b>											
$\tau$	1m	0.93 <sup>o</sup>	1.14	1.13	1.03	1.10	0.84 <sup>o</sup>	1.13*	1.13	0.97	0.91
	3m	1.10	1.22*	1.22*	1.22**	1.25**	1.10	1.18	1.14	1.10	1.14
	12m	1.14	1.18*	1.20*	1.26**	1.23**	1.12	1.10	1.10	1.15	1.11
	24m	0.96	0.97	1.00	1.01	1.04	1.00	1.00	1.00	1.04	1.08
	60m	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99 <sup>o</sup>	0.81 <sup>oo</sup>
<b>United States (52-03)</b>											
$\tau$	1m	0.84 <sup>oo</sup>	0.88 <sup>o</sup>	1.03	1.13**	1.11*	0.83 <sup>o</sup>	0.79 <sup>oo</sup>	0.87 <sup>o</sup>	0.93	1.00
	3m	0.87 <sup>o</sup>	0.95	1.19**	1.26**	1.25**	0.74 <sup>oo</sup>	0.78 <sup>oo</sup>	0.90	0.96	1.02
	12m	1.12*	1.30**	1.51**	1.61**	1.57**	0.67 <sup>oo</sup>	0.73 <sup>o</sup>	0.80 <sup>o</sup>	0.87	0.87
	24m	1.09	1.16*	1.30**	1.36**	1.39**	0.68 <sup>o</sup>	0.64 <sup>oo</sup>	0.77 <sup>o</sup>	0.77 <sup>o</sup>	0.85
	60m	0.97	1.06	1.23	1.42**	1.73**	0.62 <sup>oo</sup>	0.53 <sup>oo</sup>	0.59 <sup>o</sup>	0.74 <sup>o</sup>	0.85

*Notes:* The Table presents measures of the directional accuracy of extended estimation compared to EH-consistent constant risk premium forecasts based on hit ratios that measuring the ratio of correctly signed forecasts. Values reported are computed as the fraction of the ATSM model hit ratio relative to the EH hit ratio. Estimates with a \* indicate that the model hit ratio exceeds the 95%-percentile of the bootstrapped constant risk premium hit ratio distribution. \*\* indicates that the 5%-percentile of the model distribution is higher than the 95%-percentile of the constant risk premium hit ratio distribution. <sup>o</sup> and <sup>oo</sup> indicate analogous results when the model hit ratio is lower than the constant risk premium hit ratio. We present in- and out-of-sample results for 25 horizon ( $\tau$ ) and maturity ( $T - \tau$ ) combinations, respectively. The horizons are indicated in the rows, the maturities in the columns. For the in-sample analysis, we estimate the models using monthly data from September 1989 for Japan, January 1988 for Switzerland, and April 1987 for Germany, UK, and US. The sample period ends in March 2011 for all countries. For the US we also report results for a longer data set covering the period from January 1952 to December 2003. In the out-of-sample analysis, we estimate for each country data set the first model based on the first ten years of data available. Subsequently, we generate return forecasts every month  $t$  by re-estimating the model using only information available up to time  $t$ .

**Table 6: Statistical Accuracy and Economic Value of Bond Excess Return Forecasts with EH Priors.**

**Panel A: Imposing a Weak EH Prior**

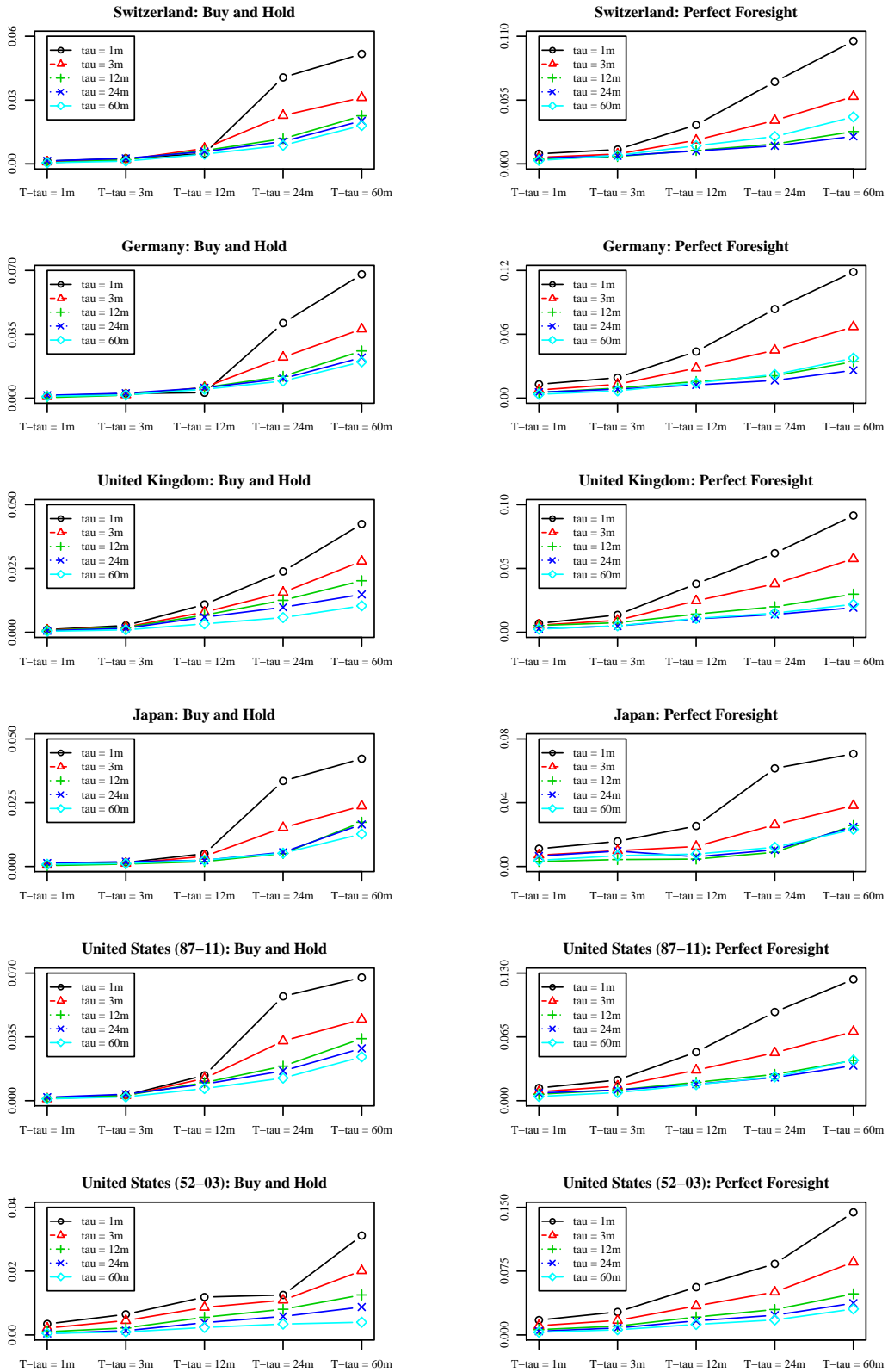
$T - \tau$		Statistical Accuracy					Relative Hit Ratios					Portfolio Excess Returns					Economic Value				
		1m	3m	12m	24m	60m	1m	3m	12m	24m	60m	1m	3m	12m	24m	60m	1m	3m	12m	24m	60m
$\tau$	1m	-0.06	-0.03	-0.01	0	-0.06	0.90 <sup>o</sup>	0.87 <sup>oo</sup>	0.92 <sup>o</sup>	1.02	0.98	111	133	150	239	297	-23	-26	0	106	-37
	3m	-0.06	-0.03	-0.01	0	-0.05	0.88 <sup>oo</sup>	0.84 <sup>o</sup>	0.97	1.07	1.04	65	77	122	182	274	-20	-27	23	63	44
	12m	0.16	0.22	0.22	0.22	0.12	0.92	0.99	1.03	1.03	0.97	38	58	96	135	213	-2	7	22	31	9
	24m	0.14	0.19	0.23	0.22	0.16	1.00	0.91	1.04	0.97	0.95	20	30	55	80	134	1	0	0	-13	-46
	60m	-0.08	-0.1	0.02	0.11	0.21	0.95	0.93	1.06	1.12	1.16	15	17	40	79	196	-3	-11	-31	-33	34

**Panel B: Imposing a Strong EH Prior**

$T - \tau$		Statistical Accuracy					Relative Hit Ratios					Portfolio Excess Returns					Economic Value				
		1m	3m	12m	24m	60m	1m	3m	12m	24m	60m	1m	3m	12m	24m	60m	1m	3m	12m	24m	60m
$\tau$	1m	-0.04 <sup>o</sup>	0	0	0	-0.01 <sup>o</sup>	1.00	1.00	1.00	0.98	1.04	134	158	140	101	345	0	0	0	-8	48
	3m	-0.01	0	0.01	0.01*	0	1.00	1.00	1.00	1.00	1.05	85	103	87	113	213	0	0	0	16	18
	12m	0	0.03	0.03*	0.02*	0.01*	1.00	1.00	1.08	1.08	1.00	39	48	96	111	133	0	0	28	19	7
	24m	0.01	0.03*	0.01	0	0.01	1.00	0.97	1.09	1.08	1.05	18	26	57	77	130	0	-2	3	10	11
	60m	-0.16 <sup>o</sup>	-0.10 <sup>o</sup>	-0.07 <sup>o</sup>	-0.06 <sup>o</sup>	0	0.93	0.96	0.91	0.91	1.11	13	23	38	56	134	-4	-4	-13	-14	13

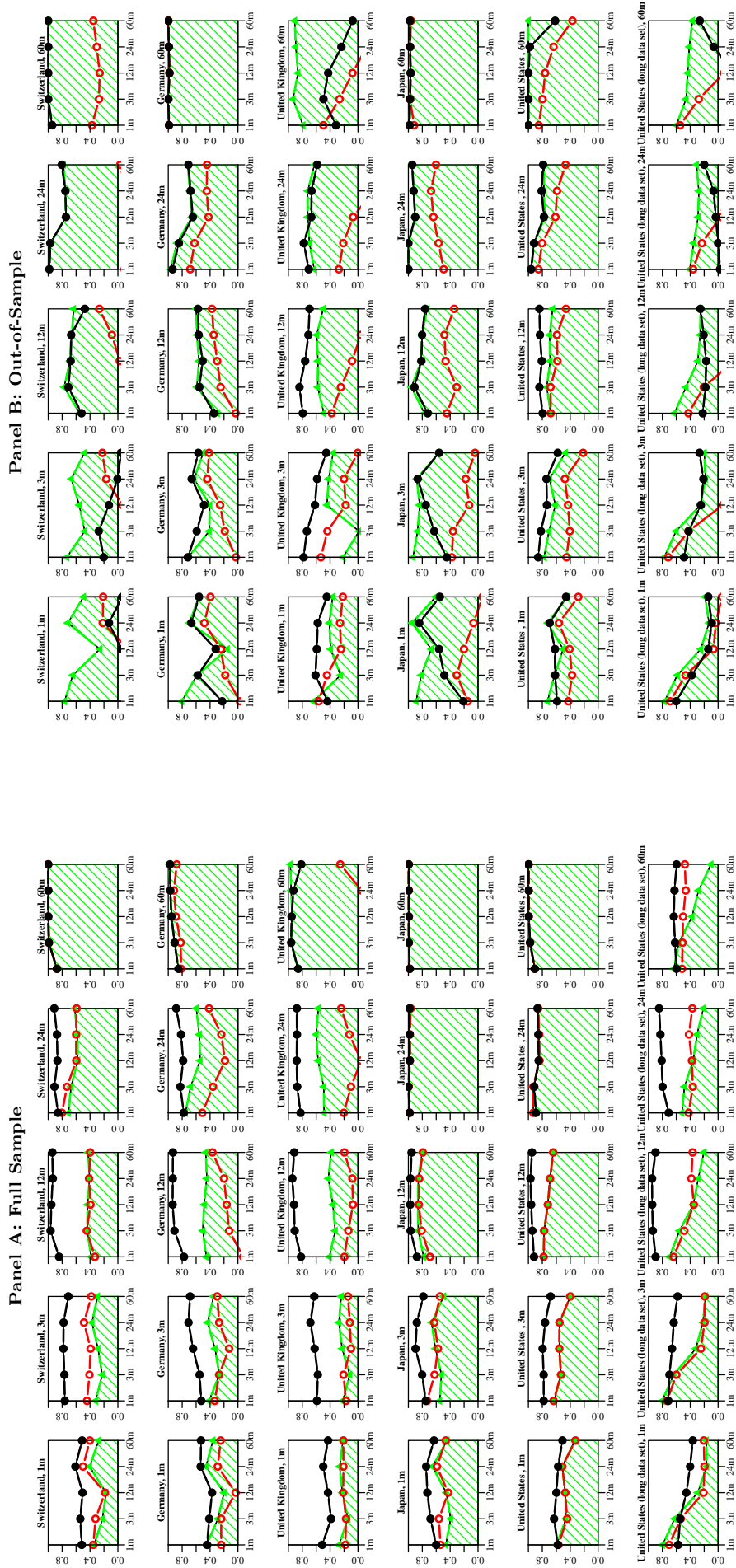
*Notes:* The Table presents results for extended estimation forecasts with EH priors imposed relative to EH-consistent constant risk premium forecasts. For descriptions of the metrics employed to measure statistical and directional accuracy as well as portfolio returns and economic value added see the notes to Tables 3, 4, and 5. Panels A and B report results for a “weak” and a “strong” EH prior defined as by  $\Xi$  identity matrix times 10,000 and times 1e12, respectively. We present in- and out-of-sample results for 25 horizon ( $\tau$ ) and maturity ( $T - \tau$ ) combinations, respectively. The horizons are indicated in the rows, the maturities in the columns. For the in-sample analysis, we estimate the models using monthly data from September 1989 for Japan, January 1988 for Switzerland, and April 1987 for Germany, UK, and US. The sample period ends in March 2011 for all countries. For the US we also report results for a longer data set covering the period from January 1952 to December 2003. In the out-of-sample analysis, we estimate for each country data set the first model based on the first ten years of data available. Subsequently, we generate return forecasts every month  $t$  by re-estimating the model using only information available up to time  $t$ .

Figure 1: Average Excess Returns.



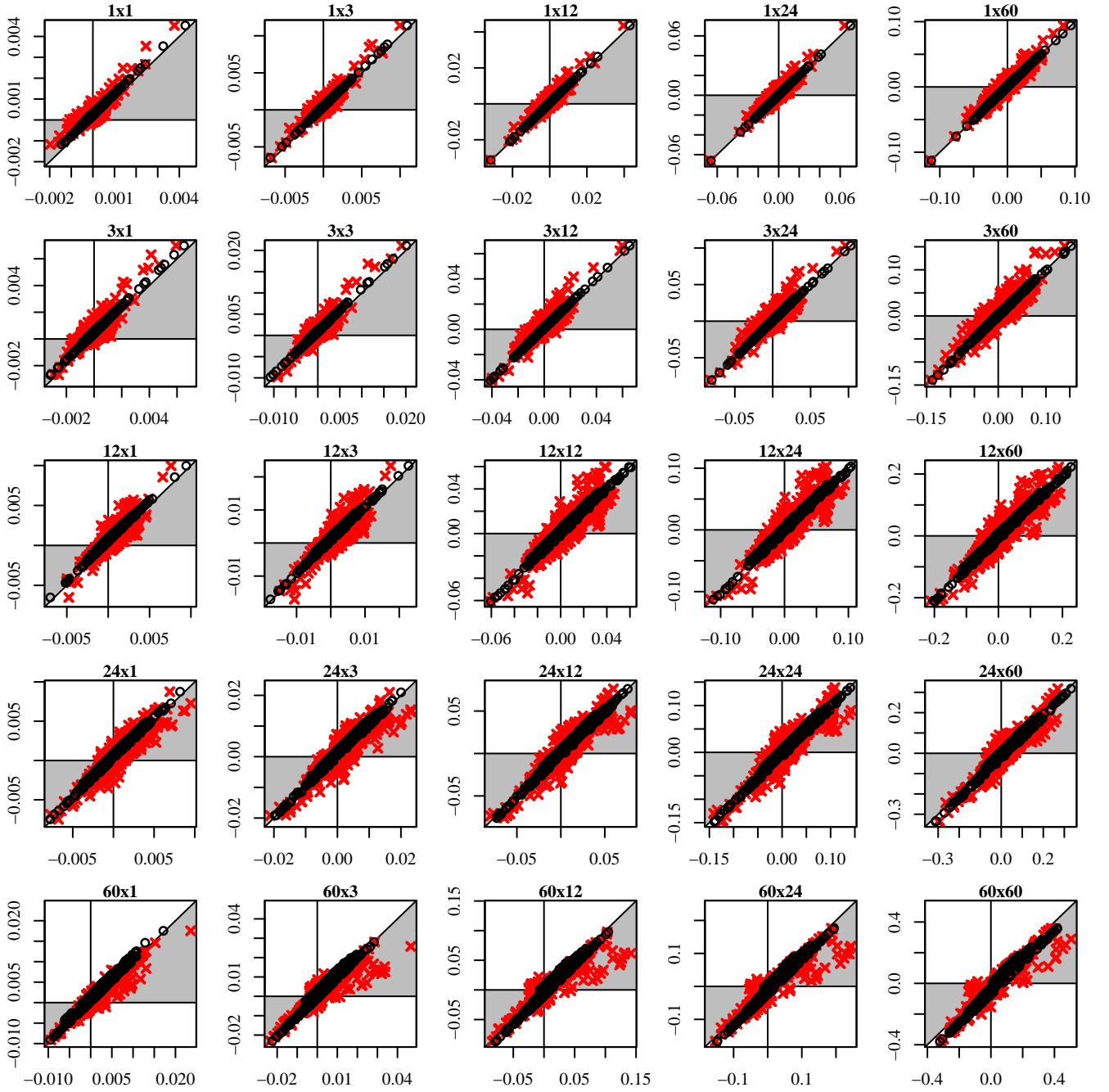
Notes: The Figure plots averages of annualized excess returns of trading the longer-term bond with maturity  $T$  over a horizon  $\tau$ . Each line represents the term structure of excess returns for a given horizon  $\tau$  for maturities indicated on the x-axis. The left column represents the excess returns of a buy and hold strategy. The right columns plots excess returns of optimal bond portfolios of investors that have perfect foresight; for an investment horizon  $\tau$ , an investor with target volatility of  $\sigma^* = 2\%$  and relative risk aversion  $\rho = 3$  optimally allocates his wealth between the risk-free bond with maturity  $\tau$  and the risky bond with maturity  $T > \tau$ . The graphs are based on monthly data from September 1989 for Japan, January 1988 for Switzerland, and April 1987 for Germany, UK, and US. The sample period ends in March 2011 for all countries. For the US we also report results for a longer data set covering the period from January 1952 to December 2003.

Figure 2: Excess Returns relative to Perfect Foresight.



Notes: The Figure plots average excess returns of bond portfolios constructed based on EH and model forecasts relative to average excess returns using perfect foresight forecasts. Portfolios using EH-consistent constant risk premium forecasts are represented by the green lines (with triangles and shaded area below). Portfolios based on model forecasts using the standard estimation procedure are plotted in red (with circles) and portfolios using extended estimation forecasts are displayed in black (with bullets). The underlying portfolios are generated by an investor with target volatility of  $\sigma^* = 2\%$  and relative risk aversion  $\rho = 3$  who optimally allocates his wealth between the risk-free bond with maturity  $T - \tau$ . For the in-sample analysis (Panel A), we estimate the models using monthly data from September 1989 for Japan, January 1988 for Switzerland, and April 1987 for Germany, UK, and US. The sample period ends in March 2011 for all countries. For the US we also report results for a longer data set covering the period from January 1952 to December 2003. In the out-of-sample analysis (Panel B), we estimate for each country data set the first model based on the first ten years of data available. Subsequently, we generate return forecasts every month  $t$  by re-estimating the model using only information available up to time  $t$ .

Figure 3: Forecast Errors and Excess Returns.



Notes: This Figure illustrates the relation between out-of-sample forecast errors (x-axis) and realized bond excess returns (y-axis). Forecast errors from using EH-consistent constant risk premium forecasts are represented by black circles and forecast errors from using extended estimation ATSM forecasts are represented by red crosses. Shaded areas represent areas where forecasts have the wrong sign and hence forecast errors that lead to (bond portfolio) losses. Each plot represents one of the 25 horizon ( $\tau$ ) and maturity ( $T - \tau$ ) combinations as indicated by the plot labels " $\tau \times (T - \tau)$ ". We use the long data set for the United States covering monthly data from January 1952 through December 2003. We estimate the first model based on the first ten years of data available. Subsequently, we generate return forecasts every month  $t$  by re-estimating the model using only information available up to time  $t$ .