Are Private Equity Investors Boon or Bane for an Economy? — A Theoretical Analysis

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In this paper, we provide a theoretical foundation for the controversial debate about the economic consequences of private equity transactions. For this purpose, we separately consider six major characteristics that typically distinguish private equity investors from standard investors. Applying a simple model framework, we compare both the maximum acquisition prices of private equity and standard investors for the takeover of a target firm as well as the subsequent optimal investment volumes. This analysis intends to reveal reasons for an inefficient behavior in the sense that private equity investors acquire a company even though afterwards they will invest less than standard investors would do. We find that most of the usually offered arguments against private equity transactions, such as a higher target return, a short-term investment perspective, a lower risk aversion or operational improvements cannot explain an inefficient behavior by private equity firms. In contrast, a high amount of leverage and informational advantages of private equity firms can result in inefficient outcomes.

JEL Classification: D40, D61, G34

Keywords: Private equity, Leveraged buyout (LBO), Takeover competition, Social welfare

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1 Introduction

Private equity firms typically follow a leveraged buyout (LBO) strategy to be involved in other firms. This special form of investment, which results in highly levered target companies after an acquisition, has been subject to a very emotional and controversial debate. A main criticism of the behavior of private equity firms refers to the high debt levels. These obligations create a heavy burden for the acquired companies which is supposed to lower the scope for profitable investments. In particular, since private equity firms typically intend to sell the portfolio companies after a certain holding period, they are regarded as corporate raiders striving for high short-term profits at the cost of the long-term prospects of the acquired companies and subsequently the well-being of their employees. A popular example for this negative opinion about private equity is expressed in the term "locusts" to describe private equity funds' behavior. This expression was introduced by a former German vice-chancellor Franz Müntefering and has received high public and media attention (see e.g. Bongaerts/Charlier (2009)). Moreover, even in capital market based economies like the USA and Great Britain, private equity activity is also critically assessed (see e.g. Jenkinson (2007) and Walker (2008)) which has brought up claims to consider a regulation of private equity firms (see e.g. Financial Services Authority (2006), Walker (2007), or OECD (2008)).

On the other side, proponents of private equity usually emphasize that private equity investors offer an additional financing source for companies which is apparently only an option but not an obligation for any firm. Furthermore, private ownership is seen as a new management model, which is superior to a public company formation with a large number of small, unorganized share holders (see e.g. Jensen et al. (2006) or Private Equity Council (2007)). Without having to consider the short-term development of the stock price and the pressure to deliver good quarterly results, the management in a private equity backed company can solely focus on long-term value increasing investments. In this way it is argued that private equity firms can help to enhance economic efficiency and deliver exceptional returns to their investors.[1]

A large body of empirical research examines the economic impact of private equity transactions (see e.g. Kaplan (1989a), Lichtenberg/Siegel (1990), Muscarella/Vetsuypens (1990), Palepu (1990), Smith (1990), Andrade/Kaplan (1998), Harris/Siegel/Wright (2005), Amess/Wright (2007), Cumming/Siegel/Wright (2007), Davis et al. (2008), Strömberg (2008) and Kaplan/Strömberg (2009)). Many of these studies consistently find that after the acquisition private equity backed companies experience a significant enhancement of operational performance in terms of an increase of both operating profit (see e.g. Kaplan (1989a), Smith (1990) and Muscarella/Vetsuypens (1990)) and productivity (see e.g. Lichtenberg/Siegel (1990) and Harris/Siegel/Wright (2005)). The effects on investment and employment in acquired companies are non-conforming across studies, but a significantly negative impact of private equity transactions is currently not documented. Instead, private

[1]Whether private equity funds actually deliver excess returns to investors is interpreted in several ways. Ljungqvist/Richardson (2003) calculate excess returns of private equity funds relative to the S&P 500 of five to eight percent. Kaplan/Schoar (2005) find that the average return of private equity funds net of fees approximately equals that of the S&P 500. Phalippou/Gottschalg (2008) report that private equity funds underperform the S&P 500 after fees by about three percent. Other studies document that a part of the returns comes from an expropriation of the incumbent debt holders that is caused by a rise of leverage (and therefore default risk) associated with a LBO (see e.g. Asquith/Wizman (1990), Warga/Welch (1993) and Renneboog/Szilagyi (2008)).

equity firms seem to foster economic restructuring. Davis et al. (2008) show that private equity backed companies experience comparably high job losses in existing establishments, but create a comparably large number of greenfield jobs. In summary, the negative image of private equity firms is not supported by the empirical literature.

However, there is a general caveat concerning all empirical studies which examine the effects of private equity transactions. This caveat refers to the fact that it is ultimately not observable how a company would have developed without the acquisition of a private equity investor. Therefore, purely empirical investigations are in general not sufficient to finally judge the impact of private equity investors on an economy because they can hardly proxy the benchmark scenario without an investment of private equity investors. Moreover, many arguments against private equity refer to the endogenous investment behavior of these companies such as a short-term investment strategy but disregard potential characteristics that might cause this behavior. In particular, to the best of our knowledge, there is no theoretical study that can endogenously explain for why private equity investors are supposed to follow inefficient investments after an acquisition.

In order to theoretically analyze the economic impact of private equity transactions, we have to define what kind of investment behavior by private equity firms constitutes a loss of social welfare and is therefore inefficient. For this purpose, we regard an investment situation as inefficient where the successful bidder in a takeover competition intends to run the acquired company after the takeover with a lower investment volume than the outbidden investor would have done. At first glance, inefficient investment situations seem to be a paradox, because it is intuitive that the investor, who invests more into the firm, will realize higher expected revenues in the future so that he is willing to pay a higher acquisition price for it. Hence, it is hard to understand why a rational private equity investor should outbid another investor to take over a target company once he intends to run the company with a "relatively low" investment volume.

The goal of this paper is to provide an academic foundation for the controversial debate whether private equity is boon or bane for an economy. If private equity investors cause harm to an economy, they must have characteristics that give them an incentive to make inefficient investment decisions. We aim at verifying which characteristics that differentiate private equity firms from standard investors do (and do not) result in an inefficiency and therefore cause a loss of social welfare. For this purpose, we develop a simple model where a private equity investor competes with a standard investor for the takeover of a target company. The successful investor can use the company's production technology and decide about the investment volume when running the firm. We compare both the acquisition and the investment decisions in the case of a successful takeover of the investors.

Our analysis shows that typical characteristics of private equity funds such as a short-term investment perspective, the ability to improve production efficiency, a lower risk aversion, and a higher target return compared to standard investors do not cause inefficient investment behavior by private equity investors. Conversely, the possibility to lever up companies and potential information asymmetries between private equity and standard investors with respect to the target company's prospects are explanations for an inefficient outcome. As a result, this analysis explains that private equity firms can exhibit inefficient investment behavior, even though it might appear to be paradoxical at first glance, but many of the usually-offered explanations *are not* the reasons for inefficient behavior.

The paper is organized as follows. Section 2 contains the model-theoretic analysis. After an outline of the structure of the model, we will evaluate the impact of each of the six distinct characteristics of private equity firms on the investment efficiency in every subsection. Section 3 concludes the paper. Technical developments are in the appendix.

2 The model

2.1 Model framework

To analyze the economic impact of private equity firms, we consider a simple, one-period model with two groups of investors. The first group of investors includes the standard participants of the market for corporate control as strategic investors or existing equity holders. The second group of investors includes typical private equity firms/buyout funds acquiring companies by leveraged buyout transactions. While the term private equity is also used to encompass both buyouts and venture capital, we focus on the buyout market and will understand under a private equity transaction a typical LBO. The investors within each group are assumed to behave competitively. Therefore, they are aggregated to one representative standard investor S and one private equity investor P.

The investors S and P compete for the takeover of a single company (or division of a company). The investor who is willing to pay a higher acquisition price acquires the company and can use the company's production technology. In the case that the standard investor represents an existing equity holder, the maximum acquisition price of S can be understood as the minimum price (reservation price) for which the equity holders would sell the company. In what follows, we will focus on the case that S is an outside investor but the competition structure is fully equivalent to the case where S represents the existing equity holders. Therefore, the results derived in this paper are valid in both cases. The time structure of the model is shown in Figure 1.

Figure 1: The time structure

At the initial date $t = 0$, the investors have to make two decisions. First, they have to decide on the maximum price $p_i,\ i \in \{S, P\}$ they are willing to pay for the acquisition. Second, the investors have to decide on the optimal investment volume $x_i^*,\ i \in \{S, P\}$ in the case of a successful takeover which affects the total outcome of the production process. The total outcome realized at $t = 1$ comes from an outcome function $O(x)$ depending on the initial investment volume x multiplied by a random variable $\widetilde{z}$. $\widetilde{z}$ is normally distributed with mean μ equal to one and standard deviation σ. Therefore, $O(x)$ represents the expected outcome of the production process in monetary units. The stochastic variable $\widetilde{z}$ causes proportional changes of the firm's total outcome, i.e. the volatility of the outcome is higher for a higher expected outcome $O(x)$. The outcome function $O(x)$ of the firm's operations exhibits the following standard properties:

$$\begin{aligned}
&O'(x) > 0, && \text{for } x > 0,\\
&O''(x) < 0, && \text{for } x > 0,\\
&O(x) = 0, && \text{for } x = 0,\\
&O'(x) \to \infty, && \text{for } x \to 0,\\
&O'(x) \to 0, && \text{for } x \to \infty.
\end{aligned} \tag{1}$$

These properties mean that a higher investment volume x increases the expected outcome $O(x)$ but the marginal increase declines with x. The last three properties imply that for any investor, who holds this production technology, it is optimal to invest a positive finite amount $x > 0$ given that alternatives with a positive return are available. At time $t = 1$, uncertainty is resolved and the investor who acquired the company obtains the outcome $z \cdot O(x^*)$ of the production process.

The investors have a target return of $r_i,\ i \in \{S, P\}$, which has the character of an opportunity cost of capital, i.e. the investors have comparable alternative investment possibilities delivering their respective (expected) target return. Therefore, the investors compare the expected return from an investment project to their target return r_i and decide for the more favorable opportunity. As a result, an investment into an available project with price p_i and investment volume x_i contributes to the wealth of the investor at time $t = 1$ as follows:

$$\widetilde{\Pi}_i = \widetilde{z} \cdot O(x_i) - (x_i + p_i) \cdot (1 + r_i),\ i \in \{S, P\}, \tag{2}$$

i.e. the investor obtains $\widetilde{z} \cdot O(x_i)$ from the investment and deploys capital which would alternatively yield an expected payoff at $t = 1$ equal to $(x_i + p_i) \cdot (1 + r_i)$. For this reason, we denote $\widetilde{\Pi}_i$ as the value contribution to investor i.

In the following analysis, we evaluate the maximum acquisition price and the optimal investment volume of the two investors. The focus of our analysis is on the detection of cases where market entry of the private equity investor causes a loss of social welfare. We measure social welfare with the volume x_i that is invested into the company after a successful takeover. Note that the investment volume x_i does not only stand for capital investment. Rather, it represents the overall amount of resources that the successful investor devotes to operating the target company such as the number of employees, skill enhancement, R&D expenditures, net fixed assets, etc. We have the notion that the economy as a whole and therefore also social welfare benefits from every additional unit of investment volume. In principle, not only the size but also the type of capital expenditure could be relevant for social welfare. For example, whether the investment in a new, growing business is more desirable than maintaining a declining business at high costs may not solely depend on the total capital deployed. The same difficulty arises when comparing a small number of high qualified jobs with a larger number of lower qualified jobs. However, since it is not obvious whether one monetary unit expenditure by a standard investor or a private equity investor is more attractive, we do not refer to this aspect explicitly. Moreover, we note that the notion of a monotonically increasing social welfare in the investment volume implicitly assumes that the alternative use of capital of the non-successful investor has no welfare effect. This is plausible in the case that the alternative investment of the private equity investor concerns another (foreign) economy while the standard investor alternatively spends his capital on a type of consumption which has only a minuscule effect upon social welfare.

As a consequence of the fact that the size of the investment volume is the relevant factor, investor i_x, who is willing to invest the highest amount

$$i_x := \arg \max_{i \in \{S,P\}} x_i^*$$

should take over the firm in order to maximize social welfare. Since in our model the bidder i_p of the maximum price acquires the firm, i.e.

$$i_p := \arg \max_{i \in \{S,P\}} p_i,$$

it is per se not obvious whether the successful bidder i_p is always the investor i_x of the highest volume. Thus, we define an investment situation as efficient if investor i_x is also the maximum bidder i_p:

$$i_x = i_p.$$

If, however, the investor with the maximum investment volume does not take over the firm, because the other investor outbids him, we define the investment situation as inefficient:

$$i_x \neq i_p.$$

At this point we note that all investors are assumed to behave rationally. Otherwise, if for some irrational reasons the standard investor is not willing to choose the optimal investment volume but rather decides for a considerably too high investment volume x_S with $x_S > x_S^*$, it is apparent that the price p_S the standard investor is willing to pay for the investment opportunity can be lower than that of a private equity investor even though the standard investor invests more. Hence, we can easily explain an inefficient investment situation introducing irrational behavior. Too avoid these trivial cases with potential inefficiencies, we will focus on rational investors throughout our analysis.[2]

In what follows, we will first consider a base case scenario that illustrates the general structure of our investigation. Then, we will extend the base case by a typical characteristic of private equity investors and examine the impact of the respective characteristic on the investment efficiency in each following subsection. This analysis aims at revealing which characteristics of private equity investors can (or cannot)

[2]Moreover, for irrational behavior a reallocation of capital usually exists that results in an increase of social welfare with a pareto improvement for all individuals.

cause inefficient investments. We will deal with the following six, often discussed characteristics of private equity investors that differentiate them from standard investors:

- a higher target return,
- the ability to realize operational improvements in the overtaken company,
- a lower risk aversion,
- a short-term investment perspective,
- a higher leverage ratio,
- superior information about the target company.

2.2 Base case: different target returns

As a base case, we analyze the model outcome for the situation that both investors are identical except that the private equity investor P has a higher target return than the standard investor S, i.e. $r_S < r_P$ holds. Private equity firms receive a substantial share of the profits of their funds' investments as performance fee (see e.g. Metrick/Yasuda (2009)). As a consequence, they must have a higher target return than standard investor in order to deliver the same expected return after fees. In addition, private equity firms usually have access to a broader range of investment possibilities than e.g. mutual funds which increases the expected return of their investment alternatives. Therefore, we consider a higher target return (opportunistic cost of capital) of the private equity investor compared to the standard investor as the standard difference between the investors for both the base case and the subsequently analyzed extensions.

In order to evaluate whether a higher target return results in an inefficient situation, we focus on a simple economy where the investors are risk-neutral and base their decisions on the expected value contribution at $t = 1$. In subsection 2.4, we will explicitly refer to the case of investors with different risk preferences. Since risk-neutrality is the simplest form of identical risk preference, we assume risk-neutrality as a base characteristic of both investors. The expected value contribution obtained by investor i at $t = 1$ for a given choice x_i and p_i equals:

$$\mathbb{E}(\widetilde{\Pi}_i) = O(x_i) - (x_i + p_i) \cdot (1 + r_i), \; i \in \{S, P\}. \tag{3}$$

We compute the required acquisition prices and investment volumes for both investors applying a backward induction, i.e. we first determine the optimal investment volumes of the investors given a successful takeover. Then, we compare the maximum acquisition prices of the investors with regard to their optimal investment volumes. This approach implies that investors anticipate their investment decision already at the time of the acquisition decision. The investors choose their maximum acquisition price and optimal investment volume by maximizing their objective function given in equation (3). The objective function shows that the optimal investment volume $x_i^*,\ i \in \{S, P\}$ is independent of the acquisition price $p_i,\ i \in \{S, P\}$, as it has the character of sunk costs. The first order condition gives an implicit expression for the optimal investment volume x_i^*:

$$O'(x_i^*) = 1 + r_i,\ i \in \{S, P\}. \tag{4}$$

As a result of the different target returns $r_S < r_P$, it must hold:

$$O'(x_S^*) < O'(x_P^*). \tag{5}$$

Equation (5) and the concavity of $O(x)$ imply that:

$$x_S^* > x_P^*. \tag{6}$$

Thus, the standard investor S would choose a higher investment volume in the case of a successful takeover than the private equity investor P, i.e. $i_x = S$.

Proposition 1 (Investment volume — different target returns) *If private equity investors differ from standard investors only in having a higher target return, they will invest less than standard investors.*

Proposition 1 implies that every situation where the private equity investor P acquires the company will be inefficient. Therefore, we now turn to the maximum price p_i the investors are willing to pay for the acquisition. The maximum acquisition price of an investor is the price for which his expected value contribution at $t = 1$ equals zero given his optimal investment volume. We assume that the investor with the higher maximum acquisition price takes over the company. Note that this assumption does not imply that the successful investor actually pays his maximum acquisition price for the takeover. Depending on the negotiation power of the buyer and the seller, the acquisition price can be somewhere between the reservation price of the seller and that of the buyer. However, we rule out the possibility of strategic bidding behavior of the investors or bidding mechanisms whereby the highest bidder is not the investor with the highest willingness to pay.

The maximum acquisition price of investor i can be derived by the condition that the expected value contribution at $t = 1$ to investor i must equal zero:

$$O(x_i^*) - (x_i^* + p_i) \cdot (1 + r_i) = 0, \;\; i \in \{S, P\}. \tag{7}$$

Thus, the maximum acquisition price of investor i is given by:

$$p_i = \frac{O(x_i^*)}{1 + r_i} - x_i^*, \;\; i \in \{S, P\}. \tag{8}$$

Inserting the expression for the optimal investment volumes leads to:

$$p_i = \frac{O(x_i^*)}{O'(x_i^*)} - x_i^*, \;\; i \in \{S, P\}. \tag{9}$$

We can see that the expressions for the maximum acquisition price have an identical structure for both investors. The only difference in the resulting prices stems from different optimal investment volumes. To evaluate this influence, we consider the first derivative of the price function with respect to x_i^*:

$$\frac{\partial p_i}{\partial x_i^*} = -\frac{O(x_i^*) \cdot O''(x_i^*)}{O'(x_i^*)^2}, \;\; i \in \{S, P\}. \tag{10}$$

For a concave function $O(x)$ with the properties stated in (1) this expression is positive, i.e. the maximum acquisition price is increasing in the optimal investment volume. As the optimal investment volume of the standard investor S is higher than the optimal investment volume of the private equity investor P, this property implies that the maximum acquisition price of S is also higher than that of P:

$$p_S > p_P. \tag{11}$$

Since the company is allocated to the investor with the higher willingness to pay, the standard investor will acquire the company, i.e. $i_p = S$. As the standard investor also decides for a higher optimal investment volume than the private equity investor would have done ($i_x = S$), the outcome will be efficient.

Proposition 2 (Acquisition price — different target returns) *If private equity investors differ from standard investors only in having a higher target return, the standard investor will acquire the company. As he also decides for a higher investment volume, the allocation will always be efficient.*

Proposition 2 has two important implications. First, the private equity investor does not participate in the market. As the standard investor always decides for a higher optimal investment volume than the private equity investor would have done, the allocation is efficient. Hence, in the base case we always obtain the plausible outcome that the investor, who is willing to invest more and therefore obtains higher expected revenues, also has a higher willingness to pay.

Even though a high target return is one of the main criticisms that private equity firms have to face, proposition 2 shows that this characteristic in isolation does not lead to an inefficient situation. This finding leads to the second important implication. As private equity firms do acquire companies, they must differ from standard investors in more aspects than just having a higher target return in order to give them an incentive to pay a higher takeover price. Therefore, in each of the following subsections, we extend the base case by one other typical characteristic of private equity investors and evaluate whether these characteristics provide a motive for private equity firms to act inefficiently.

2.3 Operational improvements

The first model extension we consider is that the private equity investor can enhance the productivity of the production process in the target company due to operational improvements. There are several reasons for why private equity firms might be able to increase the productivity in acquired companies. On the theoretical side, the main arguments are made by Jensen (1986, 1989, 2007). He argues that high leverage in private equity transactions mitigates agency problems between management and equity holders. This is because additional debt obligations motivate the management to create cash flows and lower the scope for inefficient investments. This phenomenon is also known as the control function of debt. Furthermore, the equity stake of the management is typically higher in LBOs than in public firms which also reduces corresponding incentive problems. Additionally, private equity firms are "active investors" which monitor and control management more closely than shareholders do in public companies. Jensen's arguments are supported by a large number of empirical studies. For instance, Kaplan (1989a) and Smith (1990) document a significant increase in operating income in LBO companies in the first years after the buyout which they relate to reduced agency costs. Lichtenberg/Siegel (1990) and Harris/Siegel/Wright (2005) find an over proportional increase in total factor productivity in industrial plants of private equity backed companies after the acquisition. Cotter/Peck (2001) report that private equity investors have greater board representation on smaller boards which suggests that they actively monitor managers (see also Klein/Zur (2009)).

We introduce the ability of private equity firms to enhance the productivity of acquired companies in the model by increasing the outcome of the production process for the private equity investor P from $\widetilde{z} \cdot O(x)$ to

$$a \cdot \widetilde{z} \cdot O(x), a > 1, \tag{12}$$

where the factor a is a productivity parameter that indicates the proportional increase of the total outcome that is accomplished by the private equity investor.[3] We assume that the standard investor does not have the ability to increase the outcome of the production process. Therefore, the optimal investment decision and the maximum acquisition price of the standard investor are equal to that in the base case.

Intuitively, the private equity investor now has a higher incentive to acquire the company and to invest more than in the case without the productivity enhancement due to the more attractive investment opportunity. However, it is not obvious how his optimal decisions are related to that of the standard investor and whether or not an inefficient investment situation might arise. The objective function OF_P of the private equity investor P depending on the productivity parameter a amounts to:

$$OF_P = a \cdot O(x_P) - (x_P + p_P) \cdot (1 + r_P). \tag{13}$$

Following the argumentation in subsection 2.2, the optimal investment volume of P is implicitly given by:

$$a \cdot O'(x_P^*) = 1 + r_P. \tag{14}$$

The optimal investment volume of the private equity investor now depends on the productivity parameter a. From the concavity of $O(x)$ we know that x_P^* is increasing

[3] We have evaluated the impact of various alternative ways to model a productivity increase for the private equity investor; e.g. an exponential increase $\widetilde{z} \cdot O(x)^a$ of the outcome of the production process, a proportionally better exploitation of the investment volume x with a production outcome $\widetilde{z} \cdot O(a \cdot x)$, and a decrease of the investment costs to $(1 - c) \cdot x, c < 1$. However, the qualitative results do not depend on the modeling of the operational improvements.

in a. Thus, different optimal investment volumes between the investors now do not depend solely on different target returns, but also on the productivity parameter a. As the optimal investment volume of the private equity investor is increasing in a (and has no effect on the standard investor), there must be one critical $\overline{a}$ for which both investors choose the same optimal investment volume ($x_S^* = x_P^*$). From (4) and (14) we see that $\overline{a}$ must satisfy the condition:

$$\overline{a} = \frac{1 + r_P}{1 + r_S}. \tag{15}$$

Obviously, $\overline{a}$ is greater than one due to $r_P > r_S$. For all $a < \overline{a}$, the standard investor will choose a higher investment volume ($i_x = S$), while for all $a > \overline{a}$, the private equity investor will choose a higher investment volume ($i_x = P$).

Proposition 3 (Investment volume — operational improvements) *If the degree of the productivity increase, which the private equity investor can realize in the acquired company, is sufficiently high, i.e. $a > \overline{a}$, the private equity investor chooses a higher optimal investment volume than the standard investor. Otherwise, the standard investor invests more into the firm after an acquisition.*

The maximum acquisition price of the private equity investor P is given by:

$$p_P = \frac{a \cdot O(x_P^*)}{1 + r_P} - x_P^*. \tag{16}$$

Inserting the expression for the optimal investment volume gives:

$$p_P = \frac{O(x_P^*)}{O'(x_P^*)} - x_P^*. \tag{17}$$

We see that the structure of the price function is equivalent to that in the base case. Thus, price differences between the investors still stem from different optimal investment volumes only. In line with the base case, the investor with a higher optimal investment volume is willing to pay a higher acquisition price. As the productivity parameter a determines which investor chooses a higher investment volume, a also determines which investor acquires the company. Therefore, the relation between the optimal investment volumes and the maximum acquisition prices between the investors dependent on a are given by:

$$
\begin{aligned}
&i_x = i_p = S, && \text{for } a < \overline{a}, \\
&i_x = i_p = S \text{ or } i_x = i_p = P, && \text{for } a = \overline{a}, \\
&i_x = i_p = P, && \text{for } a > \overline{a}.
\end{aligned}
$$

Proposition 4 (Acquisition price — operational improvements) *The ability to enhance the productivity of acquired companies can give the private equity investor an incentive to outbid standard investors. However, it always holds: the investor who acquires the company invests more than the other investor would have done.*

In contrast to the base case, now a situation where the private equity investor enters the market and takes over the company is possible. But as the investor for whom a higher investment volume is optimal is also willing to pay a higher maximum acquisition price, the model outcome is always efficient. Thus, the market entry of a private equity investor, who differs from other investors only in having a higher target return and by realizing a productivity increase, results in an efficient model outcome. Therefore, the ability of private equity investors to enhance the productivity of acquired companies cannot explain an inefficiency caused by private equity investors.

2.4 Risk aversion

As a next step, we consider possible differences in risk aversion between both types of investors. A reason for why private equity investors might be less risk averse than standard investors lies in the typical fee structure of private equity funds. The major source of income for private equity firms is carried interest, a share of the profits of their funds' investments. Typically private equity firms participate only in the case that the investors of the fund have at least received their initial investment plus a minimum return (hurdle rate), which ranges from 6 to 10 percent. For a detailed description of the fee structure of private equity funds we refer to Metrick/Yasuda (2009), for an economic rationale for such a structure see Axelson/Strömberg/Weisbach (2008). Because of this non-linear compensation scheme (option-like stake) private equity investors are supposed to have a higher risk appetite (lower risk aversion) than standard investors (see e.g. BIS (2008)).

In the base case, we assumed that both investors are risk-neutral. To model differences in risk aversion between the investors, we now alter this assumption. The private equity investor P remains risk-neutral and therefore has the same optimal investment volume and maximum acquisition price as in the base case. The standard investor S becomes risk averse with the preference function:

$$OF_S = \mathbb{E}(\widetilde{\Pi}_S) - b \cdot Var(\widetilde{\Pi}_S), \tag{18}$$

where $\mathbb{E}(\widetilde{\Pi}_S)$ is the expected value contribution obtained by the standard investor S at $t = 1$ as given in equation (3), $Var(\widetilde{\Pi}_S)$ is the variance of the value contribution obtained by S at $t = 1$ and b, $b > 0$, is the risk aversion parameter. Since the random variable $\widetilde{z}$ is the only source of risk in the model and normally distributed, the value contribution obtained by the standard investor S at $t = 1$ is also normally distributed. Therefore, maximizing the objective function given in (18) is equivalent to maximizing an expected utility with utility functions of the CARA type. The variance of the value contribution obtained by the standard investor S using equation (2) is given by:

$$Var(\widetilde{\Pi}_S) = O(x_S)^2 \cdot \sigma^2. \tag{19}$$

where σ^2 is the variance of $\widetilde{z}$.

Using (3) and (19) the objective function of S amounts to:

$$OF_S = O(x_S) - (x_S + p_S) \cdot (1 + r_S) - b \cdot O(x_S)^2 \cdot \sigma^2. \tag{20}$$

We see that risk aversion has a negative impact on the value of the objective function of the standard investor. Obviously, this effect will result in a lower maximum acquisition price p_S and optimal investment volume x_S^* compared to the base case where the standard investor is assumed to be risk-neutral. However, it is unclear whether this different risk appetite will cause an inefficient model outcome, i.e. whether the private equity investor will obtain an incentive to acquire the company but still invest less than the standard investor would have done.

In accordance to the previous subsections, the first order condition gives an implicit expression for the optimal investment volume of the standard investor S:

$$1 + r_S = O'(x_S^*) \cdot \left(1 - 2 \cdot b \cdot \sigma^2 \cdot O(x_S^*)\right). \tag{21}$$

Under an optimal choice x_S^*, the factor $1 - 2 \cdot b \cdot \sigma^2 \cdot O(x_S^*)$ is positive, because (21) would be violated otherwise. This relation will be helpful for the following analysis. For $\sigma = 0$, the optimal investment volume is equal to that in the base case. For

$\sigma \geq 0$, it can be shown that the optimal investment volume decreases in σ and converges to 0 for $\sigma \to \infty$. The proof for these properties is in the appendix. As the optimal investment volume of the standard investor S is decreasing in σ and S does not invest for $\sigma \to \infty$ while the optimal investment volume of the private equity investor P is independent of σ (P is still risk-neutral), there must be one critical volatility $\overline{\sigma}$ for which both investors choose the same investment volume. Using (4) and (21) the critical $\overline{\sigma}$ for which $x_S^* = x_P^*$ holds is given by:

$$\overline{\sigma} = \sqrt{\frac{r_P - r_S}{2 \cdot b \cdot O(x_S^*) \cdot (1 + r_P)}}. \tag{22}$$

Due to $r_P > r_S$, the critical volatility $\overline{\sigma}$ is positive. The fact that a positive $\overline{\sigma}$ always exists is intuitive because $x_S^* > x_P^*$ holds for $\sigma = 0$ and risk aversion mitigates the incentive of the standard investor to invest after an acquisition. The property that the optimal investment volume of the standard investor is decreasing in σ implies that for every $\sigma < \overline{\sigma}$, the standard investor will choose a higher investment volume ($i_x = S$), while for every $\sigma > \overline{\sigma}$, the private equity investor will choose a higher investment volume ($i_x = P$).

Proposition 5 (Investment volume — differences in risk aversion) *If the risk in the production process of the target company is sufficiently high, i.e. $\sigma > \overline{\sigma}$, the private equity investor chooses a higher optimal investment volume than does the standard investor. Otherwise, the standard investor invest more into the firm after an acquisition.*

Equating the objective function of the standard investor S to zero and rearranging terms leads to the maximum acquisition price of S:

$$p_S = \frac{O(x_S^*) \cdot (1 - b \cdot \sigma^2 \cdot O(x_S^*))}{1 + r_S} - x_S^*. \tag{23}$$

Using the first order condition given in (21), we can alternatively write:

$$p_S = \frac{O(x_S^*)}{O'(x_S^*)} \cdot \frac{1 - b \cdot \sigma^2 \cdot O(x_S^*)}{1 - 2 \cdot b \cdot \sigma^2 \cdot O(x_S^*)} - x_S^*. \tag{24}$$

For $\sigma = 0$, the price function is equal to that in the base case. For $\sigma \geq 0$, the maximum acquisition price of the standard investor declines in σ and converges to 0 for $\sigma \to \infty$. This intuitive property is no trivial implication of (24), because a change of σ also impacts the optimal investment volume x_S^*. The proof for this property is in the appendix. As the maximum acquisition price of the private equity investor is independent of σ, there must be a critical volatility $\overline{\overline{\sigma}}$ for which the maximum acquisition price for both investors is identical. Using (8) and (23) the critical volatility $\overline{\overline{\sigma}}$ for which $p_S = p_P$ holds is given by:

$$\overline{\overline{\sigma}} = \sqrt{\frac{-(1+r_S) \cdot O(x_P^*) + (1+r_P) \cdot ((1+r_S) \cdot (x_P^* - x_S^*) + O(x_S^*))}{b \cdot (1+r_P) \cdot O(x_S^*)^2}}. \tag{25}$$

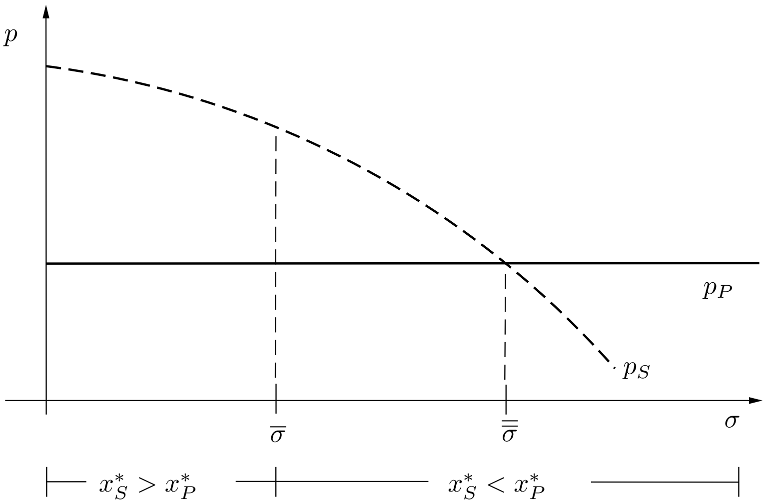
In accordance with the argumentation above, the standard investor will pay a higher takeover price for $\sigma < \overline{\overline{\sigma}}$, while the private equity investor will pay a higher takeover price for $\sigma > \overline{\overline{\sigma}}$. Therefore, a situation is possible where the private equity investor participates in the market. It is important to note that the critical volatilities $\overline{\sigma}$ and $\overline{\overline{\sigma}}$ are not necessarily identical. Both the optimal investment volume and the maximum acquisition price of the standard investor are decreasing in σ while the optimal decisions of the private equity investor are independent of σ. Thus, the relationship between $\overline{\sigma}$ and $\overline{\overline{\sigma}}$ determines whether an inefficient situation caused by a market entry of the private equity investor is possible or not. If and only if $\overline{\sigma} > \overline{\overline{\sigma}}$ holds, a situation will be possible where the private equity investor acquires the company but invests less than the standard investor would have done. This is true for a variance σ with $\overline{\overline{\sigma}} < \sigma < \overline{\sigma}$ for which $p_S < p_P$ but also $x_S^* > x_P^*$ holds.

To evaluate the model outcome, we compare the price functions of the investors p_S and p_P given in (9) and (24) for an identical optimal investment volume x^*. In other words, we regard the special case that the volatility σ equals $\overline{\sigma}$. The representations for the price functions reveal that they are identical except that the term $\frac{O(x^*)}{O'(x^*)}$ is multiplied by the factor $\frac{1-b \cdot \sigma^2 \cdot O(x^*)}{1-2 \cdot b \cdot \sigma^2 \cdot O(x^*)}$ for the standard investor. Since this factor is greater than one, the standard investor is willing to pay a higher acquisition price than the private equity investor given an identical optimal investment volume x^*. Therefore, the critical volatility $\overline{\overline{\sigma}}$ for which the prices of both investors coincide must be greater than $\overline{\sigma}$. This property combined with the fact that both the optimal investment volume and the maximum acquisition price of the standard investor are decreasing in σ implies that the relations between the optimal investment volumes and maximum acquisition prices of the investors depending on σ are given by:

$$\begin{aligned} i_x = i_p = S, &\qquad \text{for } \sigma < \overline{\sigma}, \\ i_x = P, i_p = S, &\qquad \text{for } \overline{\sigma} < \sigma < \overline{\overline{\sigma}}, \\ i_x = i_p = P, &\qquad \text{for } \sigma > \overline{\overline{\sigma}}. \end{aligned}$$

The model outcome is illustrated in Figure 2. We see that a lower risk aversion

Figure 2: Relations between acquisition prices and optimal investment volumes in the case of a risk averse standard investor



of the private equity investor does not explain an inefficient outcome where the private equity investor acquires the firm. This is because P outbids S only for a volatility greater than $\overline{\overline{\sigma}}$. In this case, he also invests more than the standard investor would have done. The standard investor, however, acquires the company for $\sigma < \overline{\overline{\sigma}}$, although the private equity investor would have chosen a higher investment volume for an intermediate risk σ with $\overline{\sigma} < \sigma < \overline{\overline{\sigma}}$.

Proposition 6 (Acquisition price — differences in risk aversion) *The private equity investor will acquire companies with high risk in their production process and will always choose an efficient investment volume. The standard investor acquires low-risk firms. An inefficient investment in the sense that the standard investor acquires the firm and invests less than the private equity investor would have done, occurs for medium volatilities. An inefficient investment, whereby the private equity investor acquires the firm, is not feasible.*

In contrast to the previous subsections, now a situation, where the successful investor invests less than the other investor would have done, is possible. However, this inefficiency does not arise because the private equity investor acquires the company, but it arises because he does *not* acquire the company. The intuition for why the standard investor chooses a lower investment volume than the private equity investor for identical acquisition prices p_S and p_P is due to the relationship between the investment volume x and the volatility of the value contribution of the investment. A higher investment volume x increases the size of the firm and therefore leads to a rise of the relevant risk for the standard investor. Hence, the risk aversion of the standard investor provides an incentive to choose a relatively low investment volume compared to the private equity investor. This is even true in some of the cases where he pays a higher acquisition price than the private equity investor. In such a situation, a private equity investor could lead to a social welfare improvement if the company were allocated to him. Thus, the so far considered characteristics are still no motive for the private equity investor to make inefficient investment decisions.

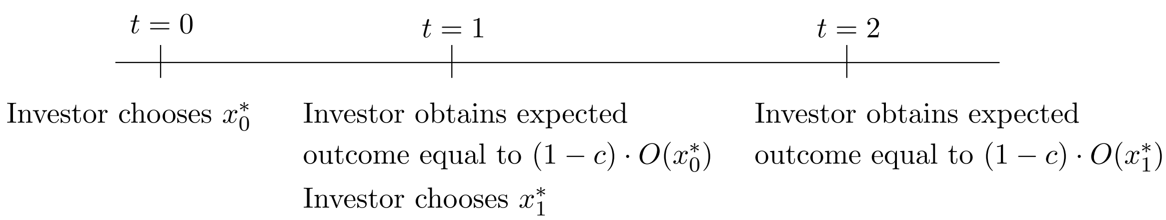
2.5 Time horizon

A further difference between private equity investors and standard investors might be the time horizon of their investments. Private equity funds usually have a limited lifespan of about ten years, whereby acquisitions are made in the first five years and the investments are exited during the last five years (see Metrick/Yasuda (2009)). Strömberg (2008) finds that the average time private equity firms hold a company in their portfolio is four to five years. Critics argue that private equity firms only focus on short-term profits disregarding long-term value enhancing investments. Therefore, we want to analyze whether the option to choose the time horizon of the investment leads to an inefficient outcome in our model.

To incorporate a decision with respect to the time horizon of the investment, we extend the model to two periods. The successful investor can choose between holding the company for one period, which we will denote as the short-term project or for two periods, which we will denote as the long-term project. The short-term project equals the situation in the base case. For x units invested at $t = 0$, the investor receives an expected outcome at $t = 1$ of $O(x)$. If for both investors the short-term project is optimal compared to the long-term choice, the model outcome will therefore be equivalent to the base case: the standard investor chooses both a higher optimal investment volume and a higher maximum acquisition price than the private equity investor, i.e. $i_x = i_p = S$ holds and the situation is efficient.

When choosing the long-term project, the successful investor has the possibility to use the production technology of the overtaken company for two periods. At $t = 0$, the investor decides for an optimal investment volume x_0^* and receives an expected outcome of the production process of $(1 - c) \cdot O(x_0^*)$ at $t = 1$. At $t = 1$, he invests x_1^* and receives an expected outcome equal to $(1 - c) \cdot O(x_1^*)$ at $t = 2$. The factor c, $0 < c < 1$, represents the relative loss of outcome in periods $t = 1$ and $t = 2$ compared to the outcome obtained from a short-term investment. Hence, we can understand c as the cost of the long-term project, where a higher c means a less attractive outcome from the long-term project. The time structure of the long-term project is shown in Figure 3.

Figure 3: Time structure of the long-term project



The expected value contribution obtained by investor i at $t = 1$ when choosing the long-term project consists of the value contribution from the outcome of the first period $t = 1$ and the contribution from the outcome of the second period $t = 2$. Since the outcome of the second period is realized at a later date, we can use the corresponding opportunity cost of capital r_i for discounting the expected outcome. Hence, the total value contribution at $t = 1$ is given by:

$$OF_i = (1-c) \cdot O(x_{0,i}) - (x_{0,i} + p_i^{LT}) \cdot (1 + r_i) + \frac{(1-c) \cdot O(x_{1,i})}{1 + r_i} - x_{1,i}, \ i \in \{S, P\}, \quad (26)$$

where p_i^{LT} is the maximum acquisition price of investor i when choosing the long-term project. Analogous to the previous sections, the optimal investment volumes of investor i can be obtained with the first order conditions and amount to:

$$(1-c) \cdot O'(x_{0,i}^*) = 1 + r_i, \ i \in \{S, P\}, \quad (27)$$

$$(1-c) \cdot O'(x_{1,i}^*) = 1 + r_i, \ i \in \{S, P\}. \quad (28)$$

Due to the time-constant target returns and investment possibilities, the optimal investment volumes in both periods are identical so that it holds:

$$x_{0,i}^* = x_{1,i}^* = x_i^{LT}, \ i \in \{S, P\}. \quad (29)$$

In the case that the long-term project is optimal for both investors, equations (27) and (28) imply that due to $r_S < r_P$ the standard investor would choose a higher investment volume than the private equity investor. This finding gives our next proposition.

Proposition 7 (Investment volume — same investment horizon) *If for both investors the same investment horizon is optimal, the standard investor will invest more than the private equity investor.*

To judge about the efficiency of this investment, we regard the prices the investors are willing to pay. The objective function given in (26) indicates an expression for the maximum acquisition price of investor i for the case that he would choose the long-term project after a successful takeover:

$$p_i^{LT} = \frac{(1-c)\cdot O(x_{0,i}^*)}{1+r_i} - x_{0,i}^* + \frac{(1-c)\cdot O(x_{1,i}^*)}{(1+r_i)^2} - \frac{x_{1,i}^*}{1+r_i}, \; i \in \{S,P\}. \tag{30}$$

Using the fact that $x_{0,i}^* = x_{1,i}^* = x_i^{LT}$, (30) can be simplified to:

$$p_i^{LT} = \left(1 + \frac{1}{1+r_i}\right) \cdot \left(\frac{(1-c_i)\cdot O(x_i^{LT})}{1+r_i} - x_i^{LT}\right), \; i \in \{S,P\}. \tag{31}$$

Inserting the expressions for the optimal investment volumes and differentiating with respect to x_i^{LT}, we can write:

$$\frac{\partial p_i^{LT}}{\partial x_i^{LT}} = -\frac{2\cdot O(x_i^{LT})\cdot O''(x_i^{LT})}{O'(x_i^{LT})^3} - \frac{\left((1-c)\cdot O(x_i^{LT}) - x_i^{LT}\right)\cdot O''(x_i^{LT})}{(1-c)\cdot O'(x_i^{LT})^2}, \; i \in \{S,P\}. \tag{32}$$

This expression is positive due to $(1-c)\cdot O(x_i^{LT}) - x_i^{LT} > 0$ and $O''(x_i^{LT}) < 0$. Thus, like for the short-term project, it also holds for the long-term project that a higher optimal investment volume implies a higher maximum acquisition price. Since the optimal investment volume x_S^{LT} for the standard investor exceeds that of the private equity investor x_P^{LT}, the standard investor will pay a higher acquisition price $p_S^{LT} > p_P^{LT}$ in the case that for both investors the long-term project is optimal.

Proposition 8 (Acquisition price — same investment horizon) *If for both investors the same investment horizon is optimal, the standard investor will acquire the company and the outcome is efficient.*

Proposition 8 implies that the private equity investor will not participate in the market given that both investors prefer the same project length. As the standard investor always invests more, the outcome will be efficient ($i_x = i_p = S$).

The analysis so far has shown that an inefficiency caused by the private equity investor can (if at all) only arise when he opts for a different project length in the case of a successful takeover than the standard investor. To examine whether such a situation is possible, we have to define how the investors choose between the two projects. We assume that a rational investor undertakes the project for which he obtains a higher expected value contribution. In our model, this choice depends on the cost c, the relative loss from holding the company for two periods. Since the expected value contribution obtained by an investor determines the maximum price he is willing to pay for the acquisition, we can evaluate the choice of the investors by comparing the maximum acquisition prices for the two project lengths p_i^{ST} and p_i^{LT}, $i \in \{S, P\}$ dependent upon the cost c for the long-term project. Obviously, the investors choose the long-term project for $c = 0$ and the short-term project for $c = 1$. By equating the expression for p_i^{LT} given in (31) with p_i^{ST} and solving for c, we derive the critical cost $\overline{c}_i$ for which investor i is indifferent between the two projects:

$$\overline{c}_i = 1 - \frac{(1 + r_i) \cdot ((2 + r_i) \cdot x_i^{LT} + (1 + r_i) \cdot p_i^{ST})}{(2 + r_i) \cdot O(x_i^{LT})}, \; i \in \{S, P\}. \tag{33}$$

If the critical cost $\overline{c}_i$ for the long-term project is the same for both investors, they would always choose the same project length. However, as the investors have different target returns and optimal investment volumes, $\overline{c}_S$ does not need to coincide with $\overline{c}_P$. In general, the relationship between $\overline{c}_S$ and $\overline{c}_P$ allows us to draw conclusions about which investor has a preference for the long-term project and which investor rather tends to the short-term project. If, for example, the critical cost $\overline{c}_P$ for the long-term project was below $\overline{c}_S$, then the standard investor would have a more pronounced preference for the long-term project than the private equity investor. This is because for every cost c ($c < \overline{c}_P$), for which the private equity investor decides for the long-term project, the standard investor also prefers the long-term project. However, even for additional costs c of the long-term project with $\overline{c}_P < c < \overline{c}_S$ the standard investor also prefers the long-term project in contrast to the private equity investor. Hence, a relation $\overline{c}_P < \overline{c}_S$ would imply that the preference for long-term projects is higher for S than for P.

To evaluate the impact of potential differences between the investors regarding the preferences for the project length on the investment efficiency, we focus on the case that the aggregated discounted investment volume of the standard investor $x_S^{LT} \cdot \left(1 + \frac{1}{1+r_S}\right)$ for the long-term project is higher than or equal to the investment volume for the short-term project x_S^{ST} given that the long-term project has a cost c

equal to the critical cost $\overline{c}_S$:[4]

$$x_S^{LT} \cdot \left(1 + \frac{1}{1+r_S}\right) - x_S^{ST} \geq 0. \tag{34}$$

Considering the value of the investment volume as a proxy for the effect on social welfare, this definition means that a long-term investment has more favorable consequences for social welfare than a short-term investment. Since x_S^{LT} is increasing with a decreasing cost c, (34) holds for every more favorable long-term project with $c \leq \overline{c}_S$.

Given that condition (34) holds, the critical cost for the long-term project $\overline{c}_P$ of the private equity investor is always below the critical cost $\overline{c}_S$ of the standard investor. As a consequence, for every c, $\overline{c}_P < c < \overline{c}_S$, the standard investor will choose the long-term project in the case of a successful takeover while the private equity investor will choose the short-term project. For other costs c, the choice of the investment horizon of the two investors coincides. This property has the important consequence that the investment horizon of the private equity investor can never exceed that of the standard investor once (34) is valid. The proof for the relation $\overline{c}_P < \overline{c}_S$ is in the appendix.

To derive this relationship, we have not assumed a specific preference of the private equity investor for short-term investments. In this subsection, the only difference between the investors still comes from a higher target return of the private equity investor. This characteristic alone gives him an incentive to opt for a shorter project length. Since the target return in our model is related to the internal rate of return (IRR) of an investment, our theoretical finding is consistent with the view presented in Phalippou (2008) who argues that private equity investors prefer shorter investment horizons in order to ensure a relatively high IRR.

In order to evaluate the relationship between the values of the optimal investment volumes in the case that the standard investor chooses the long-term project while the private equity investor opts for the short-term project, we can focus on the relevant interval of costs $\overline{c}_P < c < \overline{c}_S$ for which S and P choose different project lengths. For those costs c, condition (34) says that the value of the total investment volume of S for the long-term project is higher or equal to the investment volume he spends for the short-term project. As the private equity investor invests less than the standard investor whenever both investors follow the same investment horizon, we can conclude that the optimal investment volume of P equal to x_P^{ST} is also lower

[4]This condition holds for many feasible types of outcome functions.

than the total value of the investment volume of S equal to $x_S^{LT} \cdot \left(1 + \frac{1}{1+r_S}\right)$:

$$x_S^{LT} \cdot \left(1 + \frac{1}{1+r_S}\right) \geq x_S^{ST} > x_P^{ST}.$$

Hence, every situation where the private equity investor acquires the firm would be inefficient.

With analogous arguments as in the base case, we can conclude that the standard investor always outbids the private equity investor, because a higher investment volume implies a higher willingness to pay for the company. In the case that the investors prefer different project lengths, S voluntarily deviates from the choice of P to obtain a higher expected value contribution. Recall that if S chose the same investment horizon as P, the standard investor would still have a higher willingness to pay due to a higher investment volume.

Moreover, if we introduced an exogenous preference for a short-term investment horizon of P which differed from the optimal one from S, we would not find an incentive for the private equity investor to outbid the standard investor. A project length which is not the one that maximizes the expected value contribution lowers the willingness to pay for the acquisition of the private equity investor. Obviously, this price is lower than the maximum acquisition price of the standard investor. As a result, $i_x = i_P = S$ always holds and the outcome is efficient.

Proposition 9 (Flexible choice of investment horizons) *A flexible investment horizon does not give the private equity investor an incentive to outbid the standard investor. In some cases, the private equity investor would choose the short-term project after a successful takeover while the standard investor would choose the long-term project. However, it is impossible that the private equity investor chooses the long-term project while the standard investor would choose the short-term project given that condition (34) holds.*

As a result, having a flexible investment horizon is not a characteristic of private equity firms that is the cause of an inefficient outcome as it does not give an incentive to outbid standard investors. However, a preference for short-term investments can result in non-optimal behavior once the company is allocated to the private equity investor for some other reason.

2.6 Leverage

In this subsection, we regard a frequently criticized characteristic of private equity firms namely the use of higher leverage. As the term "leveraged buyout" implies, private equity transactions are usually financed with a high amount of debt which is assigned to the overtaken company after the acquisition. Axelson et al. (2008) examine the financial structure of 153 LBOs and report that the average leverage ratio of the companies in their sample is much higher than the average leverage ratio in public companies.[5] High leverage plays a central role for the success of private equity transactions. We have already mentioned that increased debt obligations might lower agency problems between management and investors. In addition, higher leverage can be beneficial because of tax reasons. Kaplan (1989b) finds in an analysis of 76 buyouts that tax effects due to higher leverage accounted for a major part of the value increase in the acquired companies after the buyout. The main criticism of a high leverage typically refers to a possibly increasing default risk as well as less scope for attractive investments due to high cash flow consumption of interest and redemption payments. Therefore, we want to examine whether a high leverage has a negative effect on the investment behavior of the private equity investor.

In the base case, the investors cannot use any debt. To capture differences in the leverage ratio between the investors, we assume that the private equity investor now has access to debt while the standard investor still does not have the possibility to raise any debt. At a first glance, this assumption seems to be a strong simplification. But we do not want to model the financing decisions of the investors to explain why private equity investors choose a higher leverage than other investors. Instead, we want to analyze the implications of a (given) higher leverage of the private equity investor on acquisitions and investment decisions.

As the standard investor cannot raise any debt, his maximum acquisition price and optimal investment volume are as in the base case. The private equity investor can raise debt with a nominal value of D at $t = 0$. The expected redemption payment of the private equity investors at $t = 1$ equals $D \cdot (1 + r_D)$, where r_D can be interpreted as the expected cost of debt for the private equity investor. We do not model the probability of a default explicitly, but it is implicitly captured in the expected redemption payment which can be less than the promised redemption payment. We assume that the expected cost of debt for the private equity investor

[5] For an extensive examination of the capital structure of public companies, see Rajan/Zingales (1995).

is lower than his expected target return, i.e. $r_D < r_P$. The difference between r_D and r_P represents the favorability of the credit market conditions for the financing of LBO transactions. While in the years preceding the credit crisis this difference was supposed to be high, i.e. private equity firms had access to comparably cheap debt, financing conditions have changed strongly since the middle of 2007 implying a sharp decrease in the difference between r_D and r_P (see BIS (2008)). Hence, using debt to finance the acquisition of the company implies that the private equity investor has to deploy less equity in the transaction which he can use for his alternative investment possibilities delivering an expected return of r_P. As the private equity investor is risk-neutral, the riskiness involved in the investment as well as the debt does not matter for his decision. For an exogenously given amount of debt D the expected value contribution obtained by the private equity investor at $t = 1$ amounts to:

$$\begin{aligned} OF_P &= O(x_P) - (x_P + p_P - D) \cdot (1 + r_P) - D \cdot (1 + r_D) \\ &= O(x_P) - (x_P + p_P) \cdot (1 + r_P) + D \cdot (r_P - r_D) \end{aligned} \tag{35}$$

The difference compared to the case without leverage is that a lower amount $x_P + p_P - D$ of capital at time $t = 0$ is required, but debt with an expected value equal to $D \cdot (1 + r_D)$ needs to be repaid at time $t = 1$. The representation for OF_P reveals that the optimal investment decision of the private equity investor is not affected by the use of leverage as debt and the acquisition price have the character of sunk costs. Hence, the optimal investment volume x_P^* coincides with that in the base case. Therefore, the relation between the optimal investment volumes of the investors is equal to the base case determined by the different target returns, i.e. $i_x = S$.

Proposition 10 (Investment volume — leverage) *The standard investor will choose a higher investment volume than the private equity investor in the case of a successful takeover.*

Proposition 10 implies that every situation where the private equity investor acquires the company will be inefficient. Therefore, in the case that the use of debt would give the private equity investor an incentive to outbid the standard investor, an inefficient investment situation would arise. Equating the objective function given in (35) to zero and rearranging terms leads to the maximum acquisition price of the private equity investor:

$$p_P = \frac{O(x_P^*)}{1 + r_P} - x_P^* + \frac{D \cdot (r_P - r_D)}{1 + r_P}. \tag{36}$$

We see that the acquisition price p_P of the private equity investor is increasing in the nominal debt volume D and in the difference between r_P and r_D. We can interpret $\frac{D \cdot (r_P - r_D)}{1+r_P}$ as the additional value which the private equity investor can generate by using debt. This value comes from the fact that the cost of debt r_D is relatively cheap compared to the cost of capital r_P for the private equity investor. Therefore, the use of leverage gives the private equity investor an incentive to pay a higher acquisition price without altering its investment decision. Once the debt volume D and the difference between r_P and r_D is sufficiently high, the private equity investor will pay a higher acquisition price than the standard investor although he still invests less. Hence, a situation with $i_x = S$ and $i_p = P$ is possible.

Proposition 11 (Acquisition price — leverage) *The use of debt increases the maximum acquisition price of the private equity investor without changing his investment behavior. Thus, leverage can give the private equity investor an incentive to outbid the standard investor even though he intends to invest less than the standard investor.*

Proposition 11 implies that leverage can cause inefficient situations. Since the investment decisions of both investors are equal to that in the base case, the standard investor chooses a higher investment volume in the case that he is the successful bidder. However, for a sufficiently high debt volume D that is exclusively available for the private equity investor and a high difference between r_P and r_D, the private equity investor is willing to pay a higher acquisition price than the standard investor and takes over the company which causes an inefficient allocation. The inefficiency arises because the acquisition decision of the private equity investor is not linked to its investment decision, which was the case in the previous subsections. Now, the maximum acquisition price is highly dependent on the availability and price of debt. With a greater amount of cheap debt, the private equity investor is willing to pay a higher acquisition price for the company even when the investment possibilities are unchanged. This result is supported by recent empirical studies about the investment behavior of buyout funds. Ljungqvist/Richardson/Wolfenzon (2007) find that buyout funds accelerate their investment flows when credit market conditions loosen. Axelson et al. (2008) find that the economy-wide cost of borrowing seem to drive both leverage and pricing in buyouts. In the years before the start of the credit crisis in the middle of 2007 a large increase in the number and size of buyout transactions could be observed. Our result fits to the notion that this increase was at least partly caused by the high availability of cheap debt at this time. Moreover, it suggests that for the transactions made shortly before the peak of the buyout boom, low interest rates supported very high acquisition prices by private equity investors which may have caused inefficient allocations. As a result, the exclusive possibility to use debt is one motive for why the existence of a private equity investor can result in a loss of social welfare.

2.7 Informational differences

In the last step, we consider the case that the private equity investor has better information about the target company than the standard investor. There are several reasons for possible information advantages of private equity firms. Identifying the "right" companies to invest in is the daily business of private equity funds and one of the core success factors in the private equity business. Therefore, successful private equity firms have gained an expertise in assessing the value of a business based on market intelligence (Berg/Gottschalg (2005)). Furthermore, an extensive organizational and people network combined with a strong industry knowledge can provide private equity firms with access to superior market information, giving them a competitive advantage in the selection of target companies (Fox/Marcus (1992)). There is some empirical evidence about the types of companies that private equity firms acquire. Opler/Titman (1993) find that firms acquired by leveraged buyouts have a comparatively high free cash flow and low growth options. For such firms the agency costs — as discussed in subsection 2.3 — are supposed to be relatively high. These results suggest that private equity firms have the ability to identify those types of companies for which a large value creation is possible.

To incorporate informational differences about the target company in the model, we introduce uncertainty about the expected value μ of the random variable $\widetilde{z}$. In the previous subsections, μ was normalized to one. Now, we assume that μ can have two possible values. With probability π the expected value of $\widetilde{z}$ is $\mu = z^G$, with probability $1-\pi$ the expected value of $\widetilde{z}$ is $\mu = z^B$, where $z^G > z^B$ holds. Therefore, we refer to a good company in the case of z^G, while a bad company is associated with z^B. We consider two different cases of informational asymmetries between the investors.

Case 1: First, we assume that the private equity investor always knows if the company is good or bad while the standard investor cannot differentiate between the two types of companies. Due to the fact that the private equity investor can observe the type of the company, he knows if the expected outcome of the production process will be $z^G \cdot O(x)$ or $z^B \cdot O(x)$. Therefore, the optimal investment volume and the maximum acquisition price of the private equity investor depend on the quality of the company. In the case the company is good (bad), the optimal investment volumes x_P^G (x_P^B) of P are given by:

$$1 + r_P = z^G \cdot O'(x_P^G), \qquad \text{for } \mu = z^G, \tag{37}$$

$$1 + r_P = z^B \cdot O'(x_P^B), \qquad \text{for } \mu = z^B. \tag{38}$$

As the standard investor cannot differentiate between the two types of companies, the objective function of S at $t = 1$ amounts to:

$$OF_S = \left(\pi \cdot z^G + (1 - \pi) \cdot z^B\right) \cdot O(x_S) - (x_S + p_S) \cdot (1 + r_S). \tag{39}$$

The first order condition gives an implicit expression for the optimal investment volume of the standard investor similar to the base case:

$$1 + r_S \quad = \quad O'(x_S^*), \tag{40}$$

where for simplicity purposes we normalized the factor $\pi \cdot z^G + (1 - \pi) \cdot z^B$ to one. As this factor is obviously greater than z^B due to $z^G > z^B$, the standard investor will choose a higher investment volume than the private equity investor given that the company is of the bad type, which implies:

$$x_S^* > x_P^B. \tag{41}$$

In the case the company is good, it follows from (37) and (40) that the investors have the same optimal investment volume for:

$$z^G = \frac{1 + r_P}{1 + r_S}. \tag{42}$$

The condition is similar to the one in subsection 2.3 where the private equity investor could increase the outcome of the production process by the factor a. Depending on the relation between z^G (or a respectively) and $\frac{1+r_P}{1+r_S}$ either the standard investor or the private equity investor invests more.

The maximum acquisition prices of both investors are given by the well known price function from the base case where the price is increasing in the investment volume, whereby for the private equity investor, the price also depends on the company type. Therefore, the standard investor acquires the company if it is of the bad type due to $x_S^* > x_P^B$. As he invests more than the private equity investor would have done, the outcome is efficient ($i_x = i_p = S$). In the case the company is good, the relations between the optimal investment volumes and maximum acquisition prices of the investors are given by:

$$
\begin{aligned}
&i_x = i_p = S, && \text{for } z^G < \frac{1+r_P}{1+r_S},\\
&i_x = i_p = S \text{ or } i_x = i_p = P, && \text{for } z^G = \frac{1+r_P}{1+r_S},\\
&i_x = i_p = P, && \text{for } z^G > \frac{1+r_P}{1+r_S}.
\end{aligned}
$$

Proposition 12 (Informational differences — case 1) *The private equity investor does not participate in the market if the target company is bad. Depending on the degree of his information advantage he may outbid standard investors if the target company is good. However, it always holds: the investor who acquires the company chooses the higher investment volume.*

Since the investor who acquires the company invests more than the other investor would have done, a design of information differences as considered in case 1 implies that no inefficient outcome is possible.

Case 2: In case 1 we assumed that the private equity investor always knows about the type of the company while the standard investor cannot differentiate between the two types. Now we change this assumption slightly. We still assume that the private equity investor has better information about the type of the company before the acquisition. As a difference, we now consider the case that after a takeover the successful investor has access to all available information, i.e. the acquirer will know the type of the company before making the investment decision. The private equity investor still knows the type of the company as in case 1 and therefore the optimal investment volumes and the maximum acquisition prices are equal to case 1. The standard investor does not know the type of the company at the time of the acquisition decision, but he knows the type of the company at the time of the investment decision in the case of a successful takeover. We solve the decision problem of the standard investor S using backward induction. Given S has acquired the company and identifies the company as good (bad), the optimal investment volumes x_S^G (x_S^B) are — analogous to the investment volumes of the private equity investor in case 1 — given by:

$$1 + r_S = z^G \cdot O'(x_S^G), \qquad \text{for } \mu = z^G, \tag{43}$$

$$1 + r_S = z^B \cdot O'(x_S^B), \qquad \text{for } \mu = z^B. \tag{44}$$

Comparing the optimal investment volumes of the investors, we see that the standard investor invests more than the private equity investor independent of the type of the company, i.e. $x_S^G > x_P^G$ and $x_S^B > x_P^B$ holds, which implies $i_x = S$. Therefore, every situation where the private equity investor takes over the company is inefficient. If access to superior information gave the private equity investor an incentive to outbid the standard investor, this would cause an inefficiency. To calculate the maximum acquisition price of the standard investor, we consider his expected value contribution at $t = 1$:

$$\begin{aligned} OF_S &= \pi \cdot \left(z^G \cdot O(x_S^G) - x_S^G \cdot (1 + r_S)\right) \\ &+ (1 - \pi) \cdot \left(z^B \cdot O(x_S^B) - x_S^B \cdot (1 + r_S)\right) - p_S \cdot (1 + r_S) \end{aligned} \tag{45}$$

Equating (45) to zero, rearranging terms and inserting the first order conditions given in equations (43) and (44) yields the following representation for the maximum acquisition price of S:

$$p_S = \pi \cdot \left(\frac{O(x_S^G)}{O'(x_S^G)} - x_S^G\right) + (1 - \pi) \cdot \left(\frac{O(x_S^B)}{O'(x_S^B)} - x_S^B\right) = \pi \cdot p_S^G + (1 - \pi) \cdot p_S^B, \tag{46}$$

with

$$p_S^G = \frac{O(x_S^G)}{O'(x_S^G)} - x_S^G, \tag{47}$$

$$p_S^B = \frac{O(x_S^B)}{O'(x_S^B)} - x_S^B. \tag{48}$$

(47) and (48) can be interpreted as the maximum acquisition prices in the case that the standard investor knows that the target company is of the good type or the bad type, respectively. Hence, the maximum acquisition price of the standard investor is the weighted average of p_S^G and p_S^B. p_S^G and p_S^B have the same structure as the typical price function known from the base case which increases in the optimal investment volume. Therefore, we can examine whether a situation where the private equity investor acquires the company is possible by comparing the optimal investment volumes of the investors. As both x_S^G and x_S^B are greater than x_P^B, the standard investor will pay a higher acquisition price than the private equity investor in the case the company is of the bad type, i.e. $i_x = i_p = S$ holds. Hence, the outcome is efficient.

If the company is of the good type, the situation is less apparent. As $x_S^G > x_P^G$, it holds:

$$p_S^G > p_P. \tag{49}$$

In the case also $p_S^B > p_P$ holds, the maximum acquisition price of the standard investor is obviously higher than the maximum acquisition price of the private equity investor. However, p_S^B is only greater than p_P for the extreme case that the standard investor invests more in the bad state than the private equity investor in the good state, i.e. $x_S^B > x_P^G$. This is true for:

$$\frac{z^G}{z^B} < \frac{1+r_P}{1+r_S}, \tag{50}$$

where the degree of informational inaccuracy $\frac{z^G}{z^B}$ is relatively small. In the opposite case that $p_S^B < p_P$ holds, the relation between the maximum acquisition prices of the investors depends on the probability π that the company is of the good type. From (46) we see that the critical probability $\overline{\pi}$ for which both investors choose the same acquisition price ($p_S = p_P$) is given by:

$$\overline{\pi} = \frac{p_P - p_S^B}{p_S^G - p_S^B}. \tag{51}$$

Obviously, $\overline{\pi}$ lies between zero and one due to $p_S^B < p_P < p_S^G$. As the maximum acquisition price of the standard investor is increasing in π, the relations between the maximum acquisition prices and the optimal investment volumes of the two investors in the case the company is of the good type dependent upon π are given by:

$$\begin{array}{ll} i_x = i_p = S, & \text{for } \pi > \overline{\pi}, \\ i_x = i_p = S \text{ or } i_x = S, i_p = P, & \text{for } \pi = \overline{\pi}, \\ i_x = S, i_p = P, & \text{for } \pi < \overline{\pi}. \end{array}$$

Proposition 13 (Informational differences — case 2) *The private equity investor does not participate in the market if the target company is of the bad type. Depending on his information advantage, he might outbid standard investors if the company is good although he invests less than standard investors in any case. This inefficient situation occurs for a relatively low probability for the firm being of a good type and a high degree of informational inaccuracy* $\frac{z^G}{z^B}$.

Proposition 13 shows that for a design of the information difference as considered in case 2, there is a possibility of an inefficient outcome caused by the private equity investor. Although the standard investor chooses the higher investment volume in any case, there are situations where the private equity investor has the incentive to acquire the company. The likelihood of an inefficient outcome is affected by two factors. A low probability π increases the danger for the standard investor to acquire a bad company. A high difference between z^G and z^B means a relatively low outcome for the standard investor when acquiring a bad company compared to a good company. Both effects, a lower probability π and a higher ratio $\frac{z^G}{z^B}$, decrease the expected value contribution obtained by the standard investor and thus his maximum acquisition price. The private equity investor benefits from his information advantage as he faces no uncertainty about the type of the company when making the acquisition decision. Therefore, he is able to pay a relatively high takeover price in the case the company is good. In other words, the uncertainty about the firm's quality results in lower bids by the standard investor without affecting the investment volume. For a sufficiently low probability π and high difference between z^G and z^B, the maximum acquisition price of the standard investor p_S is so low that the private equity investor acquires the company.

The results of this subsection show that the fact whether the access to superior information of the private equity investor results in an inefficient investment behavior or not depends on the structure of the informational differences between the investors. In both analyzed cases, the private equity investor acquires good firms if his information advantage is sufficiently high. However, an inefficiency arises only in case 2 where the informational differences between the investors disappear after a successful takeover. Having the same information when making the investment decision, the standard investor would choose a higher investment volume than does the private equity investor. In contrast, there is no inefficient investment in case 1 where the informational differences between the investors are permanent. The private equity investor invests more in the case of a successful takeover than the less informed standard investor would have done. Thus, access to superior information by the private equity investor does not inevitably result in a loss of social welfare. However, it can be one possible motive for the private equity investor to outbid the standard investor although he intends to invest less.

3 Conclusion

Even though there is a big debate whether private equity investors are boon or bane for an economy, there have been no convincing arguments that explain which properties of private equity firms are harmful for an economy and which are not. Hence, our analysis aims at verifying which typical characteristics of private equity investors can lead to inefficient investment behavior and therefore to a loss of social welfare and which do not. An inefficient behavior of private equity firms occurs if they have an incentive to acquire a company even though they will invest less than standard investors would do.

Our findings enrich the ongoing debate about the social consequences from private equity investors with the following insights: (i) An inefficient behavior of private equity firms in the sense that they have an incentive to acquire a company although intending to invest less than standard investors would have done, can occur. (ii) However, among the typical characteristics of private equity investors there are only a higher leverage and informational advantages that can provide a motive to act inefficiently. (iii) In contrast, most of the usually offered arguments against private equity transactions such as shorter investment horizons, lower risk aversion, operational improvements, or higher target returns cannot explain inefficient behavior of private equity firms.

In our analysis, we have considered each characteristic of private equity investors separately, while in real world they will show up simultaneously. Hence, the challenge when observing inefficient private equity transactions is to identify which (of multiple) private equity characteristics is the major driver for the inefficiency. For example, inefficient behavior in the real world can be associated with a too short investment horizon. However, as our model explains, the incentive for the private equity investor to outbid standard investors and to invest less does not come from the short-term investment horizon but is rather a result of other factors such as a higher leverage or informational asymmetries. Hence, a major strength of our model is to identify which factors are the main reasons for inefficient behavior and which factors (that are supposed to be a prominent motive for it) do not cause inefficiencies even though it is empirically difficult to distinguish between reasons that drive inefficient investment situations.

Our results have the following implications for the current debate about stricter regulation of private equity firms. To effectively prevent potential inefficient investments by private equity investors, one should focus on the characteristics that are the reason of the inefficient behavior. Therefore, measures to reduce the amount of leverage used in LBOs and to decrease potential information problems of standard investors seem to be most promising. Hence, regulative action, which aims at limiting excessive leverage and increased awareness for better transparency standards will help to limit potential negative effects caused by private equity investors.

Appendix

Proof of the relation between the optimal investment volume x_S^* of the standard investor and the volatility σ of the random variable $\widetilde{z}$.

The first order condition for the optimal investment volume of investor S is given by:

$$O'(x_S^*) \cdot \big(1 - 2 \cdot b \cdot \sigma^2 \cdot O(x_S^*)\big) - (1 + r_S) = 0. \tag{52}$$

We define the left-hand side of the first order condition as f. The partial derivative of f with respect to x_S^* is:

$$\frac{\partial f}{\partial x_S^*} = -2 \cdot b \cdot \sigma^2 \cdot O'(x_S^*)^2 + O''(x_S^*) \cdot (1 - 2 \cdot b \cdot O(x_S^*))\,. \tag{53}$$

As the term $1 - 2 \cdot b \cdot O(x_S^*)$ is positive and $O''(x_S^*)$ is negative, $\dfrac{\partial f}{\partial x_S^*}$ is negative. Therefore, there exists a unique solution for x_S^*.

Using the implicit function theorem the derivative of x_S^* with respect to σ^2 is given by:

$$\frac{dx_S^*}{d\sigma^2} = -\frac{\dfrac{\partial f}{\partial \sigma^2}}{\dfrac{\partial f}{\partial x_S^*}}. \tag{54}$$

The partial derivative of f with respect to σ^2 is:

$$\frac{\partial f}{\partial \sigma^2} = -2 \cdot b \cdot O(x_S^*) \cdot O'(x_S^*). \tag{55}$$

As both $O(x_S^*)$ and $O'(x_S^*)$ are positive, $\dfrac{\partial f}{\partial \sigma^2}$ is negative.

Hence, as both $\dfrac{\partial f}{\partial x_S^*}$ and $\dfrac{\partial f}{\partial \sigma^2}$ are negative, we know from (54) that the derivative of x_S^* with respect to σ^2 is negative. This sign implies that the optimal investment volume x_S^* is decreasing in the volatility σ of the random variable $\widetilde{z}$.

To ensure that the optimal investment volume of the standard investor can be below the optimal investment volume of the private equity investor, i.e. the critical volatility $\overline{\sigma}$ for which $x_S^* = x_P^*$ holds exists, we prove that x_S^* does not converge to a positive limit. We show this property by contradiction. Assume there exists a positive limit $\overline{x}_S$. In this case, the left-hand side of the first order condition given in (52) tends to $-\infty$ for $\sigma \to \infty$. Therefore, $\overline{x}_S > 0$ is not a feasible solution for the limit and x_S^* tends to zero for $\sigma \to \infty$ as x_S^* decreases in σ and does not have a positive limit. Hence, a unique $\overline{\sigma}$ exists.

Proof of the relation between the maximum acquisition price p_S of the standard investor and the volatility σ of the random variable $\widetilde{z}$.

The maximum acquisition price of the standard investor is given by:

$$p_S = \frac{O(x_S^*) \cdot (1 - b \cdot \sigma^2 \cdot O(x_S^*))}{1 + r_S} - x_S^*. \tag{56}$$

The derivative of p_S with respect to σ^2 can be expressed as:

$$\frac{dp_S}{d\sigma^2} = \frac{\partial p_S}{\partial x_S^*} \cdot \frac{\partial x_S^*}{\partial \sigma^2} + \frac{\partial p_S}{\partial \sigma^2}. \tag{57}$$

The partial derivative of p_S with respect to σ^2 is negative since:

$$\frac{\partial p_S}{\partial \sigma^2} = -\frac{b \cdot O(x_S^*)^2}{1 + r_S} < 0. \tag{58}$$

The partial derivative of p_S with respect to x_S^* is:

$$\frac{\partial p_S}{\partial x_S^*} = \frac{O'(x_S^*) \cdot (1 - 2 \cdot b \cdot \sigma^2 \cdot O(x_S^*)) - (1 + r_S)}{1 + r_S}. \tag{59}$$

From the first order condition given in (52) we know that $\frac{\partial p_S}{\partial x_S^*}$ equals zero. Therefore, the derivative of p_S with respect to σ^2 is negative. Hence, the maximum acquisition price of the standard investor is decreasing in the volatility σ of the random variable $\widetilde{z}$. As we know that x_S^* tends to zero for $\sigma \to \infty$, we see from (56) that p_S also tends to zero for $\sigma \to \infty$. Therefore, the maximum acquisition price of the standard investor can have values below the maximum acquisition price of the private equity investor, i.e. the critical volatility $\overline{\overline{\sigma}}$ for which $p_S = p_P$ holds exists.

Proof of the relation between the critical costs for the long-term project $\overline{c}_S$ and $\overline{c}_P$.

To evaluate the relationship between the critical costs $\overline{c}_S$ and $\overline{c}_P$, we consider the difference between the maximum acquisition prices $p_i^{LT} - p_i^{ST}$ for a long-term and short-term project from the perspective of investor i:

$$p_i^{LT} - p_i^{ST} = \left(1 + \frac{1}{1+r_i}\right) \cdot \left(\frac{(1-c) \cdot O(x_i^{LT})}{1+r_i} - x_i^{LT}\right) - \left(\frac{O(x_i^{ST})}{1+r_i} - x_i^{ST}\right). \quad (60)$$

Given a cost for the long-term project of $c = \overline{c}_i$, this difference must equal zero. In what follows, we determine how this difference for a given project cost c changes, if the investor has higher opportunity cost of capital r_i. For this purpose, we consider the partial derivative of the right-hand side of (60) with respect to r_i, taking into account that both x_i^{ST} and x_i^{LT} depend on r_i. Making use of the first order conditions (27) and (28), the derivative $\frac{\partial(p_i^{LT}-p_i^{ST})}{\partial r_i}$ results for a cost $c = \overline{c}_S$ and a target return $r_i = r_S$ in a simplified representation equal to:

$$\left.\frac{\partial(p_i^{LT} - p_i^{ST})}{\partial r_i}\right|_{\{c=\overline{c}_S, r_i=r_S\}} = (1+r_S) \cdot (O(x_S^{ST}) + x_S^{LT}) - (1-\overline{c}_S) \cdot (3+r_S) \cdot O(x_S^{LT}). \quad (61)$$

In the event that equation (61) is negative, we can interpret the sign of the derivative as follows: Due to the critical cost of the long-term project $\overline{c}_S$, the standard investor is indifferent between the two project lengths, while for the private equity investor, who faces a higher target return, the difference $p_i^{LT} - p_i^{ST}$ is negative, i.e. the private equity investor has a strict preference for the short-term project.

To evaluate the sign of (61), we recall condition (34) which says that the aggregated discounted investment volume $x_S^{LT} \cdot \left(1 + \frac{1}{1+r_S}\right)$ for the long-term project is higher than or equal to the investment volume for the short-term project given that the long-term project has a cost c equal to the critical cost $\overline{c}_S$ of the standard investor. As (60) equals zero for $c = \overline{c}_S$ by definition, adding the right-hand side of (60) to condition (34) yields the following inequality:

$$\left(1 + \frac{1}{1+r_S}\right) \cdot \frac{(1-\overline{c}_S) \cdot O(x_S^{LT})}{1+r_S} - \frac{O(x_S^{ST})}{1+r_S} \geq 0. \quad (62)$$

Since for an optimal choice of the investment volume, the expected outcome of the production process per period in the case of the long-term project $(1-\overline{c}_S) \cdot O(x_S^{LT})$ must be higher than the investment volume $x_S^{LT} \cdot (1 + r_S)$ including opportunity costs of capital, it must hold:

$$-(1-\overline{c}_S)\cdot O(x_S^{LT})+x_S^{LT}\cdot(1+r_S)<0. \tag{63}$$

Multiplying (62) by $-(1+r_S)^2$, adding (63), and rearranging terms yields the following condition:

$$(1+r_S)\cdot(O(x_S^{ST})+x_S^{LT})-(1-\overline{c}_S)\cdot(3+r_S)\cdot O(x_S^{LT})<0. \tag{64}$$

The left-hand side of condition (64) is equivalent to expression (61) for the case that the critical cost of the long-term project is $\overline{c}_S$ and the investor has a target return equal to r_S. As this expression is negative, we can conclude that from the perspective of investor P, who has a higher target return r_P, the acquisition price of the short-term project exceeds that of the long-term project for a cost c equal to $\overline{c}_S$. Therefore, the critical cost $\overline{c}_P$ for the private equity investor is lower than $\overline{c}_S$ for the standard investor.

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