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Abstract

This paper assesses the biases of four different estimators with respect to the short run and the long run parameters if a static panel model is used, although the data generating process is a dynamic error components model. We analytically derive the associated biases and provide a discussion of the determinants thereof. Our analytical and numerical results as well as Monte Carlo simulations illustrate that the asymptotic bias of both the within and the between parameter with respect to the short run and long run impact can be substantial, depending on the memory of the data generating process, the length of the time series and the importance of the cross-sectional variation in the explanatory variables.

Key words: Short run effects; Long run effects; Small sample bias; Panel econometrics

JEL classification: C23

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1 Introduction[1]

Since the seminal work of Kuh (1959) and Houthakker (1965), it is argued that in static panel models the long run effects are captured by the between estimates, while the within estimates represent short run effects. In many applications, the data comprise a panel with a large cross-section dimension, but only a few observations over time. Often static models are estimated in such short and fat panels, while a dynamic one should be appropriate. In this case, it is not clear what in fact is estimated and it is important to investigate the approximation error of static panel models with respect to the short run and the long run effects.[2]

Baltagi and Griffin (1984) consider the case of underspecified lag dynamics in an ADL(0,k) error components model. Based on Monte Carlo simulations, they conclude that "the [pooled] OLS estimator provides a robust estimator of the long run ... elasticity under alternative degrees of misspecification, variance components, and time series observations. In contrast, the within estimator offers a good estimator of the short run effects, but can severely underestimate the long run response" (ibid., p. 643).

Van den Doel and Kiviet (1994) provide analytical results on the asymptotic consequences of estimating static models (pooled OLS, fixed effects and random effects) although the true data generating process is an ADL(1,1) model, considering either a stationary or a non-stationary explanatory variable. They conclude that "static estimators usually underestimate the long

[1]We are grateful to Maurice Bun, Sylvia Kaufmann, Robert Kunst, Jan Mutl, Alain Pirotte, Ingmar Prucha, Andrea Weber and two anoymous referees for helpful comments.

[2]See Baltagi (2001) also for chaper 10.6 for an overview on this issue.

run effect", if the explanatory variable is stationary. In addition, they show that "the estimators are consistent for the long run effect" (ibid. p. 82), if the explanatory variable follows a random walk and if the impact of the initial conditions is negligible. Van den Doel and Kiviet (1995) provide extensive Monte Carlo simulations using a simple partial adjustment model with one contemporaneous explanatory variable as the data generating process. Their simulation results demonstrate that the consequences of incorrectly estimating a static model can be rather serious. Specifically, they conclude that "the long run coefficients will usually be underestimated when the Fixed Effects 'within' estimator is employed" (ibid, p. 360). In addition, they also present two exact similar tests for the presence of neglected dynamics.

Pirotte (1999) assumes a general, ADL(l,k) dynamic error components model as the data generating process and investigates, whether the pooled OLS, within and between estimates, incorrectly assuming a static model, approximate the short and long run effects properly. He calculates the corresponding probability limits numerically for a set of specific parameter values, finding that the "probability limit of the within estimator tends to [approximate, P.E., M.P.] the short run effects ..." (ibid, p. 155), if the data generating process of the explanatory variable exhibits no memory, whatever the memory of the dependent variable (ibid., p. 155). He furthermore concludes that "the probability limit of the between estimator of the static model converges, in all cases, to long run effects" and that "long run effects are obtained directly from the static relation without the need of a dynamic model" (ibid., p. 155).

This paper extends the previous literature in serval respects. Assuming a general, ADL(l,k) dynamic error components model as the data generating process, we provide further theoretical results on the asymptotic (omission) biases of the within (W), between (B), random effects (RE) and pooled OLS estimates, which incorrectly assume a static model. In the literature the within estimator is commonly associated with the short run impact. Below, we refer to the deviation of the estimated within parameter from the true short run effect as BS_W. Analogously, we denote the deviation of the estimated between parameter from the true long run parameter as BS_B. In case of one explanatory variable without memory and positive impact parameters, we proof that both BS_W and BS_B are always negative. Based on these results, the asymptotic biases of the random effects and of the pooled OLS estimator are derived. For the more general case of an explanatory variable generated by an AR(1) process, we also derive some analytical results and provide graphical illustrations. Moreover, we assess the small sample properties of the bias of these estimators with respect to both the short run and the long run impact in a Monte Carlo experiment.

We demonstrate that for fixed T and large N disregarding the dynamic process (i.e., the lagged endogenous variable) may result in inconsistent within *and* between estimates. Specifically, both BS_W and BS_B are the larger in absolute value, the slower the adjustment of the dependent variable (i.e., the higher the memory of the dependent variable) and, by and large, the lower the memory of the explanatory variable. Neither BS_W nor BS_B vanishes asymptotically for large N and fixed T. But, given a low memory of the explanatory variable, both BS_W and BS_B decrease in absolute value as T grows large. However, if this memory is pronounced, BS_W may change its sign and - at small T - even increase with T in absolute value. In our Monte Carlo experiments, the estimated within parameter is substantially biased with respect to the short run and even more so with respect to the long run impact, if the memory of both processes is high. The approximation of the long run impact by the between estimate is especially poor, if the memory of the dependent variable is high. In sum, both the estimated within *and* between parameters can substantially deviate from the true short run and long run parameters, respectively. The random effects estimator, as a weighted average of the within and between estimator, outperforms the within estimator in some cases.

The next section introduces a dynamic error components model as the data generating process and evaluates the corresponding asymptotic (lagged endogenous variable) omission biases of four different estimators, when incorrectly assuming a static model instead of the true dynamic one. Section 3 reports the analytical results and Section 4 presents the findings of the Monte Carlo simulation exercises. The last section summarizes the main findings.

2 The basic model and the omission bias

We confine our analysis to one exogenous variable (x_{it}) and assume that the data generating process is given by the following dynamic error components model:[3]

$$\Gamma(L)\,y_{it} = B(L)\,x_{it} + \mu + \alpha_i + \varepsilon_{it},\ \ i = 1,...,N;\ \ t = 1,...,T \qquad (1)$$

$\Gamma(L) = 1 - \gamma_1 L -\ ...\ -\gamma_l L^l$ (with L denoting the lag operator) is the lag polynomial describing the memory of the dependent variable and $B(L) = \beta_0 + \beta_1 L + ... + \beta_k L^k$ is the lag polynomial describing the impact of the exogenous variable. We assume that the roots of $\Gamma(L)$ are outside the unit circle so that $\Gamma(L)^{-1} = \sum_{\tau=0}^{\infty} \delta_\tau L^\tau$ exists and is finite. μ is the constant, the random individual effects are $\alpha_i \sim IID(0, \sigma_\alpha^2)$ and the remainder error is $\varepsilon_{it} \sim IID(0, \sigma_\varepsilon^2)$. We assume that x_{it} is doubly exogenous (see Cornwell et al., 1992), i.e., $E[x_{it}\alpha_i] = 0$ and $E[x_{it}\varepsilon_{it}] = 0$. Furthermore, we only look at stationary data where both the left hand side variable y_{it} and the explanatory right hand side variable x_{it} are $I(0)$. Hence, we do neither consider a unit root in x_{it} or y_{it} nor a cointegrating relationship between them. In vector form, the true model reads:

$$\Gamma(L)\,\mathbf{y} = B(L)\,\mathbf{x} + \mu\iota_{NT} + \mathbf{Z}_\alpha \alpha + \varepsilon, \qquad (2)$$

where $\mathbf{y}$ is $(NT \times 1)$, $\mathbf{x}$ is $(NT \times 1)$, α is a $(N \times 1)$ vector of random effects and $\mathbf{Z}_\alpha$ is $\mathbf{I}_N \otimes \iota_T$ with ι_T $(T \times 1)$ vector of ones. We define the lag polynomial $\Theta(L) = \frac{B(L)}{\Gamma(L)} = \sum_{\tau=0}^{\infty} \theta_\tau L^\tau$. Following Brockwell and Davies (1991) one has $\theta_0 = \beta_0$, $\theta_\tau = \beta_\tau + \sum_{0<j\leq\tau} \gamma_j \theta_{\tau-j}$ for $0 \leq \tau < \max(l,\ k+1)$ and $\theta_\tau = \sum_{0<j\leq\tau} \gamma_j \theta_{\tau-j}$ for $\tau \geq \max(l, k+1)$. For example, at $l = 1$ and $k = 1$ we

[3] The literature has predominantly concentrated on this case. The model cannot easily be generalized to one with more than one explanatory variable. Only as long as the regressors are orthogonal, the generalization is straight forward. If this is not the case, analytical results are much harder, if not impossible, to obtain.

have $\theta_\tau = \gamma_1^{\tau-1}(\gamma_1\beta_0 + \beta_1)$ for $\tau \geq 1$. We assume that in both the short run and the long run x exerts a positive effect on y, i.e., $\theta_0 > 0$ and $\Theta(1) > 0$, respectively.

Under stationarity, there exists a MA-representation of the true model (1) and we can use the lag polynomial $\Theta(L)$ to reformulate the true model (see van den Doel and Kiviet, 1994, and Pirotte, 1999).

$$\begin{aligned} y_{it} &= \Theta(L)\, x_{it} + \frac{\mu + \alpha_i + \varepsilon_{it}}{\Gamma(L)} \qquad (3) \\ &= x_{it}\theta_0 + \overline{x}_i(\Theta(1) - \theta_0) + \sum_{j=0}^{\infty}(x_{it-j-1} - \overline{x}_i)\,\theta_{j+1} + \frac{\mu + \alpha_i}{\Gamma(1)} + \frac{\varepsilon_{it}}{\Gamma(L)} \\ &= x_{it}\theta_0 + \overline{x}_i\widetilde{\varphi} + \sum_{j=0}^{\infty}(x_{it-j-1} - \overline{x}_i)\,\theta_{j+1} + \widetilde{\mu} + \widetilde{\alpha}_i + u_{it}, \end{aligned}$$

where $\widetilde{\varphi} = \Theta(1) - \theta_0$, $\widetilde{\mu} = \frac{\mu}{\Gamma(1)}$, $\widetilde{\alpha}_i = \frac{\alpha_i}{\Gamma(1)}$ and $\overline{x}_i = \frac{1}{T}\sum_{t=1}^{T} x_{it}$. This reformulation of the true model furthermore implies that the error term is now given by $u_{it} = \frac{\varepsilon_{it}}{\Gamma(L)} = \sum_{\tau=0}^{\infty}\delta_\tau\varepsilon_{it-\tau}$ with $\sigma_u^2 = \sigma_\varepsilon^2\sum_{\tau=0}^{\infty}\delta_\tau^2$.

In Pirotte (1999) there are numerical results on the probability limits of the static within, between and pooled OLS estimators available for this general case. Below, we provide analytical results for the ADL(l,k) model, but also look at the restricted cases ADL(1,1) and ADL(1,k).

We assume an infinite history of the explanatory variable, so that the source of the asymptotic bias is $\sum_{j=0}^{\infty}(x_{it-j-1} - \overline{x}_i)\,\theta_{j+1}$. It can immediately be seen that this term reduces to $\sum_{j=0}^{k}(x_{it-j-1} - \overline{x}_i)\,\beta_{j+1}$, if $\gamma_\tau = 0$, $\tau = 1, ..., l$ and the underlying model collapses to that analyzed in Baltagi and Griffin (1984). The assumption of an infinite history may not always be plausible, and there are several possibilities to introduce an initial condition (Hsiao, 2003).[4] In the following, we concentrate on an infinite history, however.

In order to assess the asymptotic bias when estimating a static model although the true one is dynamic as specified in (1), assume the within ($\widehat{\beta}_W$), between ($\widehat{\beta}_B$), random effects ($\widehat{\beta}_{RE}$) and pooled OLS estimators ($\widehat{\beta}_{OLS}$) are applied to the following static model:

$$\mathbf{y} = \mathbf{x}\beta + \mu\iota_{NT} + \mathbf{Z}_\alpha\alpha + \varepsilon \tag{4}$$

Similar to Baltagi and Griffin (1984), van den Doel and Kiviet (1994, 1995), Pirotte (1999), and others we consider a dynamic data generating process for the right hand side variable x_{it}, namely $x_{it} = \lambda x_{it-1} + \zeta_i + \eta_{it}$ with $|\lambda| < 1$, $\zeta_i \sim N\left(0, \sigma_\zeta^2\right)$, $\eta_{it} \sim N\left(0, \sigma_\eta^2\right)$, $E\left[\zeta_i \eta_{it}\right] = 0$. In Section 3, we derive the asymptotic bias of the four estimators with respect to both the short run and the long run impact for the case $\lambda = 0$ and refer to the Appendix for the more complicated derivation in the case $0 < \lambda < 1$ (i.e., x_{it} follows an AR(1) process). Based on the results in the Appendix, we numerically analyze the more general case $0 < \lambda < 1$.

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[4]In the simplest case, one would assume that $y_{i0} = \mu + \alpha_i + \varepsilon_{i0}$. For the ADL(1,1) case, the true model can be reformulated as follows: $x_{it}\beta_0 + \overline{x}_i\overline{\varphi} + \overline{\varphi}\sum_{j=0}^{t-2}\gamma_1^j \frac{1-\gamma_1}{1-\gamma_1^{t-1}}\left(x_{it-j-1} - \overline{x}_i\right) + \gamma_1^{t-1}\beta_1 x_{i0} + \gamma_1^t y_{i0} + \frac{\left(1-\gamma_1^t\right)(\mu+\alpha_i)}{1-\gamma_1} + \sum_{j=0}^{t-1}\gamma_1^j \varepsilon_{it-j} = x_{it}\beta_0 + \overline{x}_i\overline{\varphi} + \overline{\varphi}\sum_{j=0}^{t-2}\gamma_1^j \frac{1-\gamma_1}{1-\gamma_1^{t-1}}\left(x_{it-j-1} - \overline{x}_{i,t}\right) + \gamma_1^{t-1}\beta_1 x_{i0} + \frac{\left(1-\gamma_1^{t+1}\right)(\mu+\alpha_i)}{1-\gamma_1} + u_{it}$, with $\overline{\varphi} = \frac{1-\gamma_1^{t-1}}{1-\gamma_1}\left(\gamma_1\beta_0 + \beta_1\right)$, $u_{it} = \gamma_1 u_{it-1} + \varepsilon_{it}$, $u_{i0} = \varepsilon_{i0}$ and $u_{i\tau} = 0$ for $\tau < 0$.

Note, this formulation would complicate the analysis considerably. First, $\overline{\varphi}$ now depends on t. Secondly, the asymptotic bias additionally includes the starting value of the explanatory variable. Thirdly, the random effects are multiplied by $\frac{\left(1-\gamma_1^{t+1}\right)}{1-\gamma_1}$ and are not time invariant any more. Hence, there is no possibility to introduce an alternative initial condition

3 Results

Consider Case I of no memory in x: $\lambda = 0$. Comparing (3) and (4), it is evident that this static model is misspecified. To derive the probability limit of $\widehat{\beta}_W$, we apply the within transformation $\mathbf{Q} = \mathbf{I} - \mathbf{P}$ on the true model (3), with $\mathbf{P} = \mathbf{I}_N \otimes \overline{\mathbf{J}}_T / T$ and $\overline{\mathbf{J}}_T$ as a $(T \times T)$ matrix of ones:

$$\mathbf{Qy} = \mathbf{Qx}\theta_0 + \sum_{j=0}^{\infty} \theta_{j+1} \mathbf{Qx}_{-j-1} + \mathbf{Qu}, \tag{5}$$

where $\mathbf{x}_{-j-1}$ is $(NT \times 1)$ and includes the elements $(x_{i1-j-1}, x_{i2-j-1}, ..., x_{iT-j-1})'$. Applying the within estimator to static model (4) gives $\widehat{\beta}_W = (\mathbf{x}'\mathbf{Qx})^{-1} \mathbf{x}'\mathbf{Qy}$ and it can immediately be seen that this estimator suffers from an omitted variable bias from excluding $\sum_{j=0}^{\infty} \theta_{j+1} (\mathbf{x}'\mathbf{Qx})^{-1} \mathbf{x}'\mathbf{Qx}_{-j-1}$. The implied probability limit for large N can be derived as follows.

Proposition 1 *For fixed T, $\lambda = 0$, an infinite history of exogenous x_{it} and $\theta_j > 0$, $j = 0, ...T-2$*

$$\begin{aligned} p \lim_{N \to \infty} \widehat{\beta}_W &= \theta_0 - \frac{1}{T(T-1)} \sum_{j=0}^{T-2} \theta_{j+1} (T - j - 1) \\ &: = \theta_0 + BS_W \text{ with } BS_W < 0, \end{aligned} \tag{6}$$

where $j^* = int(\frac{T-3}{2})$, with int denoting the corresponding integer value. Specifically, for $l = 0$, we have $\theta_j = \beta_j$, $j = 0, ..., k$. For $l = k = 1$, it holds:

$$BS_W = -\frac{\widetilde{\varphi}}{T(T-1)(1-\gamma_1)} \left[T - \frac{1-\gamma_1^T}{1-\gamma_1} \right]. \tag{7}$$

Proof. Since $p \lim_{N \to \infty} \left[\mathbf{x}' \mathbf{Qu} \right] = 0$ and because x_{it} is exogenous and uncorrelated with the error term, it suffices to look at $p \lim_{N \to \infty} \left[\mathbf{x}' \mathbf{Qx} \right]$ and

$p\lim_{N\to\infty}\left[\mathbf{x}^{'}\mathbf{Q}\mathbf{x}_{-j-1}\right]$. For $j<T-1$, it follows that

$$p\lim_{N\to\infty}[\left(x_{it}-\frac{1}{T}\sum_{\tau=1}^{T}x_{i\tau}\right)\left(x_{it-j-1}-\frac{1}{T}\sum_{\tau=1}^{T}x_{i\tau-j-1}\right)]=$$

$$p\lim_{N\to\infty}[x_{it}x_{it-j-1}-x_{it}\frac{1}{T}\sum_{\tau=1}^{T}x_{i\tau-j-1}-x_{it-j-1}\frac{1}{T}\sum_{\tau=1}^{T}x_{i\tau}$$
$$+\frac{1}{T^2}\sum_{\tau=1}^{T}\sum_{\tau'=1}^{T}x_{i\tau}x_{i\tau'-j-1}]$$

For $j\leq j^*=int(\frac{T-3}{2})$ this probability limit amounts to

$$\begin{aligned}
0-\frac{1}{T}\sigma_\eta^2-0+\frac{T-j-1}{T^2}\sigma_\eta^2,\text{ if }t=1,...,j+1\\
0-\frac{1}{T}\sigma_\eta^2-\frac{1}{T}\sigma_\eta^2+\frac{T-j-1}{T^2}\sigma_\eta^2,\text{ if }t=j+2,...,T-j-1\\
0-0-\frac{1}{T}\sigma_\eta^2+\frac{T-j-1}{T^2}\sigma_\eta^2,\text{ if }t=T-j,...,T.
\end{aligned}$$

Summing over t gives for each i

$$\frac{\sigma_\eta^2}{T}\left(-T-(T-2j-2)+T-j-1\right)=-\frac{(T-j-1)}{T}\sigma_\eta^2.$$

For $j^*<j<T-1$, the probability limit is

$$\begin{aligned}
0-\frac{1}{T}\sigma_\eta^2-0+\frac{T-j-1}{T^2}\sigma_\eta^2,&\text{ if }t=1,...,T-j-1\\
0-0-0+\frac{T-j-1}{T^2}\sigma_\eta^2,&\text{ if }t=T-j,..,j+1\text{ and }j>int(\tfrac{T-1}{2})\\
0-0-\frac{1}{T}\sigma_\eta^2+\frac{T-j-1}{T^2}\sigma_\eta^2,&\text{ if }t=j+2,...,T.
\end{aligned}$$

Summing over t gives for each i

$$\frac{\sigma_\eta^2}{T}\left(-2\left(T-j-1\right)+T-j-1\right)=-\frac{(T-j-1)}{T}\sigma_\eta^2.$$

For $j \geq T-1$, the probability limit is 0. Furthermore,

$$\begin{aligned}
& p\lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} \left[\left(x_{it} - \frac{1}{T} \sum_{\tau=1}^{T} x_{i\tau} \right) \left(x_{it} - \frac{1}{T} \sum_{\tau=1}^{T} x_{i\tau} \right) \right] \\
= & \left(T\sigma_{\zeta}^2 + T\sigma_{\eta}^2 - 2T\sigma_{\zeta}^2 - 2\sigma_{\eta}^2 + T\sigma_{\zeta}^2 + \sigma_{\eta}^2 \right) = (T-1)\,\sigma_{\eta}^2 .
\end{aligned}$$

Combining these two results, the probability limit of the asymptotic bias BS_W of the within parameter amounts to

$$\begin{aligned}
BS_W &= p\lim_{N\to\infty} \left[\sum_{j=0}^{T-2} \theta_{j+1} (\mathbf{x}'\mathbf{Q}\mathbf{x})^{-1} \mathbf{x}'\mathbf{Q}\mathbf{x}_{-j-1} \right] \\
&= \sum_{j=0}^{T-1} \theta_{j+1} p\lim_{N\to\infty} \left((\mathbf{x}'\mathbf{Q}\mathbf{x})^{-1} \right) p\lim_{N\to\infty} \left(\mathbf{x}'\mathbf{Q}\mathbf{x}_{-j-1} \right) \\
&= -\frac{1}{T\,(T-1)} \sum_{j=0}^{T-2} \theta_{j+1}\,(T-j-1) < 0,
\end{aligned}$$

since $1 \leq j \leq T-2$ and $\theta_j > 0$ for all j. Note that $\Theta(1) > 0$ is neither a necessary nor a sufficient condition for $\sum_{j=0}^{T-1} \theta_{j+1} > 0$.

If $l = 0$, we have $\theta_j = \beta_j$ for $j \leq k$ and $\beta_j = 0$ for $j > k$. If $l = 1$ and $k = 1$, we have $\theta_0 = \beta_0$, $\theta_1 = \beta_1 + \beta_0\gamma_1$, $\theta_\tau = \gamma_1^{\tau-1}\theta_{\tau-1}$ for $\tau > 1$, so that BS_W is given by

$$\begin{aligned}
BS_W &= -\frac{1}{T\,(T-1)} \sum_{j=0}^{T-2} \theta_{j+1}\,(T-j-1) \\
&= -\frac{\beta_1 + \beta_0\gamma_1}{T\,(T-1)} \sum_{j=0}^{T-2} \gamma_1^j\,(T-j-1) \\
&= -\frac{\widetilde{\varphi}}{T\,(T-1)\,(1-\gamma_1)} \left[T - \frac{1-\gamma_1^T}{1-\gamma_1} \right] < 0,
\end{aligned}$$

since $\beta_1 + \beta_0\gamma_1 > 0$ by assumption and $j < T-1$. Note that in this case $\widetilde{\varphi} = \frac{\beta_1+\beta_0\gamma_1}{1-\gamma_1}$. ■

For large N, fixed T, and $\theta_j > 0$ for all j (if $l = k = 1$ for $\widetilde{\varphi} > 0$ and $\gamma_1 > 0$), the within estimate tends to underestimate the true short run impact, especially, in short panels (i.e., small T) and with high persistence parameters θ_j. However, it can immediately be seen that the bias vanishes, if T approaches infinity as long as all roots of Θ lie outside the unit circle (i.e., for $l = k = 1$, $|\gamma_1| < 1$).[5]

The probability limit of $\widehat{\beta}_B$ is derived by applying the between transformation to (3):

$$\begin{aligned}\mathbf{Py} &= \mathbf{Px}^* \begin{pmatrix} \theta_0+\widetilde{\varphi} \\ \mu \end{pmatrix} + \sum_{j=0}^{\infty} \theta_{j+1} \mathbf{P} \left(\mathbf{x}_{-j-1} - \mathbf{Px} \right) \\ &\quad + \mathbf{PZ}_{\alpha} \widetilde{\alpha} + \mathbf{Pu}, \end{aligned} \tag{8}$$

defining the $NT \times 2$ matrix $\mathbf{x}^* = [\mathbf{x}, \iota_{NT}]$. The between estimator based on (4) is given by $\left(\widehat{\beta}_B, \widehat{\mu}_B\right)' = \left(\mathbf{x}^{*\prime}\mathbf{Px}^*\right)^{-1} \mathbf{x}^{*\prime}\mathbf{Py}$ and it is also asymptotically biased.

Proposition 2 *For fixed T, $\lambda = 0$, an infinite history of exogenous x_{it}, and $\theta_j > 0$ for all j*

$$\begin{aligned} p \lim_{N\to\infty} \left[\widehat{\beta}_B\right] &= \theta_0 + \widetilde{\varphi} - \frac{\sigma_\eta^2}{T\sigma_\zeta^2 + \sigma_\eta^2} \left[\sum_{j=0}^{T-2} \theta_{j+1} \frac{j+1}{T} + \sum_{j=T-1}^{\infty} \theta_{j+1} \right] \\ : &= \theta_0 + \widetilde{\varphi} + BS_B \text{ with } BS_B < 0. \end{aligned} \tag{9}$$

For $l = k = 1$, we have

$$BS_B = - \left[\frac{1-\gamma_1^T}{1-\gamma_1} \right] \frac{\sigma_\eta^2 \widetilde{\varphi}}{T \left(T\sigma_\zeta^2 + \sigma_\eta^2 \right)}.$$

[5] Since $-(T-j-1) < -1$ for all $j < T-1$, we have:
$-\lim_{T\to\infty} \left[\frac{1}{T(T-1)} \sum_{j=0}^{T-2} \theta_{j+1} (T-j-1) \right] < -\lim_{T\to\infty} \left[\frac{1}{T(T-1)} \right] \lim_{T\to\infty} \left[\sum_{j=0}^{T-2} \theta_{j+1} \right] = 0\widetilde{\varphi} = 0.$

Proof. In addition to $p\lim_{N\to\infty}\left[\mathbf{x}'\mathbf{Pu}\right]=0$, $p\lim_{N\to\infty}\left[\mathbf{x}'\mathbf{PZ}_{\alpha}\widetilde{\alpha}\right]=0$ holds by assumption.

$$p\lim_{N\to\infty}\begin{bmatrix}\widehat{\beta}_B\\ \widehat{\mu}\end{bmatrix}=\begin{bmatrix}\theta_0+\widetilde{\varphi}\\ \mu\end{bmatrix}+p\lim_{N\to\infty}\left[\sum_{j=0}^{\infty}\theta_{j+1}\left((\mathbf{x}^{*\prime}\mathbf{Px}^{*})^{-1}\mathbf{x}^{*\prime}\mathbf{Px}_{-j-1}-\iota_2\right)\right] \tag{11}$$

Now use the formula of the partitioned inverse (Hsiao, 2003, pp. 36-37) and look at the first row, $p\lim_{N\to\infty}\left[\widehat{\beta}_B\right]$. For $j<T-1$, one obtains:

$$\begin{aligned}p\lim_{N\to\infty}\left[\frac{1}{N}\sum_{i=1}^{N}\left(\frac{1}{T}\sum_{\tau=1}^{T}x_{i\tau}\right)\left(\frac{1}{T}\sum_{\tau=1}^{T}x_{i\tau-j-1}\right)\right] &= \sigma_\zeta^2+\frac{T-j-1}{T^2}\sigma_\eta^2,\\ p\lim_{N\to\infty}\left[\frac{1}{N}\sum_{i=1}^{N}\left(\frac{1}{T}\sum_{\tau=1}^{T}x_{i\tau}\right)\left(\frac{1}{T}\sum_{\tau=1}^{T}x_{i\tau}\right)\right] &= \sigma_\zeta^2+\frac{1}{T}\sigma_\eta^2,\end{aligned}$$

while for $j\geq T-1$ we have

$$p\lim_{N\to\infty}\left[\frac{1}{N}\sum_{i=1}^{N}\left(\frac{1}{T}\sum_{\tau=1}^{T}x_{i\tau}\right)\left(\frac{1}{T}\sum_{\tau=1}^{T}x_{i\tau-j-1}\right)\right]=\sigma_\zeta^2.$$

Furthermore,

$$p\lim_{N\to\infty}\left[\frac{1}{NT}\sum_{i=1}^{N}\sum_{\tau=1}^{T}x_{i\tau}\right]=p\lim_{N\to\infty}\left[\frac{1}{NT}\sum_{i=1}^{N}\sum_{\tau=1}^{T}x_{i\tau-j-1}\right]=0.$$

Collecting terms yields

$$p\lim_{N\to\infty}\left[\sum_{j=0}^{\infty}\theta_{j+1}\left(\frac{\frac{1}{N}\sum_{i=1}^{N}\overline{x}_i\overline{x}_{i(-j-1)}-\left(\frac{1}{N}\sum_{i=1}^{N}\overline{x}_i\right)\left(\frac{1}{N}\sum_{i=1}^{N}\overline{x}_{i(-j-1)}\right)}{\frac{1}{N}\sum_{i=1}^{N}\overline{x}_i^2-\left(\frac{1}{N}\sum_{i=1}^{N}\overline{x}_i\right)^2}-1\right)\right]=$$

$$\sum_{j=0}^{T-2}\theta_{j+1}\left(\frac{\sigma_\zeta^2T^2+(T-j-1)\,\sigma_\eta^2}{T^2\sigma_\zeta^2+T\sigma_\eta^2}-1\right)+\sum_{j=T-1}^{\infty}\theta_{j+1}\left(\frac{T\sigma_\zeta^2}{T\sigma_\zeta^2+\sigma_\eta^2}-1\right)=$$

$$-\frac{\sigma_\eta^2}{T\sigma_\zeta^2+\sigma_\eta^2}\left[\sum_{j=0}^{T-2}\theta_{j+1}\frac{j+1}{T}+\sum_{j=T-1}^{\infty}\theta_{j+1}\right]<0,$$

if $\theta_{j+1} > 0$ for all j. For $l = k = 1$, we have

$$BS_B = -\widetilde{\varphi}(1-\gamma_1)[\sum_{j=0}^{T-2}\gamma_1^j\left(\frac{\sigma_\zeta^2 T^2 + (T-j-1)\,\sigma_\eta^2}{T^2\sigma_\zeta^2 + T\sigma_\eta^2} - 1\right) + \sum_{j=T-1}^{\infty}\gamma_1^j\left(\frac{T\sigma_\zeta^2}{T\sigma_\zeta^2+\sigma_\eta^2} - 1\right)]$$

$$= -\frac{\sigma_\eta^2(1-\gamma_1)\widetilde{\varphi}}{T\sigma_\zeta^2+\sigma_\eta^2}\left[\sum_{j=0}^{T-2}\frac{\gamma_1^j(j+1)}{T} + \sum_{j=T-1}^{\infty}\gamma_1^j\right]$$

$$= -\left[\gamma_1\left(\frac{1-\gamma_1^{T-1}}{1-\gamma_1}\right) - (T-1)\,\gamma_1^{T-1} + \left(1-\gamma_1^{T-1}\right) + T\gamma_1^{T-1}\right]\frac{\sigma_\eta^2\widetilde{\varphi}}{T\left(T\sigma_\zeta^2+\sigma_\eta^2\right)}$$

$$= -\left[\frac{1-\gamma_1^T}{1-\gamma_1}\right]\frac{\sigma_\eta^2\widetilde{\varphi}}{T\left(T\sigma_\zeta^2+\sigma_\eta^2\right)} < 0.$$

■

The asymptotic bias of the constant (μ) can be analogously derived. It is evident that in absolute terms BS_B shrinks, if T grows large, if the persistence parameter θ_j (γ_1 in the case of $l = k = 1$) becomes smaller or if the between variation in the right hand side variable (σ_ζ^2) increases. If T approaches infinity, BS_B tends to zero under the present assumptions.[6]

Now assume a random effects estimator is applied to model (4), although the dynamic one (3) is true. Defining $\psi = \frac{\sigma_u^2}{T\sigma_\alpha^2+\sigma_u^2}$, the random effects estimator for known ψ is $\left(\widehat{\beta}_{RE}, \widehat{\mu}_{RE}\right)' = \left(\mathbf{x}^{*\prime}\left(\mathbf{Q}+\psi\mathbf{P}\right)\mathbf{x}^*\right)^{-1}\mathbf{x}^{*\prime}\left(\mathbf{Q}+\psi\mathbf{P}\right)\mathbf{y}$. From Propositions 1 and 2, we can easily derive the asymptotic bias of this estimator. Setting $\psi = 1$ gives the pooled OLS estimator. Hence, the following proposition also applies to pooled OLS.

[6]It can immediately seen that - $\lim_{T\to\infty}\left[-\frac{\sigma_\eta^2}{T\sigma_\zeta^2+\sigma_\eta^2}\right] = 0$. Since $j+1 < T$, $\lim_{T\to\infty}\left[\sum_{j=0}^{T-2}\theta_{j+1}\frac{j+1}{T} + \sum_{j=T-1}^{\infty}\theta_{j+1}\right] < \lim_{T\to\infty}\left[\sum_{j=0}^{T-2}\theta_{j+1} + \sum_{j=T-1}^{\infty}\theta_{j+1}\right] = \Theta(1) - \theta_0$, which is finite. Hence, the limit of BS_W is 0 as T grows large.

Proposition 3 *For fixed* T*,* $\lambda = 0$*, an infinite history of exogenous* x_{it}*, and* $\theta_j > 0$*,* $j = 0, ...T-1$

$$p \lim_{N\to\infty} \left[\widehat{\beta}_{RE}\right] = F_W\theta_0 + (1 - F_W)\,\Theta(1) + F_W BS_W + (1 - F_W)\,BS_B, \quad (12)$$

where $F_W = (\mathbf{x}'\mathbf{Q}\mathbf{x}+\psi(\mathbf{x}-\overline{\overline{\mathbf{x}}})'\mathbf{P}(\mathbf{x}-\overline{\overline{\mathbf{x}}}))^{-1}\mathbf{x}'\mathbf{Q}\mathbf{x}$ *with* $\overline{\overline{\mathbf{x}}} = \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T} x_{it}$.

Proof. Applying $(\mathbf{Q}+\psi\mathbf{P})$ to the static model (4) leads to

$$p \lim_{N\to\infty} \left(\widehat{\beta}_{RE}, \widehat{\mu}_{RE}\right)' = p \lim_{N\to\infty} \left(\mathbf{x}^{*\prime}\left(\mathbf{Q}+\psi\mathbf{P}\right)\mathbf{x}^{*}\right)^{-1}\mathbf{x}^{*\prime}\left(\mathbf{Q}+\psi\mathbf{P}\right)\mathbf{y}).$$

Using the formula of the partitioned inverse (Hsiao, 2003, pp. 36-37), define

$$\begin{aligned} F_W &= \left(\mathbf{x}'\mathbf{Q}\mathbf{x}+\psi(\mathbf{x}-\overline{\overline{\mathbf{x}}})'\mathbf{P}\left(\mathbf{x}-\overline{\overline{\mathbf{x}}}\right)\right)^{-1}\mathbf{x}'\mathbf{Q}\mathbf{x} := (S_{xx}^{W} + \psi S_{xx}^{B})^{-1}S_{xx}^{W} \\ &= 1-\psi(S_{xx}^{W} + \psi S_{xx}^{B})^{-1}S_{xx}^{B}, \end{aligned}$$

where $S_{xx}^{W} = \mathbf{x}'\mathbf{Q}\mathbf{x}$ and $S_{xx}^{B} = (\mathbf{x}-\overline{\overline{\mathbf{x}}})'\mathbf{P}\left(\mathbf{x}-\overline{\overline{\mathbf{x}}}\right)$ and $\overline{\overline{\mathbf{x}}} = \dfrac{1}{NT}\displaystyle\sum_{i=1}^{N}\sum_{t=1}^{T} x_{it}$.

Therefore,

$$\begin{aligned} p \lim_{N\to\infty} \widehat{\beta}_{RE} &= \theta_0 F_W + (1 - F_W)\,\Theta(1) \\ &+ \sum_{j=0}^{\infty} \theta_{j+1} F_W \left(\mathbf{x}'\mathbf{Q}\mathbf{x}\right)^{-1}\mathbf{x}'\mathbf{Q}\mathbf{x}_{-j-1} \\ &+ \sum_{j=0}^{\infty} \theta_{j+1}\left(1 - F_W\right)\left((\mathbf{x}-\overline{\overline{\mathbf{x}}})'\mathbf{P}\left(\mathbf{x}-\overline{\overline{\mathbf{x}}}\right)\right)^{-1} \\ &\cdot \left[(\mathbf{x}-\overline{\overline{\mathbf{x}}})'\mathbf{P}\left(\mathbf{x}_{-j-1}-\overline{\overline{\mathbf{x}}}_{-j-1}\right) - 1\right] \\ &= F_W\theta_0 + (1 - F_W)\,\Theta(1) + F_W BS_W + (1 - F_W)\,BS_B. \end{aligned}$$

■

The following table summarizes the results for $\lambda = 0$ and the ADL(1,1) model as the data generating process:

Estimator	As. deviation from θ_0	As. deviation from $\Theta(1)$
$\widehat{\beta}_W$	BS_W	$BS_W - \widetilde{\varphi}$
$\widehat{\beta}_B$	$BS_B + \widetilde{\varphi}$	BS_B
$\widehat{\beta}_{RE}$	$F_W BS_W + (1 - F_W)\,(BS_B + \widetilde{\varphi})$	$F_W(BS_W - \widetilde{\varphi}) + (1 - F_W)\,BS_B$
$\widehat{\beta}_{OLS}$	$\frac{1}{2}(BS_W + BS_B) + \frac{1}{2}\widetilde{\varphi}$	$\frac{1}{2}(BS_W + BS_B) - \frac{1}{2}\widetilde{\varphi}$

With an ADL(1,1) model as the underlying data generating process both $BS_W < 0$ and $BS_B < 0$ are ensured. The asymptotic downward bias of $\widehat{\beta}_W$ with respect to $\Theta(1)$ is even stronger than with respect to θ_0. Hence, $\widehat{\beta}_W$ is closer to the short run impact than to the long run impact at $\lambda = 0$.

Similarly, $\widehat{\beta}_B$ always underestimates $\Theta(1)$, but it tends to overestimate θ_0, if $\widetilde{\varphi} > -BS_B$. If $\widetilde{\varphi} > -2BS_B$, the squared asymptotic bias of $\widehat{\beta}_B$ with respect to $\Theta(1)$ is always smaller than that with respect to θ_0. From Proposition 2 we see that $\widehat{\beta}_B$ tends to capture the more likely the long run effect, the lower $\widetilde{\varphi}$, σ^2_η and γ_1 and the larger T and σ^2_ζ.

The performance of $\widehat{\beta}_{RE}$ depends on both the size of BS_W and BS_B and their relative weight. The larger F_W (the lower ψ), the closer the probability limit of $\widehat{\beta}_{RE}$ is to $\theta_0 + BS_W$. Under ADL(1,1), a sufficient condition for $\widehat{\beta}_{RE}$ to outperform $\widehat{\beta}_W$ with respect to θ_0 in terms of the squared bias is $F_W < \frac{BS_W^2 - (BS_B + \widetilde{\varphi})^2}{2BS_W(BS_B + \widetilde{\varphi})}$ if $\widetilde{\varphi} > -BS_B$, and $F_W > \frac{BS_W^2 - (BS_B + \widetilde{\varphi})^2}{2BS_W(BS_B + \widetilde{\varphi})}$ if $\widetilde{\varphi} < -BS_B$.[7]

[7]The difference between the squared asymptotic bias of $\widehat{\beta}_{RE}$ and $\widehat{\beta}_W$ with respect to θ_0 is given by $(F_W BS_W + (1 - F_W)\,(BS_B + \widetilde{\varphi}))^2 - {BS_W}^2 = (1 - F_W)(2F_W BS_W(BS_B + \widetilde{\varphi}) - ({BS_W}^2 - (BS_B + \widetilde{\varphi})^2) - F_W({BS_W}^2 + (BS_B + \widetilde{\varphi})^2)) < (1 - F_W)(2F_W BS_W(BS_B + \widetilde{\varphi}) - ({BS_W}^2 - (BS_B + \widetilde{\varphi})^2))$, which is always smaller than 0 if the condition given in the text holds.

The squared asymptotic bias of $\widehat{\beta}_{RE}$ with respect to θ_0 is smaller than that with respect to $\Theta(1)$, if $(1-2F_W)\widetilde{\varphi} < -2(F_W BS_W + (1-F_W)BS_B)$. Since both BS_W and BS_B are negative under ADL(1,1), the right hand side of this inequality is positive. Hence, a sufficient condition for the squared asymptotic bias of $\widehat{\beta}_{RE}$ to be smaller with respect to θ_0 than with respect to $\Theta(1)$ is $F_W \geq \frac{1}{2}$. As a result, under ADL(1,1) the pooled OLS estimator is always closer to θ_0 than to $\Theta(1)$ at $\lambda = 0$, since $F_W = \frac{1}{2}$ in this case.

The analysis so far concentrated on a data generating process of x_{it} without a memory. In Case II, $0 < \lambda < 1$, the analytical derivation of both the short run and the long run bias is much more complicated (see the Appendix) and the sign and extent of the asymptotic bias have to be computed numerically as in Pirotte (1999). In Figures 1a - 3f the asymptotic bias of the within estimator is expressed in percent of θ_1 and that of the between estimator in percent of $\Theta(1)$. From (14) and (15) in the Appendix, we know that both BS_W and BS_B linearly depend on θ_j, $j \geq 1$. Hence, in absolute terms the biases in all figures increase in θ_j, even if measured in percent of θ_0 or $\Theta(1)$, respectively. The biases implied by the ADL(1,1) model are plotted for $\beta_0 = \beta_1 = 1$. We additionally analyze a ADL(1,3) model and a ADL(3,3) model with $\beta_0 = \beta_1 = \beta_2 = \beta_3 = 1$ and $\gamma_2 = 0.05$, $\gamma_3 = 0.025$, respectively. The time dimension of the panel is $T = 5$ and $T = 10$, respectively. We compute the asymptotic biases for different combinations of γ_1 and λ.[8]

[8]The presented surfaces of the asymptotic bias in the figures are based on the derivations in the Appendix, summing over $j = 1, ..., J$ and $t = 1, ..., T$. We impose $\Delta\left(\frac{100 \cdot BS_W}{\theta_0}\right) \leq 0.001$ and $\Delta\left(\frac{100 \cdot BS_B}{\Theta(1)}\right) \leq 0.001$, respectively, as the termination criterion for the summation of the series. Note, values outside the interval $[-600, 600]$, including those where a finite value of a bias does not exist, are clipped.

Figure 1 displays the asymptotic bias of the within estimator with respect to the short run parameter, and Figures 2 and 3 that of the between estimator with respect to the long run parameter. In Figure 2 we assume $\sigma_\zeta^2 = 1$ for the between error component, whereas $\sigma_\zeta^2 = 4$ in Figure 3.

*** Insert Figures 1.a - 3.f about here ***

In relation to θ_0, the within estimate is asymptotically downward biased, if λ is relatively small and/or γ_1 is relatively large, where we should again emphasize that we only consider the case of $\theta_j > 0$ for all j. In contrast to Case I, BS_W turns to the positive, if λ gets sufficiently large. An increase in γ_1 generally tends to reduce BS_W for small enough λ. However, at intermediate values of γ_1 and high λ we observe the opposite. The asymptotic bias increases and becomes positive. At small T, BS_W even grows with T.[9] If the explanatory variable enters with additional lags (e.g., ADL(1,3)), the downward bias is reinforced for large γ_1 and small λ, but also the upward bias gets more severe, if λ is large. With an ADL(3,3) model BS_W gets even more pronounced. Figures 1.d-1.f illustrate that a larger time dimension is helpful as long as λ is not too large, as shown in Proposition 1. Summing up, in contrast to Case I the within estimator can be either negatively (low λ) or positively (high λ) biased with respect to the short run parameter. However, at low λ a large γ_1 always leads to an underestimation of the short run effect. With the present parametrization, we observe deviations of more than 600% in both directions, so the asymptotic bias could be rather severe, especially if γ_1 and/or λ are near 1.

[9] From the derivations in the Appendix, it can easily be seen that for fixed t and j the elements in (14) tend to zero, if T grows large.

The asymptotic bias of the between estimate with respect to $\Theta(1)$ is negative for all combinations of γ_1 and λ, and Proposition 2 seems to hold in the more general case. In absolute terms, the asymptotic bias decreases in σ_ζ^2 and in T, but it increases in γ_1. With respect to λ, we observe a u-shaped pattern. The asymptotic bias first increases in absolute terms, but at high values of λ (in Figures 2.a-2.f at around $\lambda = 0.8$) it gets smaller again. This pattern is more pronounced in the ADL(1,3) or ADL (3,3) models. At the chosen parameters, the between estimator generally exhibits a smaller bias and leads to a good approximation of the long run impact for a large subset of (γ_1, λ), even more so, if T is not too small or the between variation of the explanatory variable (σ_ζ^2) is high. However, if the roots of $\Gamma(L)$ are near the unit circle, also the between estimator can be heavily downward biased.

4 Monte Carlo simulation

Our Monte Carlo simulation set-up assesses the sensitivity of the approximation of the short run and long run effect for the finite sample case ($N = 100$). Again, we consider the within, the between, the random effects and the pooled OLS-estimator.

The Monte Carlo experiments are designed as follows. The parameters of the ADL(1,1) and ADL(3,3) are chosen as in Figures 1-3. We assume $\varepsilon_{it} \sim IID\ N(0,1)$, $\eta_{it} \sim IID\ N(0,1)$, $\zeta_i \sim IID\ N(0,\sigma_\zeta^2)$ and $\alpha_i \sim IID\ N(0,\sigma_\alpha^2)$. The remaining parameters vary in the following form:

- Memory of the autoregressive process in the basic equation, i.e. the coefficient of the lagged dependent variable: $\gamma_1 = 0.2,\ 0.8$.

- Relative importance of the between variance component in the data generating process of y_{it} : $\psi = \frac{\sigma_\varepsilon}{T\sigma_\alpha+\sigma_\varepsilon} = 0.25$, 0.01, hence σ_α is implicitly defined.

- Cross-section versus time variation in the data generating process of x_{it}: $x_{it} = \lambda x_{it-1} + \zeta_i + \eta_{it}$, with $\sigma^2_\zeta = 1$ and 4.

- Memory of the autoregressive process in x_{it}: $\lambda = 0$, 0.9.

- Time dimension: $T = 5$, 10.

We set $N = 100$ and replicate each of the 64 experiments 10000 times. Tables 1 and 2 contain the results of the ADL(1,1) model, and Tables 3 and 4 those of the ADL(3,3) model. The tables provide information on the true parameters, the average estimated short run and long run parameter in each model (Av.), its variance, the squared bias and the mean square error (MSE). For example, the variance of $\widehat{\beta}_l$, $l = W, B, RE, OLS$ is calculated as $\left(\frac{1}{10000}\sum_{k=1}^{10000}\widehat{\beta}_{kl} - \overline{\widehat{\beta}}_l\right)^2$, the corresponding squared bias is defined as $\left(\overline{\widehat{\beta}}_l - \theta_0\right)^2$ and mean squared error (MSE$= \frac{1}{10000}\sum_{k=1}^{10000}\left(\widehat{\beta}_{kl} - \theta_0\right)^2$), where k indexes the replication. The corresponding figures with respect to the long run effect ($\Theta(1)$) have been analogously calculated for all four estimators. Since our focus is on short panels, we only briefly discuss the effects of an increase in T.

*** Insert Tables 1 - 4 about here ***

Consistent with the theoretical results derived above, the within estimator tends to underestimate the short run parameter in both the ADL(1,1) and

ADL(3,3) model, if there is no autocorrelation in the explanatory variable ($\lambda = 0$). The approximation of the short run impact is particularly weak, if γ_1 is large. At $\lambda = 0$, the bias decreases with rising T and it seems independent of the cross-sectional variation (θ or σ_ζ^2). At $\lambda = 0.9$ we find a substantial upward bias and especially so, if T is large. Again, the bias is independent of σ_ζ^2. With the ADL(3,3) model, the bias is more pronounced, although the same pattern can be found. Hence, in short panels the within estimate only proxies the short run impact (θ_0) well, if the memory in the autoregressive processes of both the dependent and the explanatory variable(s) is not too strong (γ_1 and/or λ is low). Otherwise, the deviation can be substantial and the within estimator can even better approximate the long run impact, if λ is sufficiently large (for instance, in experiments #50 or #54).

With the ADL(1,1) model the between estimator approximates the long run effect quite well for all analyzed time dimensions, if γ_1 is sufficiently low. This seems to hold independently of the relative importance of the cross-sectional variation as represented by ψ and σ_ζ^2. The memory in the explanatory variable λ tends to improve the quality of the between approximation of the long run effect in many cases. Otherwise, the bias may be substantial if γ_1 is large. If the ADL(3,3) model is the data generating process, the performance is even worse at large γ_1. For instance, in experiments #51 or #55 the between estimate is even closer to the short run than to the long run impact.

In some cases the random effects model outperforms the within estimator in terms of the bias with respect to θ_0. Especially, if γ_1 is high and ψ is low, one may be better off with the random effects model (for instance, see all experiments except #2 in Table 1). However, at high λ the random effects estimator is substantially upward biased with respect to the short run effect (see Table 2).

At $\lambda = 0$, the pooled OLS estimate always tends to capture the short run effect (see the Table 1, especially so at large γ_1), but it never approximates the short run impact better than the within estimator. Pooled OLS captures the long effect at high λ, but never beats the between estimator in this respect.

The presented Monte Carlo design cannot be easily compared to existing studies. First, Baltagi and Griffin (1984) provide Monte Carlo simulations for the ADL(0,k) model ($N = 18$, $T = 14$, $\gamma_1 = 0$, $0.9 \leq \lambda \leq 1$). They assume a data generating process corresponding to the estimates in their earlier work on gasoline demand (Baltagi and Griffin, 1983). They estimate the short run and long run price elasticitity of gasoline demand using the within, the between, the pooled OLS estimator and several random effects estimators under an underspecified lag structure of the right hand side variable. According to the RMSE criterion, the random effects estimators perform best in estimating the long run price elasticity, followed by pooled OLS and the between estimator. The within estimator is heavily downward biased in this regard. Concerning the short run elasticity, the order is reversed and the within is estimator performs best, followed by the random effects estimators. As far as the bias is concerned, the within estimator approximates the short run impact and the between estimator the long run impact best as in our simulations. Furthermore and similar to our results, the downward bias of the within estimator critically depends on the number of included lags of the right hand side variable as well as on its memory.

Second, van den Doel and Kiviet (1995) assume a partial adjustment ADL(1,0) error component model ($N = 100$, $T = 3$, $\gamma_1 = 0.6$, $\lambda = 0.99$). The right hand side variable follows an AR(1) process with a constant and a trend term. Their Monte Carlo results demonstrate that the within estimator is much more sensitive to neglected dynamics and downward biased with respect to the long run impact, but upward biased with respect the short run impact multiplier. In contrast to our simulations, the distribution functions of the other estimators turn out remarkably stable.

Summing up, both mentioned studies provide some important Monte Carlo evidence. However, neither of them considers a general ADL(l,k) model as the data generating process. They do not investigate the combined impact of γ_1 and λ on the bias in relation to both the short and the long run impact.

5 Conclusions

The estimation of short run and long run effects in static panels with a short time series dimension as an approximation of a dynamic model depends crucially on the parameter of the lagged dependent variable of the underlying autoregressive distributed lag (ADL) model. We derive analytical solutions for the case of zero memory of the explanatory variable and provide numerical calculations for the more general case with non-zero memory. Specifically, we investigate the asymptotic bias of four estimators (within, between, random effects, and pooled OLS) with respect to both the short run and long run impact, assuming a static model although a dynamic one is true.

In relation to a positive true short run parameter, we find that the within estimate is downward (upward) biased, if the memory of the explanatory variable is low (high). At a high memory of the explanatory variable, (i) the within estimator gets even closer to the long run than to the short run impact, and (ii) at small T its bias with respect to the short run may even increase as T rises.

The between estimate is always downward biased with respect to the (positive) long run impact. In absolute terms, the bias gets very high, if the memory of the dependent variable is high and both the long run impact and T are small. Then, the between estimate may be even closer to the short run than to the long run impact.

At no memory in the explanatory variable two additional conclusions arise. First, the random effects estimator is closer to the short run impact the more important the within variation, and there are cases where it even outperforms the within estimator in this regard. Second, pooled OLS tends to capture the short run impact in this case.

Practitioners still often use static models, while a dynamic one should be appropriate. We demonstrate that the quality of the approximation of the short run impact by the within estimator and the long run effect by the between estimator may be quite bad. In fact, the biases may amount to several hundred percent of the true parameter. Hence, it is a prerequisite to test for underspecified dynamics as suggested by van den Doel and Kiviet (1995), when associating within estimates with short run effects and between parameters with long run effects in static models.

6 Appendix

6.1 Asymptotic bias of $\widehat{\beta}_W$ with respect to θ_0

(i) Since $|\lambda| < 1$, $x_{it} = \sum_{u=0}^{\infty} \lambda^u B^u (\zeta_i + \eta_{it}) = \frac{\zeta_i}{1-\lambda} + \sum_{u=0}^{\infty} \lambda^u \eta_{it-u}$, and the autocovariance of x_{it} for $|\lambda| \neq 0$ is given by

$$\begin{aligned} g_x(\tau) &= g_x(-\tau) = Cov\left(x_{it}, x_{it+\tau}\right) = \sum_{u=0}^{\infty}\sum_{v=0}^{\infty} \lambda^u \lambda^v (\zeta_i + \eta_{it-u})(\zeta_i + \eta_{it+\tau-v}) \\ &= \frac{\sigma_\zeta^2}{(1-\lambda)^2} + \sigma_\eta^2 \sum_{u=0}^{\infty} \lambda^u \lambda^{\tau+u} = \frac{\sigma_\zeta^2}{(1-\lambda)^2} + \sigma_\eta^2 \frac{\lambda^\tau}{1-\lambda^2}. \end{aligned}$$

(ii)

$$g_x(j+1) = \frac{\sigma_\zeta^2}{(1-\lambda)^2} + \frac{\sigma_\eta^2}{1-\lambda^2}\lambda^{j+1} = \frac{\sigma_\zeta^2}{(1-\lambda)^2} + \frac{\sigma_\eta^2}{1-\lambda^2} A_j,$$

where $A_j = \lambda^{j+1}$.

(iii)

$$\begin{aligned} &\frac{1}{T}\sum_{\tau=1}^{T} g_x(t-\tau+j+1) \\ &= \frac{1}{T}\left[\sum_{\tau=1}^{t+j} g_x(t-\tau+j+1) + \sum_{\tau=t+j+1}^{T} g_x(\tau-t-j-1)\right] \\ &= \frac{\sigma_\zeta^2}{(1-\lambda)^2} + \frac{\sigma_\eta^2}{(1-\lambda^2)\,T}\left[\sum_{\tau=1}^{t+j} \lambda^{t-\tau+j+1} + \sum_{\tau=t+j+1}^{T} \lambda^{\tau-t-j-1}\right] \\ &= \frac{\sigma_\zeta^2}{(1-\lambda)^2} + \frac{\sigma_\eta^2}{(1-\lambda^2)}\left[\underbrace{\frac{\lambda(1-\lambda^{t+j})}{(1-\lambda)\,T}}_{B_{tj,1}} + \underbrace{\frac{\left(1-\lambda^{T-t-j}\right)}{(1-\lambda)\,T}}_{B_{tj,2}}\right] \\ &= \frac{\sigma_\zeta^2}{(1-\lambda)^2} + \frac{\sigma_\eta^2}{(1-\lambda^2)} B_{tj} \end{aligned}$$

with $B_{tj,2} = B_{tj,1} + B_{tj,2}$ and $B_{tj,2} = 0$ if $t \geq T - j$.

(iv)

$$\begin{aligned}
& \frac{1}{T}\sum_{\tau=1}^{T} g_x(\tau - t + j + 1) \\
= & \frac{1}{T}\left[\sum_{\tau=1}^{t-j-1} g_x(t - j - 1 - \tau) + \sum_{\tau=t-j}^{T} g_x(\tau - t + j + 1)\right] \\
= & \frac{\sigma_\zeta^2}{(1-\lambda)^2} + \frac{\sigma_\eta^2}{(1-\lambda^2)\,T}\left[\sum_{\tau=1}^{t-j-1}\lambda^{t-j-1-\tau} + \sum_{\tau=t-j}^{T}\lambda^{\tau-t+j+1}\right] \\
= & \frac{\sigma_\zeta^2}{(1-\lambda)^2} + \frac{\sigma_\eta^2}{1-\lambda^2}\left[\underbrace{\frac{1-\lambda^{t-j-1}}{(1-\lambda)\,T}}_{C_{tj,1}} + \underbrace{\frac{\lambda\left(1-\lambda^{T-t+j+1}\right)}{(1-\lambda)\,T}}_{C_{tj,2}}\right] \\
= & \frac{\sigma_\zeta^2}{(1-\lambda)^2} + \frac{\sigma_\eta^2}{1-\lambda^2}C_{tj},
\end{aligned}$$

with $C_{tj} = C_{tj,1} + C_{tj,2}$ and $C_{tj,1} = 0$ if $t \leq j + 1$.

(v)

$$\frac{1}{T^2}\sum_{\tau=1}^{T}\sum_{\tau'=1}^{T} g_x(\tau - \tau' + j + 1) = \frac{\sigma_\zeta^2}{(1-\lambda)^2} + \frac{\sigma_\eta^2 D_j}{(1-\lambda^2)} \qquad (13)$$

with

$$\begin{aligned}
D_j = & \frac{1}{T^2}\sum_{\tau=1}^{T}\sum_{\tau'=1}^{T}\lambda^{|\tau-\tau'+j+1|} = \underbrace{\sum_{u=1-T}^{-(j+2)}(T+u)\,\lambda^{-u-j-1}}_{D_{tj,1}} + \underbrace{\sum_{u=-(j+1)}^{-1}(T+u)\,\lambda^{u+j+1}}_{D_{tj,2}} + \\
& \underbrace{\lambda^{j+1}\sum_{u=0}^{T-1}(T-u)\,\lambda^{u+j+1}}_{D_{tj,3}}
\end{aligned}$$

and

$$D_{j,1} = \frac{1}{T^2}\lambda \sum_{k=1}^{T-j-2} \frac{1-\lambda^{T-j-1-k}}{1-\lambda} = \frac{1}{T^2}\frac{\lambda}{1-\lambda}\left[(T-j-2) - \frac{1-\lambda^{T-j-1}}{1-\lambda}\right]$$

$$D_{j,2} = \frac{1}{T^2}\left((T-j-1)\frac{1-\lambda^{j+1}}{1-\lambda} + \sum_{k=1}^{j}\lambda^k \frac{1-\lambda^{j-k+1}}{1-\lambda}\right) =$$

$$\frac{1}{T^2}\left(\frac{(T-j-1)\left(1-\lambda^{j+1}\right)}{1-\lambda} - \frac{\lambda\left(1-\lambda^j\right)}{(1-\lambda)^2} - \frac{j\lambda^{j+1}}{1-\lambda}\right)$$

$$D_{j,3} = \frac{1}{T^2}\frac{1}{1-\lambda}\sum_{k=1}^{T} 1-\lambda^{T-k+1} = \frac{1}{T^2}\frac{\lambda^{j+1}}{1-\lambda}\left[T+\lambda\left(\frac{1-\lambda^T}{1-\lambda}\right)\right],$$

where $D_j = D_{j,1} + D_{j,2} + D_{j,3}$ and $D_{j,2} = 0$, if $j > T-2$ or $j < 0$. Collecting terms and summarizing over T, the bias becomes

$$BS_W = \sum_{j=0}^{\infty} \theta_{j+1} \sum_{t=1}^{T} \frac{A_j - B_{tj} - C_{tj} + D_j}{A_{-1} - B_{t,-1} - C_{t,-1} + D_{t,-1}}. \tag{14}$$

Noteworthy, the asymptotic bias of $\widehat{\beta}_W$ with respect to θ_0 is independent of σ_ζ^2 and σ_η^2 as in Proposition 1. Since, A_j, B_{tj}, C_{tj} and D_j are all positive, there exist parameter constellations, where the asymptotic bias is positive as shown in the figures in the text.

6.2 Asymptotic bias of $\widehat{\beta}_B$ with respect to $\Theta(1)$

$$BS_B =$$

$$= p\lim_{N\to\infty}\left[\widetilde{\varphi}(1-\gamma_1)\sum_{j=0}^{\infty}\gamma_1^j\left(\frac{\frac{1}{N}\sum_{i=1}^{N}\overline{x}_i\overline{x}_{i(-j-1)} - \left(\frac{1}{N}\sum_{i=1}^{N}\overline{x}_i\right)\left(\frac{1}{N}\sum_{i=1}^{N}\overline{x}_{i(-j-1)}\right)}{\frac{1}{N}\sum_{i=1}^{N}\overline{x}_i^2 - \left(\frac{1}{N}\sum_{i=1}^{N}\overline{x}_i\right)^2} - 1\right)\right].$$

Since $p\lim_{N\to\infty}\left[\frac{1}{NT}\sum_{i=1}^{N}\sum_{\tau=1}^{T}x_{i\tau}\right] = p\lim_{N\to\infty}\left[\frac{1}{NT}\sum_{i=1}^{N}\sum_{\tau=1}^{T}x_{i\tau-j-1}\right] = 0$, we have

$$\begin{aligned}
p\lim_{N\to\infty}\left[\frac{1}{N}\sum_{i=1}^{N}\overline{x}_i\overline{x}_{i(-j-1)}\right] &= p\lim_{N\to\infty}\left[\frac{1}{NT^2}\sum_{i=1}^{N}\sum_{\tau=1}^{T}\sum_{\tau'=1}^{T}x_{i\tau}x_{i\tau'-j-1}\right] \\
&= \frac{\sigma_\zeta^2}{(1-\lambda)^2} + \frac{\sigma_\eta^2 D_j}{(1-\lambda^2)} \\
p\lim_{N\to\infty}\left[\frac{1}{N}\sum_{i=1}^{N}\overline{x}_i^2\right] &= \frac{\sigma_\zeta^2}{(1-\lambda)^2} + \frac{\sigma_\eta^2 D_{-1}}{(1-\lambda^2)},
\end{aligned}$$

Combining terms, we obtain

$$\begin{aligned}
BS_B &= \sum_{j=0}^{\infty}\theta_{j+1}\sum_{t=1}^{T}\left(\frac{\frac{\sigma_\zeta^2}{(1-\lambda)^2}+\frac{\sigma_\eta^2 D_j}{(1-\lambda^2)}}{\frac{\sigma_\zeta^2}{(1-\lambda)^2}+\frac{\sigma_\eta^2 D_{-1}}{(1-\lambda^2)}} - 1\right) \\
&= \sum_{j=0}^{\infty}\theta_{j+1}\sum_{t=1}^{T}\left(\frac{\sigma_\zeta^2+\sigma_\eta^2 D_j}{\sigma_\zeta^2+\sigma_\eta^2 D_{-1}} - 1\right) \\
&= \sigma_\eta^2\sum_{j=0}^{\infty}\theta_{j+1}\sum_{t=1}^{T}\frac{D_j - D_{-1}}{\sigma_\zeta^2+\sigma_\eta^2 D_{-1}}. \qquad (15)
\end{aligned}$$

7 References:

Baltagi, B.H., 2001, Econometric Analysis of Panel Data, John Wiley, Chichester, 2nd edition.

Baltagi, B.H., Griffin, J.H., 1983, Gasoline Demand in the OECD: An Application of Pooling and Testing Procedures, European Economic Review 22(2), pp. 117-137.

Baltagi, B.H., Griffin, J.H., 1984, Short and Long Run Effects in Pooled Models, International Economic Review 25(3), 631-645,.

Brockwell, P.J., Davies, R.A., 1991, Time Series: Theory and Methods, Springer Series in Statistics, 2nd ed., New York.

Cornwell, Ch., Schmidt, P., Wyhowski, D. J., 1992, Simultaneous Equations and Panel Data, Journal of Econometrics 51, pp. 151-181.

Doel van den, I.T., Kiviet, J.F., 1994, Asymptotic Consequences of Neglected Dynamics in Individual Effects Models, Statistica Neerlandica 48, pp. 71-85.

Doel van den, I.T., Kiviet, J.F., 1995, Neglected Dynamics in Panel Data Models; Consequences and Detection in Finite Samples, Statistica Neerlandica 49, pp. 343-361.

Houthakker, H.S, 1965, New Evidence on Demand Elasticities, Econometrica 33, pp. 277-288.

Hsiao, Ch., 2003, Analysis of Panel Data, Cambridge University Press, Cambridge, 2nd edition.

Kuh, E., 1959, The Validity of Cross-sectionally Estimated Behavior Equations in Time Series Applications, Econometrica 27, pp. 197-214.

Pirotte, A., 1999, Convergence of the Static Estimation Toward Long Run Effects of the Dynamic Panel Data Models, Economics Letters 63, pp. 151-158.

Figure 1: Bias of the within estimator with respect to β_0

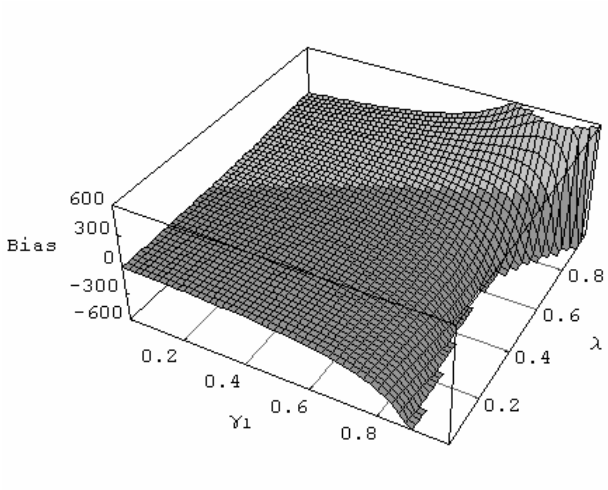

Figure 1.a – T = 5, ADL(1,1)

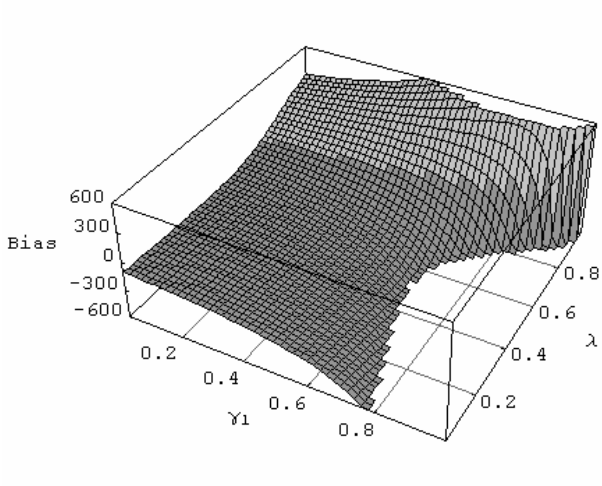

Figure 1.b – T = 5, ADL(1,3)

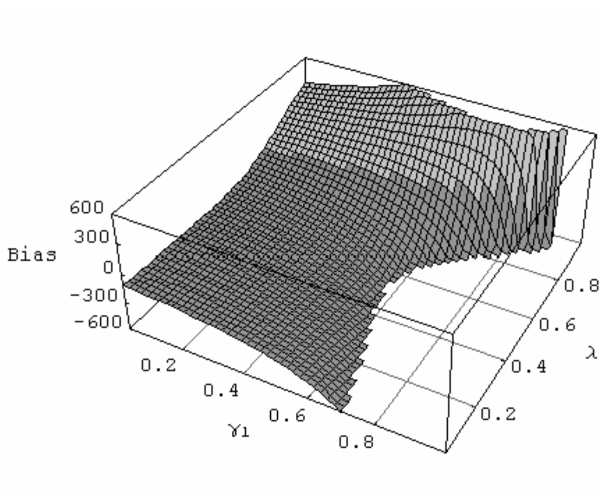

Figure 1.c – T = 5, ADL(3,3)

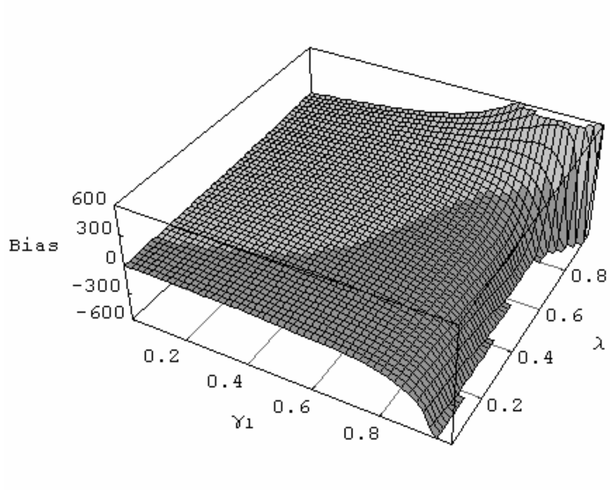

Figure 1.d – T = 10, ADL(1,1)

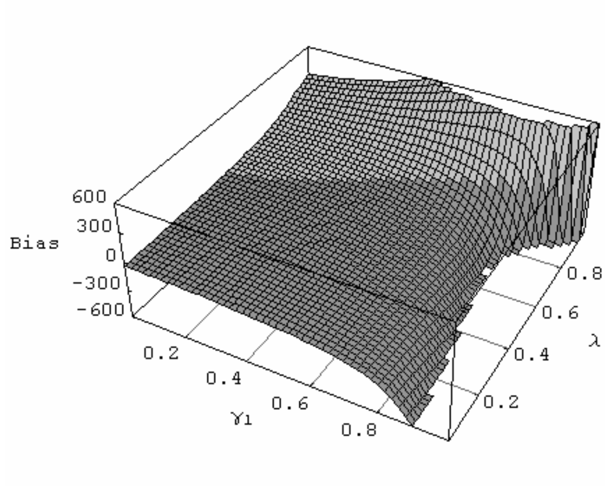

Figure 1.e – T = 10, ADL(1,3)

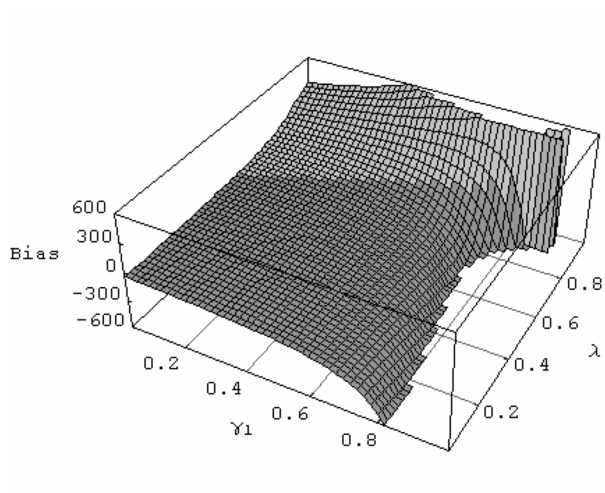

Figure 1.f – T = 10, ADL(3,3)

ADL(1,1): $\beta_0=\beta_1=1$. ADL(1,3): $\beta_0=\beta_1=\beta_2=\beta_3=1$. ADL(3,3): $\beta_0=\beta_1=\beta_2=\beta_3=1$; $\gamma_2=0.05$; $\gamma_3=0.025$. Bias values outside the interval [-600,600] have been clipped. In all Figures, the bias is measured in percent of the short run parameter β_0.

Figure 2: Bias of the between estimator with respect to *Θ(1)*; $\sigma_{\zeta}^{2} = 1$

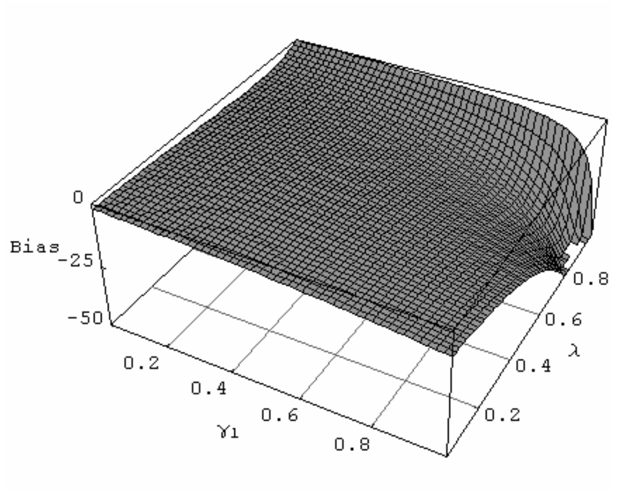

Figure 2.a – T = 5, ADL(1,1)

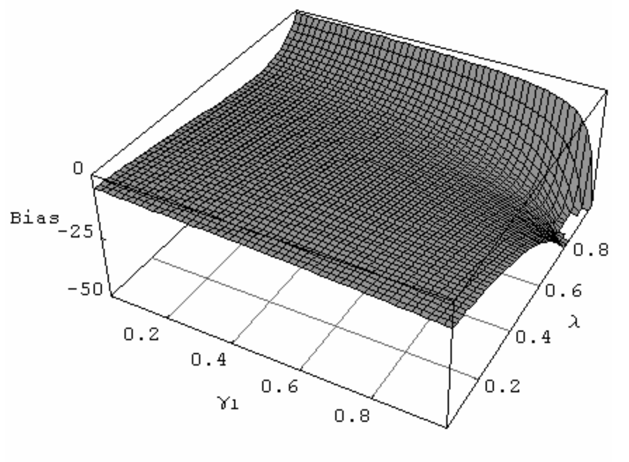

Figure 2.b – T = 5, ADL(1,3)

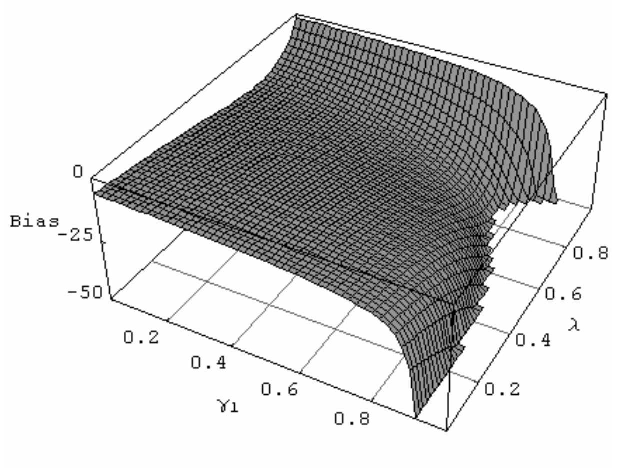

Figure 2.c – T = 5, ADL(3,3)

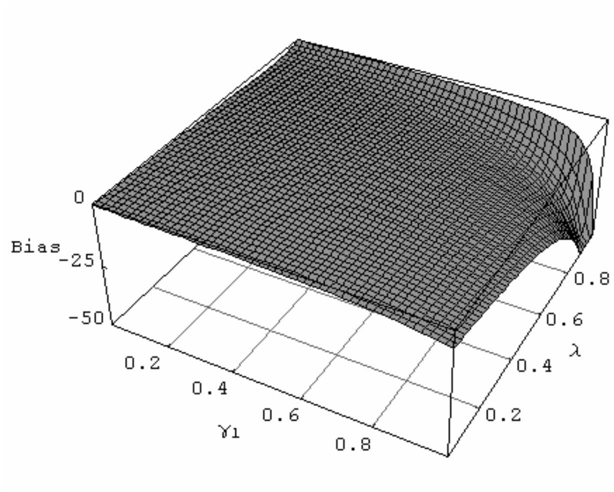

Figure 2.d – T = 10, ADL(1,1)

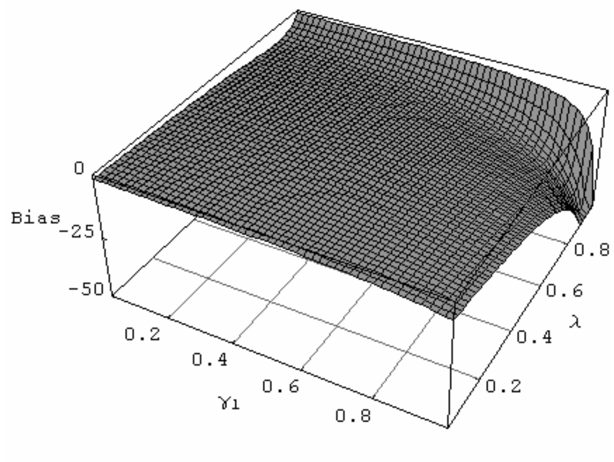

Figure 2.e – T = 10, ADL(1,3)

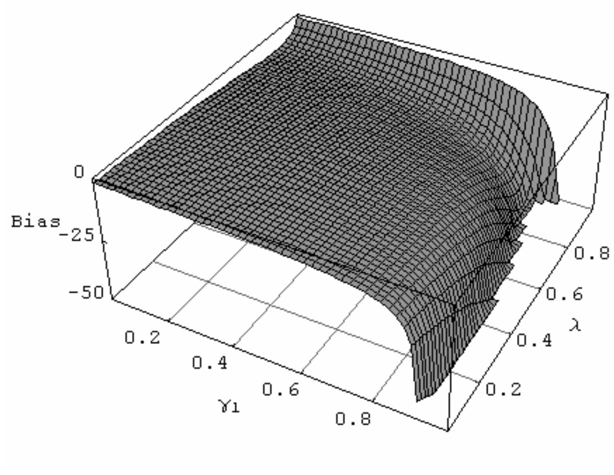

Figure 2.f – T = 10, ADL(3,3)

ADL(1,1): $\beta_0=\beta_1=1$. ADL(1,3): $\beta_0=\beta_1=\beta_2=\beta_3=1$. ADL(3,3): $\beta_0=\beta_1=\beta_2=\beta_3=1$; $\gamma_2=0.05$; $\gamma_3=0.025$. Bias values outside the interval [-600,600] have been clipped. In all Figures, the bias is measured in percent of the long run parameter Θ(1).

Figure 3: Bias of the between estimator with respect to *Θ(1)*; $\sigma_{\zeta}^{2} = 4$

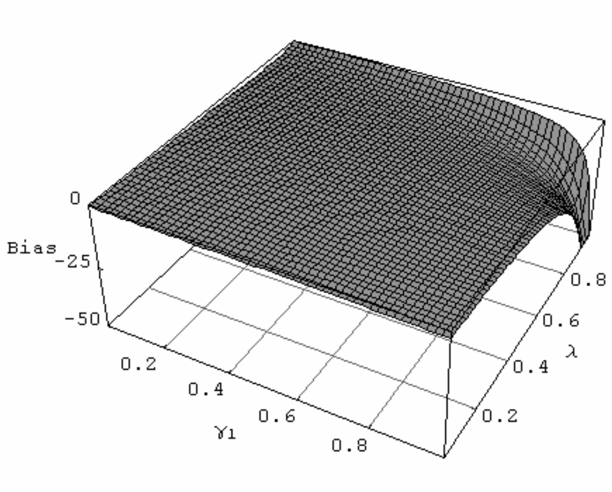

Figure 3.a – T = 5, ADL(1,1)

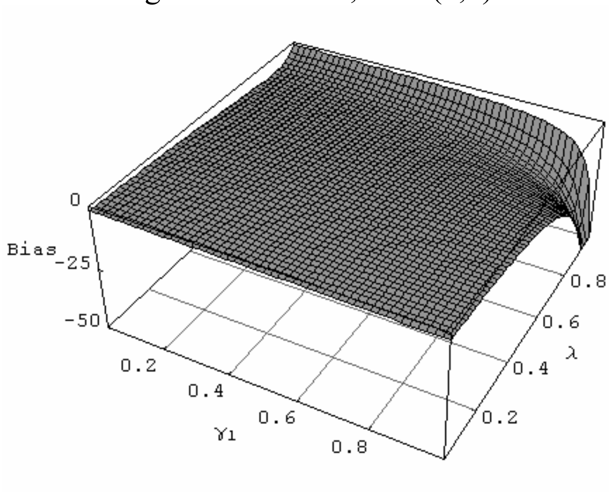

Figure 3.b – T = 5, ADL(1,3)

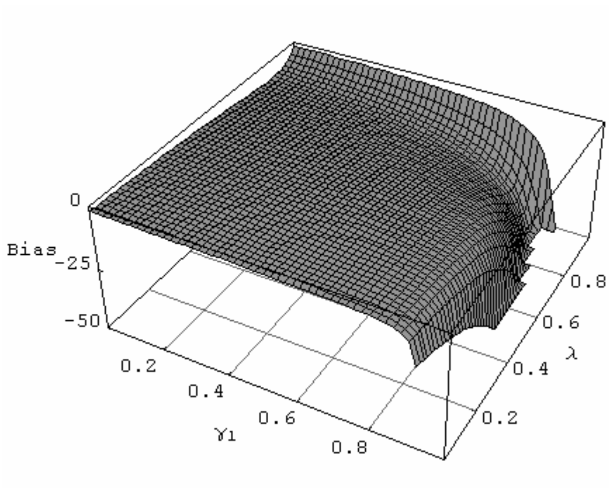

Figure 3.c – T = 5, ADL(3,3)

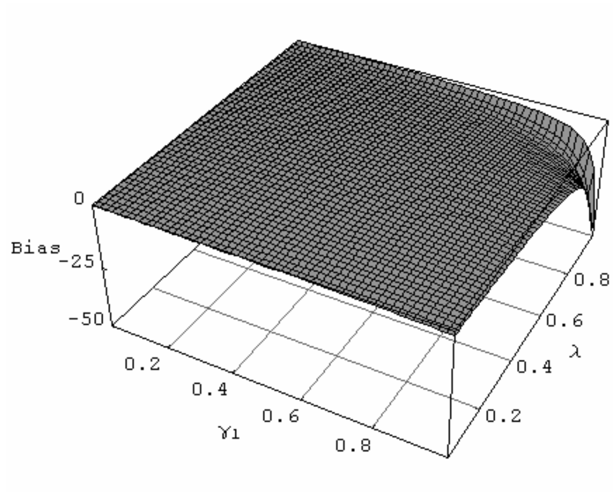

Figure 3.d – T = 10, ADL(1,1)

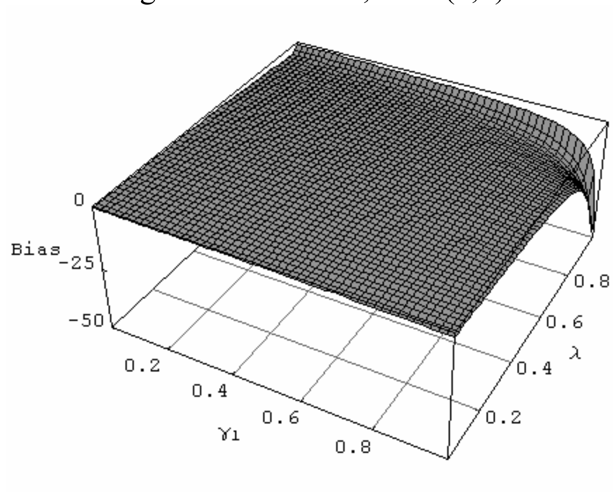

Figure 3.e – T = 10, ADL(1,3)

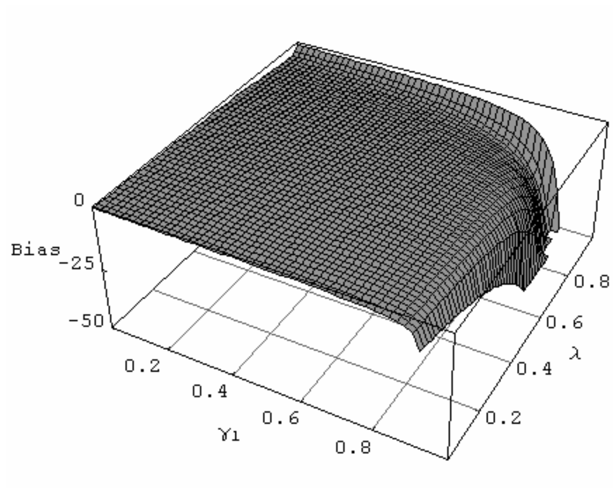

Figure 3.f – T = 10, ADL(3,3)

ADL(1,1): $\beta_0=\beta_1=1$. ADL(1,3): $\beta_0=\beta_1=\beta_2=\beta_3=1$. ADL(3,3): $\beta_0=\beta_1=\beta_2=\beta_3=1$; $\gamma_2=0.05$; $\gamma_3=0.025$. Bias values outside the interval [-600,600] have been clipped. In all Figures, the bias is measured in percent of the long run parameter Θ(1).

Table 1 - Monte Carlo Results I: ADL(1,1) with x_{it}=ζ_i+η_{it}

				w.r.t. θ_0		w.r.t. $\Theta(1)$		True parameters				
		Mean	Var.	Bias	MSE	Bias	MSE	T	γ	ψ	σ^2_ζ	$\Theta(1)$
1	$\hat{\beta}_W$	0.718	0.005	0.080	0.084	3.176	3.181	5	0.2	0.25	1	2.5
	$\hat{\beta}_B$	2.438	0.012	2.069	2.082	0.004	0.016	5	0.2	0.25	1	2.5
	$\hat{\beta}_{RE}$	1.271	0.010	0.073	0.084	1.511	1.521	5	0.2	0.25	1	2.5
	$\hat{\beta}_{OLS}$	1.742	0.010	0.550	0.560	0.575	0.584	5	0.2	0.25	1	2.5
2	$\hat{\beta}_W$	0.854	0.002	0.021	0.024	2.708	2.710	10	0.2	0.25	1	2.5
	$\hat{\beta}_B$	2.483	0.006	2.199	2.206	0.000	0.007	10	0.2	0.25	1	2.5
	$\hat{\beta}_{RE}$	1.352	0.007	0.124	0.131	1.319	1.326	10	0.2	0.25	1	2.5
	$\hat{\beta}_{OLS}$	1.742	0.006	0.550	0.556	0.575	0.581	10	0.2	0.25	1	2.5
3	$\hat{\beta}_W$	0.263	0.007	0.543	0.550	94.811	94.818	5	0.8	0.25	1	10
	$\hat{\beta}_B$	8.046	0.199	49.650	49.849	3.817	4.016	5	0.8	0.25	1	10
	$\hat{\beta}_{RE}$	0.625	0.014	0.141	0.154	87.889	87.902	5	0.8	0.25	1	10
	$\hat{\beta}_{OLS}$	4.897	0.159	15.184	15.343	26.044	26.204	5	0.8	0.25	1	10
4	$\hat{\beta}_W$	0.447	0.005	0.306	0.310	91.260	91.264	10	0.8	0.25	1	10
	$\hat{\beta}_B$	8.953	0.110	63.254	63.364	1.096	1.206	10	0.8	0.25	1	10
	$\hat{\beta}_{RE}$	0.960	0.017	0.002	0.019	81.730	81.747	10	0.8	0.25	1	10
	$\hat{\beta}_{OLS}$	5.082	0.132	16.665	16.797	24.184	24.316	10	0.8	0.25	1	10
5	$\hat{\beta}_W$	0.719	0.005	0.079	0.083	3.171	3.175	5	0.2	0.01	1	2.5
	$\hat{\beta}_B$	2.439	0.268	2.071	2.339	0.004	0.272	5	0.2	0.01	1	2.5
	$\hat{\beta}_{RE}$	0.756	0.005	0.060	0.064	3.043	3.048	5	0.2	0.01	1	2.5
	$\hat{\beta}_{OLS}$	1.742	0.099	0.551	0.649	0.574	0.673	5	0.2	0.01	1	2.5
6	$\hat{\beta}_W$	0.854	0.002	0.021	0.024	2.711	2.713	10	0.2	0.01	1	2.5
	$\hat{\beta}_B$	2.485	0.142	2.205	2.347	0.000	0.142	10	0.2	0.01	1	2.5
	$\hat{\beta}_{RE}$	0.884	0.002	0.013	0.016	2.611	2.614	10	0.2	0.01	1	2.5
	$\hat{\beta}_{OLS}$	1.743	0.046	0.552	0.598	0.573	0.619	10	0.2	0.01	1	2.5
7	$\hat{\beta}_W$	0.264	0.008	0.542	0.550	94.798	94.806	5	0.8	0.01	1	10
	$\hat{\beta}_B$	8.073	3.580	50.024	53.604	3.714	7.294	5	0.8	0.01	1	10
	$\hat{\beta}_{RE}$	0.287	0.008	0.508	0.516	94.337	94.345	5	0.8	0.01	1	10
	$\hat{\beta}_{OLS}$	4.910	1.331	15.288	16.619	25.908	27.238	5	0.8	0.01	1	10
8	$\hat{\beta}_W$	0.446	0.005	0.307	0.312	91.288	91.293	10	0.8	0.01	1	10
	$\hat{\beta}_B$	8.955	2.073	63.289	65.363	1.091	3.165	10	0.8	0.01	1	10
	$\hat{\beta}_{RE}$	0.475	0.005	0.276	0.281	90.724	90.729	10	0.8	0.01	1	10
	$\hat{\beta}_{OLS}$	5.081	0.707	16.657	17.363	24.194	24.900	10	0.8	0.01	1	10
9	$\hat{\beta}_W$	0.720	0.005	0.079	0.083	3.169	3.174	5	0.2	0.25	4	2.5
	$\hat{\beta}_B$	2.440	0.012	2.072	2.084	0.004	0.016	5	0.2	0.25	4	2.5
	$\hat{\beta}_{RE}$	1.273	0.010	0.074	0.085	1.506	1.516	5	0.2	0.25	4	2.5
	$\hat{\beta}_{OLS}$	1.744	0.009	0.554	0.563	0.571	0.580	5	0.2	0.25	4	2.5
10	$\hat{\beta}_W$	0.853	0.002	0.022	0.024	2.711	2.714	10	0.2	0.25	4	2.5
	$\hat{\beta}_B$	2.482	0.006	2.197	2.203	0.000	0.006	10	0.2	0.25	4	2.5
	$\hat{\beta}_{RE}$	1.351	0.007	0.123	0.130	1.320	1.328	10	0.2	0.25	4	2.5
	$\hat{\beta}_{OLS}$	1.741	0.006	0.550	0.556	0.575	0.582	10	0.2	0.25	4	2.5
11	$\hat{\beta}_W$	0.263	0.007	0.544	0.550	94.814	94.820	5	0.8	0.25	4	10
	$\hat{\beta}_B$	8.058	0.201	49.810	50.011	3.773	3.973	5	0.8	0.25	4	10
	$\hat{\beta}_{RE}$	0.625	0.013	0.140	0.154	87.883	87.897	5	0.8	0.25	4	10
	$\hat{\beta}_{OLS}$	4.908	0.156	15.274	15.429	25.927	26.083	5	0.8	0.25	4	10
12	$\hat{\beta}_W$	0.447	0.005	0.306	0.311	91.267	91.271	10	0.8	0.25	4	10
	$\hat{\beta}_B$	8.956	0.103	63.298	63.401	1.090	1.193	10	0.8	0.25	4	10
	$\hat{\beta}_{RE}$	0.960	0.017	0.002	0.019	81.714	81.731	10	0.8	0.25	4	10
	$\hat{\beta}_{OLS}$	5.086	0.132	16.692	16.825	24.151	24.284	10	0.8	0.25	4	10
13	$\hat{\beta}_W$	0.718	0.005	0.080	0.084	3.176	3.181	5	0.2	0.01	4	2.5
	$\hat{\beta}_B$	2.443	0.274	2.082	2.356	0.003	0.277	5	0.2	0.01	4	2.5
	$\hat{\beta}_{RE}$	0.754	0.005	0.060	0.065	3.048	3.053	5	0.2	0.01	4	2.5
	$\hat{\beta}_{OLS}$	1.744	0.101	0.553	0.654	0.572	0.673	5	0.2	0.01	4	2.5
14	$\hat{\beta}_W$	0.854	0.002	0.021	0.024	2.709	2.711	10	0.2	0.01	4	2.5
	$\hat{\beta}_B$	2.479	0.146	2.188	2.333	0.000	0.146	10	0.2	0.01	4	2.5
	$\hat{\beta}_{RE}$	0.884	0.002	0.013	0.016	2.611	2.613	10	0.2	0.01	4	2.5
	$\hat{\beta}_{OLS}$	1.740	0.047	0.547	0.595	0.578	0.625	10	0.2	0.01	4	2.5
15	$\hat{\beta}_W$	0.263	0.008	0.543	0.551	94.809	94.817	5	0.8	0.01	4	10
	$\hat{\beta}_B$	8.075	3.594	50.052	53.645	3.707	7.300	5	0.8	0.01	4	10
	$\hat{\beta}_{RE}$	0.287	0.008	0.509	0.517	94.347	94.355	5	0.8	0.01	4	10
	$\hat{\beta}_{OLS}$	4.913	1.349	15.315	16.664	25.873	27.222	5	0.8	0.01	4	10
16	$\hat{\beta}_W$	0.447	0.005	0.306	0.311	91.262	91.267	10	0.8	0.01	4	10
	$\hat{\beta}_B$	8.955	2.108	63.275	65.384	1.093	3.201	10	0.8	0.01	4	10
	$\hat{\beta}_{RE}$	0.477	0.005	0.274	0.279	90.696	90.701	10	0.8	0.01	4	10
	$\hat{\beta}_{OLS}$	5.083	0.710	16.670	17.380	24.178	24.888	10	0.8	0.01	4	10

Parametrization: $\beta_0 = \beta_1 = 1$; $\sigma^2_u = 1$; $\sigma^2_\eta = 1$.

Table 2 - Monte Carlo Results II: ADL(1,1) with x_{it}=$0.9x_{i(t-1)} + \zeta_i$+η_{it}

				w.r.t. θ_0		w.r.t. $\Theta(1)$		True parameters				
		Mean	Var.	Bias	MSE	Bias	MSE	T	γ	ψ	σ_ζ^2	$\Theta(1)$
17	$\hat{\beta}_{FE}$	1.717	0.008	0.514	0.522	0.613	0.621	5	0.2	0.25	1	2.5
	$\hat{\beta}_{BE}$	2.426	0.000	2.034	2.034	0.005	0.006	5	0.2	0.25	1	2.5
	$\hat{\beta}_{RE}$	2.390	0.000	1.932	1.932	0.012	0.012	5	0.2	0.25	1	2.5
	$\hat{\beta}_{OLS}$	2.415	0.000	2.002	2.003	0.007	0.008	5	0.2	0.25	1	2.5
18	$\hat{\beta}_{FE}$	2.051	0.003	1.104	1.107	0.202	0.205	10	0.2	0.25	1	2.5
	$\hat{\beta}_{BE}$	2.446	0.000	2.091	2.091	0.003	0.003	10	0.2	0.25	1	2.5
	$\hat{\beta}_{RE}$	2.418	0.000	2.012	2.012	0.007	0.007	10	0.2	0.25	1	2.5
	$\hat{\beta}_{OLS}$	2.435	0.000	2.060	2.060	0.004	0.004	10	0.2	0.25	1	2.5
19	$\hat{\beta}_{FE}$	3.639	0.234	6.964	7.198	40.462	40.696	5	0.8	0.25	1	10
	$\hat{\beta}_{BE}$	7.431	0.011	41.362	41.373	6.598	6.609	5	0.8	0.25	1	10
	$\hat{\beta}_{RE}$	7.015	0.024	36.186	36.210	8.907	8.931	5	0.8	0.25	1	10
	$\hat{\beta}_{OLS}$	7.371	0.010	40.596	40.606	6.909	6.920	5	0.8	0.25	1	10
20	$\hat{\beta}_{FE}$	5.317	0.245	18.633	18.878	21.935	22.179	10	0.8	0.25	1	10
	$\hat{\beta}_{BE}$	8.036	0.007	49.506	49.512	3.857	3.864	10	0.8	0.25	1	10
	$\hat{\beta}_{RE}$	7.712	0.016	45.055	45.071	5.234	5.249	10	0.8	0.25	1	10
	$\hat{\beta}_{OLS}$	7.963	0.006	48.481	48.487	4.150	4.156	10	0.8	0.25	1	10
21	$\hat{\beta}_{FE}$	1.717	0.008	0.514	0.522	0.613	0.621	5	0.2	0.01	1	2.5
	$\hat{\beta}_{BE}$	2.426	0.006	2.034	2.039	0.005	0.011	5	0.2	0.01	1	2.5
	$\hat{\beta}_{RE}$	2.078	0.005	1.163	1.168	0.178	0.183	5	0.2	0.01	1	2.5
	$\hat{\beta}_{OLS}$	2.415	0.005	2.002	2.007	0.007	0.013	5	0.2	0.01	1	2.5
22	$\hat{\beta}_{FE}$	2.052	0.003	1.108	1.111	0.200	0.203	10	0.2	0.01	1	2.5
	$\hat{\beta}_{BE}$	2.446	0.002	2.090	2.092	0.003	0.005	10	0.2	0.01	1	2.5
	$\hat{\beta}_{RE}$	2.216	0.002	1.479	1.481	0.081	0.082	10	0.2	0.01	1	2.5
	$\hat{\beta}_{OLS}$	2.435	0.002	2.059	2.062	0.004	0.007	10	0.2	0.01	1	2.5
23	$\hat{\beta}_{FE}$	3.639	0.235	6.966	7.201	40.458	40.693	5	0.8	0.01	1	10
	$\hat{\beta}_{BE}$	7.435	0.081	41.409	41.490	6.579	6.660	5	0.8	0.01	1	10
	$\hat{\beta}_{RE}$	5.683	0.184	21.931	22.115	18.636	18.820	5	0.8	0.01	1	10
	$\hat{\beta}_{OLS}$	7.375	0.078	40.642	40.720	6.890	6.968	5	0.8	0.01	1	10
24	$\hat{\beta}_{FE}$	5.321	0.240	18.671	18.911	21.893	22.133	10	0.8	0.01	1	10
	$\hat{\beta}_{BE}$	8.037	0.039	49.513	49.553	3.855	3.895	10	0.8	0.01	1	10
	$\hat{\beta}_{RE}$	6.850	0.117	34.218	34.335	9.925	10.041	10	0.8	0.01	1	10
	$\hat{\beta}_{OLS}$	7.963	0.037	48.490	48.527	4.147	4.185	10	0.8	0.01	1	10
25	$\hat{\beta}_{FE}$	1.685	0.008	0.470	0.478	0.664	0.672	5	0.2	0.25	4	2.5
	$\hat{\beta}_{BE}$	2.436	0.000	2.063	2.063	0.004	0.004	5	0.2	0.25	4	2.5
	$\hat{\beta}_{RE}$	2.401	0.000	1.963	1.964	0.010	0.010	5	0.2	0.25	4	2.5
	$\hat{\beta}_{OLS}$	2.426	0.000	2.032	2.033	0.006	0.006	5	0.2	0.25	4	2.5
26	$\hat{\beta}_{FE}$	2.022	0.003	1.044	1.047	0.229	0.232	10	0.2	0.25	4	2.5
	$\hat{\beta}_{BE}$	2.453	0.000	2.111	2.111	0.002	0.002	10	0.2	0.25	4	2.5
	$\hat{\beta}_{RE}$	2.425	0.000	2.031	2.031	0.006	0.006	10	0.2	0.25	4	2.5
	$\hat{\beta}_{OLS}$	2.442	0.000	2.081	2.081	0.003	0.003	10	0.2	0.25	4	2.5
27	$\hat{\beta}_{FE}$	3.381	0.227	5.668	5.895	43.814	44.041	5	0.8	0.25	4	10
	$\hat{\beta}_{BE}$	7.624	0.010	43.877	43.887	5.646	5.656	5	0.8	0.25	4	10
	$\hat{\beta}_{RE}$	7.181	0.025	38.200	38.224	7.949	7.974	5	0.8	0.25	4	10
	$\hat{\beta}_{OLS}$	7.564	0.010	43.080	43.090	5.936	5.946	5	0.8	0.25	4	10
28	$\hat{\beta}_{FE}$	4.977	0.237	15.815	16.053	25.232	25.470	10	0.8	0.25	4	10
	$\hat{\beta}_{BE}$	8.193	0.006	51.736	51.742	3.266	3.272	10	0.8	0.25	4	10
	$\hat{\beta}_{RE}$	7.826	0.017	46.599	46.616	4.725	4.741	10	0.8	0.25	4	10
	$\hat{\beta}_{OLS}$	8.114	0.006	50.612	50.618	3.556	3.562	10	0.8	0.25	4	10
29	$\hat{\beta}_{FE}$	1.684	0.008	0.468	0.476	0.666	0.674	5	0.2	0.01	4	2.5
	$\hat{\beta}_{BE}$	2.436	0.005	2.063	2.068	0.004	0.009	5	0.2	0.01	4	2.5
	$\hat{\beta}_{RE}$	2.084	0.005	1.175	1.180	0.173	0.178	5	0.2	0.01	4	2.5
	$\hat{\beta}_{OLS}$	2.426	0.005	2.032	2.037	0.006	0.011	5	0.2	0.01	4	2.5
30	$\hat{\beta}_{FE}$	2.022	0.003	1.044	1.047	0.229	0.232	10	0.2	0.01	4	2.5
	$\hat{\beta}_{BE}$	2.454	0.002	2.113	2.116	0.002	0.004	10	0.2	0.01	4	2.5
	$\hat{\beta}_{RE}$	2.210	0.002	1.465	1.467	0.084	0.086	10	0.2	0.01	4	2.5
	$\hat{\beta}_{OLS}$	2.443	0.002	2.083	2.085	0.003	0.005	10	0.2	0.01	4	2.5
31	$\hat{\beta}_{FE}$	3.374	0.233	5.636	5.869	43.903	44.136	5	0.8	0.01	4	10
	$\hat{\beta}_{BE}$	7.627	0.072	43.914	43.986	5.632	5.705	5	0.8	0.01	4	10
	$\hat{\beta}_{RE}$	5.744	0.201	22.504	22.705	18.115	18.316	5	0.8	0.01	4	10
	$\hat{\beta}_{OLS}$	7.566	0.070	43.118	43.188	5.922	5.993	5	0.8	0.01	4	10
32	$\hat{\beta}_{FE}$	4.979	0.240	15.834	16.075	25.208	25.448	10	0.8	0.01	4	10
	$\hat{\beta}_{BE}$	8.188	0.038	51.672	51.710	3.282	3.321	10	0.8	0.01	4	10
	$\hat{\beta}_{RE}$	6.832	0.127	34.009	34.137	10.038	10.165	10	0.8	0.01	4	10
	$\hat{\beta}_{OLS}$	8.110	0.036	50.551	50.588	3.572	3.609	10	0.8	0.01	4	10

Parametrization: $\beta_0 = \beta_1 = 1$; $\sigma_u^2 = 1$; $\sigma_\eta^2 = 1$.

Table 3 - Monte Carlo Results III: ADL(3,3) with $x_{it}=\zeta_i+\eta_{it}$

				w.r.t. θ_0		w.r.t. $\Theta(1)$		True parameters				
		Mean	Var.	Bias	MSE	Bias	MSE	T	γ	ψ	σ^2_ζ	$\Theta(1)$
33	$\hat\beta_W$	0.414	0.007	0.343	0.350	26.044	26.051	5	0.2	0.25	1	5.52
	$\hat\beta_B$	5.152	0.035	17.243	17.278	0.133	0.168	5	0.2	0.25	1	5.52
	$\hat\beta_{RE}$	1.318	0.026	0.101	0.127	17.637	17.663	5	0.2	0.25	1	5.52
	$\hat\beta_{OLS}$	3.236	0.045	5.002	5.047	5.202	5.247	5	0.2	0.25	1	5.52
34	$\hat\beta_W$	0.622	0.004	0.143	0.146	23.960	23.964	10	0.2	0.25	1	5.52
	$\hat\beta_B$	5.415	0.013	19.495	19.508	0.010	0.024	10	0.2	0.25	1	5.52
	$\hat\beta_{RE}$	1.943	0.042	0.889	0.932	12.774	12.817	10	0.2	0.25	1	5.52
	$\hat\beta_{OLS}$	3.236	0.037	4.998	5.035	5.205	5.242	10	0.2	0.25	1	5.52
35	$\hat\beta_W$	-0.175	0.024	1.382	1.406	1035.262	1035.286	5	0.8	0.25	1	32.00
	$\hat\beta_B$	19.502	0.906	342.334	343.240	156.193	157.100	5	0.8	0.25	1	32.00
	$\hat\beta_{RE}$	0.378	0.041	0.387	0.427	999.934	999.975	5	0.8	0.25	1	32.00
	$\hat\beta_{OLS}$	11.539	0.875	111.060	111.935	418.672	419.547	5	0.8	0.25	1	32.00
36	$\hat\beta_W$	-0.156	0.019	1.336	1.355	1034.007	1034.026	10	0.8	0.25	1	32.00
	$\hat\beta_B$	23.256	0.506	495.311	495.816	76.465	76.971	10	0.8	0.25	1	32.00
	$\hat\beta_{RE}$	0.915	0.085	0.007	0.093	966.266	966.352	10	0.8	0.25	1	32.00
	$\hat\beta_{OLS}$	12.598	0.887	134.503	135.390	376.455	377.342	10	0.8	0.25	1	32.00
37	$\hat\beta_W$	0.414	0.007	0.344	0.351	26.047	26.054	5	0.2	0.01	1	5.52
	$\hat\beta_B$	5.158	0.350	17.287	17.637	0.129	0.479	5	0.2	0.01	1	5.52
	$\hat\beta_{RE}$	0.525	0.007	0.225	0.232	24.918	24.926	5	0.2	0.01	1	5.52
	$\hat\beta_{OLS}$	3.240	0.155	5.016	5.171	5.188	5.343	5	0.2	0.01	1	5.52
38	$\hat\beta_W$	0.622	0.004	0.143	0.147	23.967	23.971	10	0.2	0.01	1	5.52
	$\hat\beta_B$	5.409	0.184	19.441	19.625	0.012	0.196	10	0.2	0.01	1	5.52
	$\hat\beta_{RE}$	0.754	0.005	0.061	0.065	22.691	22.696	10	0.2	0.01	1	5.52
	$\hat\beta_{OLS}$	3.232	0.088	4.981	5.068	5.224	5.311	10	0.2	0.01	1	5.52
39	$\hat\beta_W$	-0.177	0.032	1.386	1.418	1035.368	1035.400	5	0.8	0.01	1	32.00
	$\hat\beta_B$	19.525	6.530	343.183	349.712	155.621	162.151	5	0.8	0.01	1	32.00
	$\hat\beta_{RE}$	-0.075	0.032	1.156	1.189	1028.827	1028.859	5	0.8	0.01	1	32.00
	$\hat\beta_{OLS}$	11.553	2.875	111.359	114.234	418.093	420.968	5	0.8	0.01	1	32.00
40	$\hat\beta_W$	-0.156	0.023	1.336	1.358	1033.986	1034.008	10	0.8	0.01	1	32.00
	$\hat\beta_B$	23.214	4.192	493.483	497.675	77.186	81.378	10	0.8	0.01	1	32.00
	$\hat\beta_{RE}$	0.006	0.025	0.989	1.014	1023.640	1023.665	10	0.8	0.01	1	32.00
	$\hat\beta_{OLS}$	12.588	1.994	134.273	136.267	376.840	378.834	10	0.8	0.01	1	32.00
41	$\hat\beta_W$	0.414	0.007	0.344	0.350	26.046	26.053	5	0.2	0.25	4	5.52
	$\hat\beta_B$	5.153	0.035	17.249	17.284	0.133	0.168	5	0.2	0.25	4	5.52
	$\hat\beta_{RE}$	1.317	0.027	0.100	0.127	17.642	17.669	5	0.2	0.25	4	5.52
	$\hat\beta_{OLS}$	3.235	0.045	4.994	5.040	5.209	5.255	5	0.2	0.25	4	5.52
42	$\hat\beta_W$	0.622	0.004	0.143	0.147	23.966	23.970	10	0.2	0.25	4	5.52
	$\hat\beta_B$	5.416	0.014	19.502	19.515	0.010	0.024	10	0.2	0.25	4	5.52
	$\hat\beta_{RE}$	1.944	0.043	0.892	0.934	12.766	12.809	10	0.2	0.25	4	5.52
	$\hat\beta_{OLS}$	3.237	0.036	5.003	5.040	5.200	5.237	10	0.2	0.25	4	5.52
43	$\hat\beta_W$	-0.177	0.024	1.385	1.409	1035.349	1035.373	5	0.8	0.25	4	32.00
	$\hat\beta_B$	19.750	0.960	351.574	352.534	150.055	151.016	5	0.8	0.25	4	32.00
	$\hat\beta_{RE}$	0.362	0.039	0.407	0.447	1000.970	1001.009	5	0.8	0.25	4	32.00
	$\hat\beta_{OLS}$	11.689	0.924	114.250	115.174	412.546	413.470	5	0.8	0.25	4	32.00
44	$\hat\beta_W$	-0.157	0.019	1.339	1.358	1034.087	1034.106	10	0.8	0.25	4	32.00
	$\hat\beta_B$	23.475	0.520	505.123	505.643	72.677	73.197	10	0.8	0.25	4	32.00
	$\hat\beta_{RE}$	0.899	0.083	0.010	0.093	967.291	967.373	10	0.8	0.25	4	32.00
	$\hat\beta_{OLS}$	12.729	0.920	137.571	138.490	371.370	372.289	10	0.8	0.25	4	32.00
45	$\hat\beta_W$	0.414	0.007	0.343	0.350	26.039	26.045	5	0.2	0.01	4	5.52
	$\hat\beta_B$	5.154	0.349	17.254	17.602	0.132	0.481	5	0.2	0.01	4	5.52
	$\hat\beta_{RE}$	0.526	0.007	0.225	0.232	24.917	24.924	5	0.2	0.01	4	5.52
	$\hat\beta_{OLS}$	3.234	0.156	4.990	5.146	5.214	5.370	5	0.2	0.01	4	5.52
46	$\hat\beta_W$	0.621	0.004	0.144	0.147	23.973	23.976	10	0.2	0.01	4	5.52
	$\hat\beta_B$	5.414	0.185	19.483	19.668	0.011	0.196	10	0.2	0.01	4	5.52
	$\hat\beta_{RE}$	0.753	0.005	0.061	0.066	22.695	22.699	10	0.2	0.01	4	5.52
	$\hat\beta_{OLS}$	3.235	0.086	4.995	5.081	5.209	5.294	10	0.2	0.01	4	5.52
47	$\hat\beta_W$	-0.178	0.031	1.387	1.418	1035.398	1035.429	5	0.8	0.01	4	32.00
	$\hat\beta_B$	19.770	6.552	352.330	358.882	149.562	156.114	5	0.8	0.01	4	32.00
	$\hat\beta_{RE}$	-0.076	0.031	1.159	1.190	1028.901	1028.932	5	0.8	0.01	4	32.00
	$\hat\beta_{OLS}$	11.701	2.853	114.517	117.370	412.039	414.892	5	0.8	0.01	4	32.00
48	$\hat\beta_W$	-0.158	0.022	1.340	1.362	1034.118	1034.140	10	0.8	0.01	4	32.00
	$\hat\beta_B$	23.490	4.180	505.801	509.981	72.420	76.600	10	0.8	0.01	4	32.00
	$\hat\beta_{RE}$	0.002	0.024	0.997	1.021	1023.891	1023.915	10	0.8	0.01	4	32.00
	$\hat\beta_{OLS}$	12.726	2.005	137.506	139.512	371.475	373.481	10	0.8	0.01	4	32.00

Parametrization: $\beta_0 = \beta_1 = \beta_2 = \beta_3 = 1$; $\gamma_2 = 0.05$; $\gamma_3 = 0.025$; $\sigma^2_u = 1$; $\sigma^2_\eta = 1$.

Table 4 - Monte Carlo Results IV: ADL(3,3) with x_{it}=$0.9x_{i(t-1)} + \zeta_i$+η_{it}

				w.r.t. θ_0		w.r.t. $\Theta(1)$			True parameters			
		Mean	Var.	Bias	MSE	Bias	MSE	T	γ	ψ	σ^2_ζ	$\Theta(1)$
49	$\hat{\beta}_{FE}$	2.332	0.068	1.774	1.842	10.146	10.214	5	0.2	0.25	1	5.52
	$\hat{\beta}_{BE}$	5.021	0.002	16.168	16.171	0.246	0.249	5	0.2	0.25	1	5.52
	$\hat{\beta}_{RE}$	4.812	0.004	14.533	14.537	0.497	0.501	5	0.2	0.25	1	5.52
	$\hat{\beta}_{OLS}$	4.979	0.002	15.830	15.832	0.290	0.292	5	0.2	0.25	1	5.52
50	$\hat{\beta}_{FE}$	3.398	0.047	5.749	5.796	4.492	4.539	10	0.2	0.25	1	5.52
	$\hat{\beta}_{BE}$	5.158	0.001	17.285	17.286	0.129	0.130	10	0.2	0.25	1	5.52
	$\hat{\beta}_{RE}$	5.025	0.002	16.200	16.201	0.242	0.244	10	0.2	0.25	1	5.52
	$\hat{\beta}_{OLS}$	5.111	0.001	16.896	16.897	0.165	0.166	10	0.2	0.25	1	5.52
51	$\hat{\beta}_{FE}$	7.641	2.218	44.105	46.323	593.354	595.572	5	0.8	0.25	1	32.00
	$\hat{\beta}_{BE}$	16.330	0.089	235.008	235.097	245.550	245.639	5	0.8	0.25	1	32.00
	$\hat{\beta}_{RE}$	15.471	0.162	209.414	209.576	273.203	273.365	5	0.8	0.25	1	32.00
	$\hat{\beta}_{OLS}$	16.192	0.085	230.799	230.883	249.891	249.976	5	0.8	0.25	1	32.00
52	$\hat{\beta}_{FE}$	12.478	3.205	131.742	134.947	381.113	384.319	10	0.8	0.25	1	32.00
	$\hat{\beta}_{BE}$	18.941	0.069	321.890	321.959	170.530	170.599	10	0.8	0.25	1	32.00
	$\hat{\beta}_{RE}$	18.229	0.136	296.842	296.977	189.638	189.774	10	0.8	0.25	1	32.00
	$\hat{\beta}_{OLS}$	18.765	0.062	315.606	315.668	175.157	175.220	10	0.8	0.25	1	32.00
53	$\hat{\beta}_{FE}$	2.330	0.065	1.770	1.835	10.157	10.222	5	0.2	0.01	1	5.52
	$\hat{\beta}_{BE}$	5.020	0.009	16.163	16.172	0.247	0.256	5	0.2	0.01	1	5.52
	$\hat{\beta}_{RE}$	4.399	0.024	11.551	11.575	1.251	1.276	5	0.2	0.01	1	5.52
	$\hat{\beta}_{OLS}$	4.978	0.008	15.825	15.834	0.291	0.299	5	0.2	0.01	1	5.52
54	$\hat{\beta}_{FE}$	3.399	0.046	5.753	5.800	4.489	4.535	10	0.2	0.01	1	5.52
	$\hat{\beta}_{BE}$	5.157	0.004	17.283	17.287	0.130	0.133	10	0.2	0.01	1	5.52
	$\hat{\beta}_{RE}$	4.749	0.011	14.054	14.064	0.590	0.601	10	0.2	0.01	1	5.52
	$\hat{\beta}_{OLS}$	5.110	0.004	16.893	16.897	0.166	0.169	10	0.2	0.01	1	5.52
55	$\hat{\beta}_{FE}$	7.644	2.220	44.149	46.370	593.191	595.412	5	0.8	0.01	1	32.00
	$\hat{\beta}_{BE}$	16.338	0.203	235.248	235.451	245.305	245.508	5	0.8	0.01	1	32.00
	$\hat{\beta}_{RE}$	14.618	0.456	185.437	185.893	302.151	302.607	5	0.8	0.01	1	32.00
	$\hat{\beta}_{OLS}$	16.200	0.195	231.040	231.234	249.641	249.835	5	0.8	0.01	1	32.00
56	$\hat{\beta}_{FE}$	12.502	3.154	132.305	135.459	380.157	383.311	10	0.8	0.01	1	32.00
	$\hat{\beta}_{BE}$	18.943	0.131	321.958	322.088	170.481	170.611	10	0.8	0.01	1	32.00
	$\hat{\beta}_{RE}$	17.762	0.325	280.965	281.290	202.720	203.045	10	0.8	0.01	1	32.00
	$\hat{\beta}_{OLS}$	18.768	0.122	315.719	315.841	175.073	175.195	10	0.8	0.01	1	32.00
57	$\hat{\beta}_{FE}$	2.174	0.061	1.378	1.439	11.179	11.240	5	0.2	0.25	4	5.52
	$\hat{\beta}_{BE}$	5.087	0.002	16.701	16.704	0.185	0.188	5	0.2	0.25	4	5.52
	$\hat{\beta}_{RE}$	4.869	0.004	14.968	14.972	0.420	0.425	5	0.2	0.25	4	5.52
	$\hat{\beta}_{OLS}$	5.045	0.002	16.365	16.367	0.223	0.225	5	0.2	0.25	4	5.52
58	$\hat{\beta}_{FE}$	3.236	0.044	5.001	5.045	5.203	5.246	10	0.2	0.25	4	5.52
	$\hat{\beta}_{BE}$	5.204	0.001	17.671	17.672	0.098	0.099	10	0.2	0.25	4	5.52
	$\hat{\beta}_{RE}$	5.061	0.002	16.491	16.493	0.208	0.210	10	0.2	0.25	4	5.52
	$\hat{\beta}_{OLS}$	5.156	0.001	17.270	17.271	0.131	0.132	10	0.2	0.25	4	5.52
59	$\hat{\beta}_{FE}$	6.974	2.324	35.690	38.015	626.293	628.618	5	0.8	0.25	4	32.00
	$\hat{\beta}_{BE}$	17.128	0.086	260.114	260.199	221.175	221.261	5	0.8	0.25	4	32.00
	$\hat{\beta}_{RE}$	16.196	0.171	230.924	231.095	249.760	249.931	5	0.8	0.25	4	32.00
	$\hat{\beta}_{OLS}$	16.983	0.082	255.447	255.528	225.519	225.601	5	0.8	0.25	4	32.00
60	$\hat{\beta}_{FE}$	11.478	3.259	109.785	113.044	421.158	424.417	10	0.8	0.25	4	32.00
	$\hat{\beta}_{BE}$	19.648	0.068	347.756	347.824	152.567	152.634	10	0.8	0.25	4	32.00
	$\hat{\beta}_{RE}$	18.814	0.150	317.329	317.480	173.877	174.028	10	0.8	0.25	4	32.00
	$\hat{\beta}_{OLS}$	19.447	0.061	340.299	340.360	157.573	157.635	10	0.8	0.25	4	32.00
61	$\hat{\beta}_{FE}$	2.173	0.063	1.375	1.437	11.187	11.250	5	0.2	0.01	4	5.52
	$\hat{\beta}_{BE}$	5.088	0.008	16.711	16.720	0.184	0.192	5	0.2	0.01	4	5.52
	$\hat{\beta}_{RE}$	4.433	0.026	11.783	11.809	1.176	1.202	5	0.2	0.01	4	5.52
	$\hat{\beta}_{OLS}$	5.047	0.008	16.376	16.384	0.221	0.229	5	0.2	0.01	4	5.52
62	$\hat{\beta}_{FE}$	3.239	0.043	5.012	5.055	5.192	5.235	10	0.2	0.01	4	5.52
	$\hat{\beta}_{BE}$	5.203	0.004	17.668	17.672	0.099	0.102	10	0.2	0.01	4	5.52
	$\hat{\beta}_{RE}$	4.763	0.011	14.163	14.174	0.568	0.579	10	0.2	0.01	4	5.52
	$\hat{\beta}_{OLS}$	5.155	0.004	17.268	17.271	0.131	0.134	10	0.2	0.01	4	5.52
63	$\hat{\beta}_{FE}$	7.003	2.299	36.039	38.338	624.839	627.138	5	0.8	0.01	4	32.00
	$\hat{\beta}_{BE}$	17.123	0.192	259.952	260.144	221.324	221.516	5	0.8	0.01	4	32.00
	$\hat{\beta}_{RE}$	15.283	0.489	203.995	204.485	279.469	279.958	5	0.8	0.01	4	32.00
	$\hat{\beta}_{OLS}$	16.978	0.185	255.310	255.495	225.648	225.834	5	0.8	0.01	4	32.00
64	$\hat{\beta}_{FE}$	11.476	3.267	109.737	113.005	421.253	424.520	10	0.8	0.01	4	32.00
	$\hat{\beta}_{BE}$	19.649	0.124	347.769	347.893	152.558	152.682	10	0.8	0.01	4	32.00
	$\hat{\beta}_{RE}$	18.248	0.363	297.508	297.871	189.106	189.469	10	0.8	0.01	4	32.00
	$\hat{\beta}_{OLS}$	19.447	0.116	340.290	340.406	157.579	157.695	10	0.8	0.01	4	32.00

Parametrization: $\beta_0 = \beta_1 = \beta_2 = \beta_3 = 1$; $\gamma_2 = 0.05$; $\gamma_3 = 0.025$; $\sigma^2_u = 1$; $\sigma^2_\eta = 1$.

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