Price-Directed Search and Collusion

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Motivation I/II

- Many (online) markets: **Consumers freely observe competing products' prices, but need to further investigate products at positive cost** ("search cost") **to find out how much they like them**
- **E.g. hotels on online platform:** Prices (and some other attributes) can easily be compared – but need to click on listed hotels to check photos, customer ratings, specific details
- In a given product category (e.g., with some filters set), all products
  - have **prices readily observable**
  - **satisfy "base" need, but some are better then others**
  - ex-ante identical: **optimal to search in increasing order of price**

What is **equilibrium pricing and search behavior** with (for tractability) **binary match values?**

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Motivation II/II

- **Price transparency in online markets** makes it easy for firms to monitor rivals' prices. **Facilitates collusion?**
- Lately, **investigations by European Commission** on "algorithmic pricing" (firms using pricing algorithms which take competitors' prices into account)
- Argue that **pricing algorithms facilitate explicit and implicit, tacit collusive agreements**

Main application of model: study **collusion in (online) markets characterized by price-directed search**

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Main application of model: study **collusion in (online) markets characterized by price-directed search**

Main Findings

- **Four types of pricing equilibria**, of which three are in mixed strategies; the other one leads to the Bertrand-outcome
- **Inefficiencies in two types of mixed-strategy-equilibria**: Not all consumers (always) served, even though consumers' valuations always exceed marginal cost
- **Characterization of payoff-dominant symmetric collusive equilibrium** (and cartel solution)
- **Collusion/cartelization may improve market performance** if search costs relatively low

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Related Literature

- **Price-directed consumer search: Ding and Zhang (2018)**, Choi et al. (2018), Haan et al. (2018), Armstrong and Zhou (2011)
- **Consumer search / market transparency and collusion: Petrikaite (2016)**, Campbell et al. (2005), Schultz (2005), Nilsson (1999)
- Ordered consumer search and prominence: Arbatskaya (2007), Armstrong et al. (2009), Zhou (2011), Armstrong (2017)
- Position auctions and consumer search in online platforms: Chen and He (2011), Athey and Ellison (2011), Anderson and Renault (2016)
- Consumer search with differentiated products and unobservable prices: Wolinsky (1986), Anderson and Renault (1999)

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Model Setup I/III

- **n $\geq$ 2** symmetric risk-neutral **firms each selling a single differentiated product** at $MC = 0$
- **Simultaneously set prices $p_i \geq 0$**
- **Mass 1 of risk-neutral consumers**; unit demand; outside option value 0
- **Freely observe each firm's price**, but do not know ex-ante how much they value its product: **uncertain match values**
- Firms appear ex-ante homogeneous: **For each consumer-firm pair**
  - **Prob. $\alpha \in (0, 1)$: "full match", valuation $v_H > 0$**
  - **Prob. $1 - \alpha$: "partial match", valuation $v_L \in [0, v_H]$**

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Model Setup II/III

- **Consumers need to incur search cost s $\geq$ 0** to find out whether any given product is a full or partial match for them
- **Sequential search with free recall**
- Can only buy from any firm after inspecting its product
- Utility of consumer m when buying at firm i:

$$u_{mi} = matchvalue_{mi} - price_i - totalsearchcost$$

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Model Setup III/III

Equilibrium:

- **Consumers search and purchase optimally**, given observed prices p_i, known market fundamentals (v_H, v_L, s, α, n), and so-far obtained match values
- **Firms price optimally**, given rivals' pricing strategies and consumers' optimal search behavior

Attention restricted to **symmetric equilibria**

Optimal Search I/III

- Prices freely observable, products ex-ante identical: Intuitive that consumers find it **optimal to search in ascending order of prices**
- W.l.o.g.: Let $p_1 \leq p_2 \leq \ldots \leq p_n$
- Our setting with binary match values: have to distinguish **two cases**

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Optimal Search II/III

- **Case I: $\mathbf{p_1 > v_L}$**
  - Search in ascending order of prices and purchase immediately if full match is found – “fresh” demand
  - Only search firms $i \geq 1$ for which the expected gains from search are non-negative, i.e. $\alpha(v_H - p_i) - s \geq 0 \Leftrightarrow p_i \leq v_H - \frac{s}{\alpha}$
  - **Take outside option** if no more suitable firms left

Optimal Search III/III

- **Case II: $\mathbf{p_1 \leq v_L}$**
  - Search in ascending order of prices and purchase immediately if full match is found – "fresh" demand
  - Consumers who found a partial match at firm 1 hold a purchasing option with positive net utility.
  - Only search firms $i > 1$ for which the expected gains from search are non-negative, i.e.

    $\alpha((v_H - p_i) - (v_L - p_1)) - s \geq 0 \Leftrightarrow p_i \leq p_1 + (v_H - v_L - \frac{s}{\alpha})$
  - **Return to purchase at firm 1** (with only partial match) if no more suitable firms left – **"returning" demand**

Equilibrium Analysis: Preliminaries

- **No pure-strategy equilibrium exists where firms make positive profits**
  - profitable to undercut slightly in order to be searched first
- For similar reasons: in any symmetric mixed-strategy equilibrium, **no mass points at positive prices can exist**
  - in any symmetric equlibrium in which firms make positive profits, prices must be drawn from an atomless CDF

High-Price Equilibrium

- **In "high-price equilibrium", all firms always price strictly above $\mathbf{v_L}$**: no "returning" demand.
- **Upper support bound** equals highest price that still induces search: $\overline{p}_H = v_H - \frac{s}{\alpha}$. Associated demand $(1-\alpha)^{n-1}\alpha$
  $\rightarrow \boldsymbol{\pi_H^*} = \left(v_H - \frac{s}{\alpha}\right)(1-\alpha)^{n-1}\alpha$
- Equilibrium CDF $F_H(\cdot)$ satisfies $p \underbrace{(1-\alpha F_H(p))^{n-1}\alpha}_{\text{Expected "fresh" demand}} = \pi_H^*$
- **No firm has an incentive to deviate to a price $\mathrm{p} \leq \mathrm{v_L}$ if and only if $\mathrm{v_L}/\mathrm{v_H}$ is relatively low (high product differentiation)**

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Low-Price + Gap Equilibrium: Conditions

- **In both "low-price" and "gap equilibrium" firms price strictly below $\mathbf{v_L}$ with positive probability** $\Rightarrow$ returning demand
- In either case, equilibrium support $[\underline{p}, \overline{p}]$ and profit π^* determined as follows:

1. **Support-boundary condition:** Consumer with partial match at lowest possible price $\underline{p}$ exactly indifferent between stopping and searching firm with highest possible price $\overline{p}$. **Hence:**

$$\overline{p} \stackrel{!}{=} \underline{p} + (v_H - v_L - \frac{s}{\alpha})$$

2. **Profit indifference:** Profit at lowest price $\underline{p}$ must equal profit at highest price $\overline{p}$:

$$\underline{p}[\alpha + (1-\alpha)^n] \stackrel{!}{=} \overline{p}(1-\alpha)^{n-1}\alpha$$

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Low-Price Equilibrium

- **For $\mathbf{v_L/v_H}$ relatively high** (low differentiation): $\overline{p} \leq v_L$ – **low-price equilibrium**
  - Equilibrium CDF $F_L(\cdot)$ implicitly defined by

$$\pi_i(p) = p\Big[\underbrace{(1-\alpha F_L(p))^{n-1}\alpha}_{\text{“fresh” demand}} + \underbrace{(1-F_L(p))^{n-1}(1-\alpha)^n}_{\text{“returning” demand}}\Big] \stackrel{!}{=} \pi^*$$

Gap Equilibrium

- **For $\mathbf{v_L/v_H}$ intermediate** (intermediate differentiation): $\overline{p} > v_L$. Then support has gap in some range $(v_L, \underline{p}'_M)$ – **gap equilibrium**
  - **Reason:** Discrete loss of (returning) demand when pricing above v_L
  - Equilibrium CDF has two parts: lower part defined as above, upper part as above but without returning demand
  - **Any given firm prices below v_L only with probability $\kappa \in (0, 1)$**

Bertrand Equilibrium

- **For low probability of full matches, or very low differentiation**: even for $p_2 \approx p_1$, consumers would optimally stop at firm 1
  - Hence, **no consumer ever searches beyond lowest-priced firm**
  - **Bertrand-type competition leads to marginal-cost pricing** and zero profits in the unique symmetric equilibrium

Illustration of Pricing Equilibria

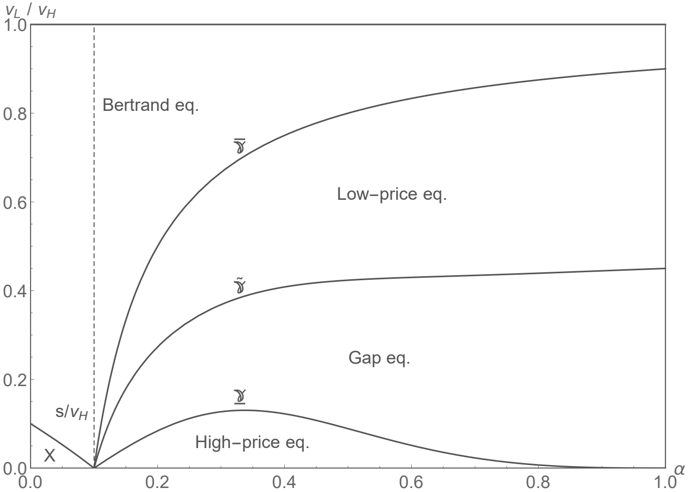

Figure: Equilibrium regions for $\frac{s}{v_H} = 0.1$, $n = 4$.

Welfare

- **Prices paid are pure transfers; equilibrium search behavior optimal** from society's point of view. However:
- **Deterministic welfare loss** of $(1-\alpha)^n v_L$ **in high-price equilibrium**; **expected welfare loss** of $(1-\alpha)^n v_L (1-\kappa)^n$ **in gap equilibrium**

Welfare comparative statics

Price-Directed Search and Collusion I/II

- Consider **repeated version of outlined game:**
  - Consumers get replaced each period
  - Firms discount future profits with common discount factor $\delta \in (0, 1)$
  - **Focus on optimal symmetric collusive scheme**

Price-Directed Search and Collusion II/II

- **Main result 1: The optimal symmetric collusive price p^C is**
  - $< v_L$ in Bertrand region
  - $\mathbf{= v_L}$ **in non-Bertrand regions with high** $\mathbf{v_L}$
  - $\mathbf{= v_H - \frac{s}{\alpha} > v_L}$ **in non-Bertrand regions with low** $\mathbf{v_L}$
- **Main result 2: Welfare may strictly increase under collusion**
  - Two prerequisites: **(1) Differentiation relatively high**, v_L/v_H small, such that **gap equilibrium** played in baseline game
  - **(2) But:** $\mathbf{v_L/v_H}$ **not too small such that still** $\mathbf{p^C = v_L}$
  - For any n, region exists if s sufficiently small; α, v_L intermediate

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Price-Directed Search and Collusion: Welfare Effects

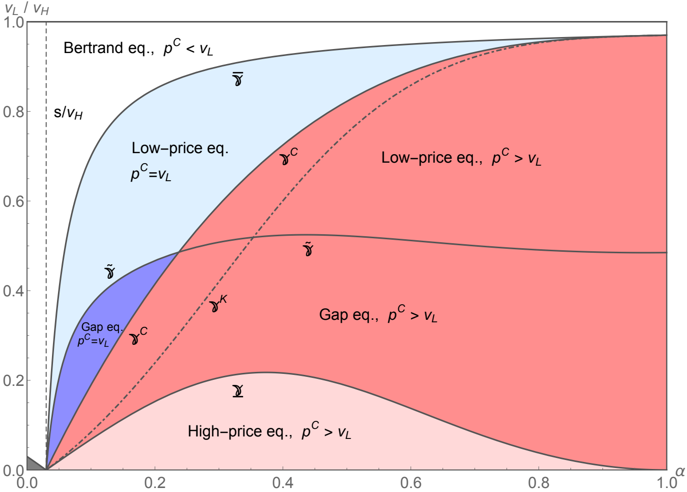

Figure: Welfare effects of optimal collusion for $\frac{s}{v_H} = 0.03$, $n = 3$.

Conclusion

- We provide **tractable model of competition with price-directed consumer search**
- Full equilibrium characterization: **Three types of mixed-strategy equilibria + Bertrand equilibrium**
- **Welfare losses** in high-price and gap equilibrium
- Characterization of **optimal symmetric collusive scheme**
- **Collusion may improve market outcome** if s is not too high, probability of full matches α intermediate, v_L intermediate

Welfare Comparative Statics

- We show: Apart from gap equilibrium, **welfare comparative statics "intuitive"**: increases in v_H, v_L, α and n; decreases in s
- **In gap equilibrium**, increases in v_H and decreases in s induce firms to price less aggressively; κ falls. **Negative strategic effect may dominate positive direct effect.** Not true for v_L
- **Individual firm profits** strictly increase in v_H and strictly decrease in v_L, s, n. Ambiguous in α (due to differentiation)
- **Industry profit** may increase in n. Reason: market expansion
- **Consumer surplus** may (locally) decrease when v_H or n increases, or s decreases. Again, adverse effect on firms' pricing may dominate positive direct effect

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