Incorporating parameter uncertainty into the setup of EWMA control charts monitoring normal mean and variance

Sven Knoth[1]
Institute of Mathematics and Statistics, Helmut Schmidt University Hamburg, Germany

Most of the literature concerned with the design of control charts relies on perfect knowledge of the distribution for at least the good (so-called in-control) process. Some papers treated the handling of EWMA charts monitoring normal mean in case of unknown parameters — refer to Jones, Champ and Rigdon (2001) for a good introduction. In Jensen, Jones-Farmer, Champ, and Woodall (2006): "Effects of Parameter Estimation on Control Chart Properties: A Literature Review" a nice overview was given. In the simple case of monitoring normal mean, one usually assumes that the in-control mean μ_0 and the variance σ (typically assumed to be fix) are known. Then the standard EWMA looks like this:

$$Z_0 = z_0 = \mu_0 = 0\,, \quad Z_i = (1-\lambda) Z_{i-1} + \lambda X_i\,, \quad i = 1,2,\dots\,,$$
$$L = \inf\left\{ i \in \mathbb{N} : |Z_i - \mu_0| > c_E \sqrt{\frac{\lambda}{2-\lambda}}\sigma \right\}\,.$$

The parameters $\lambda \in (0,1]$ and $c_E > 0$ are chosen to enable a certain useful detection performance (not too much false alarms and quick detection of changes). The most popular performance measure is the so-called Average Run Length (ARL), that is $E_\mu(L)$ for the true mean μ. If the distribution parameters, μ_0 and σ, have to be estimated by sampling data during a pre-run phase, then these uncertain parameters effect, of course, the behavior of the applied control chart. Most of the papers about characterizing the uncertainty impact deal with the changed ARL patterns and possible adjustments. Here, a different way of designing the chart is treated: Setup the chart through specifying a certain false alarm probability such as $P_{\mu_0}(L \le 1000) \le \alpha$. This results in a specific c_E. Here we describe a feasible way to determine this value c_E also in case of unknown parameters for a pre-run series of size N.

Jensen et al. (2006) ask for an evaluation and treatment of these effects for variance control charts. Again we consider EWMA charts. Given a sequence of sub-samples of size n, $\{X_{ij}\}$, $i = 1,2,\dots$ and $j = 1,2,\dots,n$ utilize the following EWMA control chart:

$$Z_0 = z_0 = \sigma_0^2 = 1\,, \quad Z_i = (1-\lambda) Z_{i-1} + \lambda S_i^2\,, \quad i = 1,2,\dots\,,$$
$$S_i^2 = \frac{1}{n-1}\sum_{i=1}^{n} (X_{ij} - \bar{X}_i)^2\,,\ \bar{X}_i = \frac{1}{n}\sum_{i=1}^{n} X_{ij}\,,$$
$$L = \inf\left\{ i \ge 1 : Z_i > c_u \sigma_0^2 \right\}\,.$$

As above, σ_0 has to be estimated by sampling data during a pre-run phase. Calculation of the ARL and subsequently the threshold c_u (providing $P_{\sigma_0}(L \le 1000) \le \alpha$) is a more subtle numerical task than in the mean case. A two-sided version of the introduced EWMA scheme is analyzed as well.

[1]e-mail: knoth@hsu-hh.de, web: www.hsu-hh.de/compstat, telephone: +49(0)40 6541 3400