Optimal taxation with current and future cohorts

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Abstract

This paper quantitatively characterizes optimal tax systems when the short-run welfare effects of tax reforms are explicitly considered. Taking the recent study of Conesa et al. (2009) as an example, we show that the optimal tax system under such an aggregate welfare approach significantly differs from tax systems that are designed to maximize long-run welfare only.

In order to isolate the aggregate efficiency consequences of a tax reform from the effects of intergenerational redistribution we develop several aggregate welfare measures. Based on these indicators, we find a much lower capital income tax rate and a significantly less progressive labor income tax schedule to be optimal. As we demonstrate, the optimality of capital income taxation is explained by the low interest elasticity of precautionary savings compared to that of life-cycle savings.

JEL Classifications: C68, H21, D91

Keywords: stochastic OLG model, precautionary savings, intragenerational risk sharing and redistribution

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1 Introduction

In his famous Hicks lecture "Supply Side Economics: An Analytical Review", Robert E. Lucas, Jr. thoroughly discusses why capital income should not be taxed at all. After supporting the results of Chamley (1986) and Judd (1985), namely that along a balanced growth path the Ramsey efficient tax scheme comprises a positive tax on labor income and a zero tax rate on capital income, he argues that "there is a good deal more to the story than can be told on the basis of balanced growth analysis". He further states that "the passage from the current balanced path to an efficient one [...] will involve a long period of reduced consumption or reduced leisure or both, partially offsetting the welfare gains enjoyed on the new balanced path", see Lucas (1990, p. 309). In order to incorporate this issue, he computes the welfare changes along a transition path towards a new long run equilibrium, resulting from a sudden elimination of the existing US capital income tax. He finds that "the welfare gains reported for balanced paths [...] overstate the actual gains by a factor of five, or perhaps more."

The recent literature on optimal taxation in dynamic general equilibrium models yet has changed the view on capital income taxation. In their seminal paper "Taxing Capital? Not a Bad Idea After All!", Conesa, Kitao and Krueger quantitatively characterize the optimal capital and labor tax schedule in a model of overlapping generations in which households face substantial idiosyncratic income uncertainty and borrowing constraints. They find that, based on long-run welfare comparisons, the optimal capital income tax rate is significantly positive at 36 percent and the optimal progressive labor income tax combines a flat tax of 23 percent and a deduction of $7,200. Given these results, two natural question along the lines of Lucas' analysis certainly arise: (i) how are generations on the adjustment path to that tax system affected and (ii) taking their welfare effects into account, what are the overall efficiency gains we can expect from such a tax reform?

In order to address these questions, we take the same model and calibration as Conesa et al. (2009). Yet, we do not only compare long-run equilibria, but compute entire transitions paths resulting from immediate (and unannounced) changes in the tax structure. In this framework, we develop several welfare measures that take into account welfare effects of any generation affected by the proposed tax reforms, including lump sum redistribution in the spirit of Auerbach and Kotlikoff (1987) and Nishiyama and Smetters (2005, 2007). Since these welfare measures are independent of any intergenerational redistribution that might be going on in the model, we can interpret them as measures of overall efficiency in the spirit of Lucas (1990).

Switching from the current US tax system to the one that guarantees highest long-run welfare gains induces strong welfare losses among current generations. The big losers of such a reform are the generations in or close to retirement. Their retirement savings are suddenly due to higher taxation. In addition, the increase in the capital income tax rate distorts their remaining life-cycle savings behavior. We find that these current losses outweigh future welfare gains so that a switch to a high capital income tax regime will be efficiency deteriorating for all our different aggregate welfare measures. Next, we characterize the optimal tax schedule taking the welfare effects of all affected cohorts into account. In this schedule, capital income is still due to taxation, but at a much lower rate of 14 percent. The marginal tax rate on labor is 17 percent and the government levies a lump-sum tax of $712 per household and year. If lump sum taxation was not available, optimal capital and labor tax rates would increase to 17 and 19 percent, respectively. The rationale behind a positive optimal capital tax in this model is the low interest rate elasticity of precautionary savings as compared to life-cycle savings. We finally demonstrate the robustness of our results with respect to several model assumptions.

Our paper therefore contributes to one of the oldest, most controversial and most policy relevant topics in public finance. The discussion combines both the debate about the tax base, e.g. whether capital income should be taxed or not, and the question of the optimal tax schedule, especially its progressivity. Recent surveys by Mankiw et al. (2009) as well as Diamond and Saez (2011) in the *Journal of Economic Perspectives* document the ongoing debate and explain the logic underlying the seemingly contradictory results. As the distribution of individual abilities, the life-cycle income process, and individual preferences are the major determinants of the optimal income tax schedule, drawing robust conclusions from quantitative analyses is not a simple task.

Nevertheless, analyzing the optimal tax structure in numerical studies has a long tradition in the literature. The majority of papers are testing the sensitivity of results with respect to certain model assumptions. Our special interest lies with models of overlapping generations with households facing both borrowing constraints and uncertainty about future labor earnings. Studies in this framework include İmrohoroğlu (1998), Conesa and Krueger (2006) or Conesa et al. (2009) which all aim to quantitatively characterize the optimal capital and/or labor income tax. A number of recent studies have extended the benchmark model of Conesa at al. (2009) in various directions in order to test the sensitivity of their results. Nakajima (2010) incorporates a housing asset and shows that in this case the optimal capital income tax rate is close to zero. Kitao (2010) demonstrates that it may be optimal to reduce the capital income tax rate when labor supply rises. Fukushima (2010) highlights the optimality of age- and history-dependent income taxes whereas Kumru and Piggott (2012) analyze the implications of means-tested pension benefits for optimal capital income taxation. While all these studies exclusively focus on long-run welfare changes as a measure of optimality, our study takes into account welfare effects of any generation that is affected by a tax reform. Furthermore we are, to the best of our knowledge, the first to quantitatively characterize optimal tax schedules in such a setup with this type of efficiency measure.

The remainder of the paper is arranged as follows: the next section describes our model and its calibration. Section 3 presents simulation results, section 4 concludes.

2 The model economy

The description of our simulation model's structure and calibration follows closely that of Conesa et al. (2009).

2.1 Demographics

The model economy is populated by J overlapping generations. At any discrete point t in time a new generation is born, the mass of which grows at rate n. Agents survive from age j to age $j+1$ with probability ψ_j, where $\psi_J = 0$. Since we abstract from annuity markets, individuals may leave accidental bequests Tr_t that are distributed in a lump-sum manner across the currently alive. Agents retire at age j_r and start to receive social security payments SS_t, which are financed by proportional payroll taxes at rate $\tau_{SS,t}$ that are paid up to an income threshold $\bar{y}$. In the following, we omit the time index t for simplicity reasons wherever possible.

2.2 Endowments and preferences

Individuals enter the economy with zero assets $a_1 = 0$ and are not allowed to run into debt throughout their whole life, i.e. $a_j \geq 0$. During their working phase, they supply part of their maximum time endowment of one unit per period as labor to the market. The remainder of time is consumed as leisure.

Households are heterogeneous along three dimensions that affect their labor productivity. First, average labor productivity ϵ_j varies with age, governing the average wage of a cohort. Second, households are born with permanent differences in productivity, standing in for differences in education and innate abilities. We consider two ability types α_1 and α_2 with equal mass. Finally, workers of same age and ability face idiosyncratic shocks $\eta \in \mathcal{E}$ with respect to their individual labor productivity. The stochastic process for labor productivity status is identical and independent across agents and follows a finite-state Markov chain with stationary transitions over time, i.e.

$$\Pr(\eta' \in \mathcal{E}|\eta) = Q(\eta, \mathcal{E}). \tag{1}$$

Since Q consists of strictly positive entries only, there exists a unique, strictly positive, invariant distribution associated with Q denoted by Π. All individuals start their life with average stochastic productivity $\bar{\eta} = \sum_\eta \eta \Pi(\eta)$, where $\bar{\eta} \in \mathcal{E}$ and $\Pi(\eta)$ is the probability of η under the stationary distribution. Different realizations of the stochastic process then give rise to cross-sectional productivity distributions that become more dispersed as a cohort ages.

At any given time households are characterized by (a, η, i, j), where a are current holdings of assets, η is the stochastic labor productivity status, i is ability type, and j is age. A household of type (a, η, i, j) working l hours commands pre-tax labor income $y = \epsilon_j \alpha_i \eta l w_t$, where w_t is the wage per efficiency unit of labor. Let $\Phi_t(a, \eta, i, j)$ denote the measure of agents of type (a, η, i, j) at date t.

Preferences over consumption c_j and and leisure $1 - l_j$ are assumed to be representable by a time-separable utility function of the form

$$W(c, 1-l) = E\left\{\sum_{j=1}^{J} \beta^{j-1} u(c_j, 1-l_j)\right\},$$

where β is the time discount factor. Expectations are taken with respect to the stochastic processes governing idiosyncratic labor productivity and mortality. Due to additive separability, we can formulate the individual optimization problem recursively:

$$v_t(a, \eta, i, j) = \max_{c,l,a'} \left\{ u(c, 1-l) + \beta \psi_j \int v_{t+1}(a', \eta', i, j+1) Q(\eta, \mathrm{d}\eta') \right\}$$

The dynamic budget constraint reads

$$(1 + \tau_{c,t})c + a' = [1 + r_t(1 - \tau_{k,t})](a + Tr_t) + y + SS_t - \tau_{SS,t} \min\{y, \bar{y}\} - T_t(y_{\text{tax}}),$$

where savings a' and consumption expenditure (including consumption taxes) are financed out of current assets and inheritances (including capital income net of capital taxes at rate τ_k), gross income from labor y or pensions SS_t net of payroll taxes and income taxes according to the tax schedule $T_t(\cdot)$ in period t. y_{tax} is taxable income, see below.

2.3 Technology

We let the production technology be represented by a Cobb-Douglas production function. The aggregate resource constraint is given by

$$C_t + K_{t+1} - (1-\delta)K_t + G_t \leq K_t^\alpha N_t^{1-\alpha}, \tag{2}$$

where K_t, C_t, G_t and N_t measure the aggregate capital stock, aggregate private and public consumption and aggregate labor input (in efficiency units) in period t, and α defines the capital share. The depreciation rate for physical capital is denoted by δ.

2.4 Government policy

In each period t, the government engages in three activities: it spends resources, levies taxes and runs a social security system. The social security system collects contributions up to a maximum labor income level $\bar{y}$ from working households and pays benefits SS_t to retirees, independent of their earnings history. The payroll tax rate $\tau_{SS,t}$ is used to balance the budget of the system. The social security system is exogenously given and not subject to the optimization of the policymaker.

Furthermore the government faces an exogenously given consumption path $\{G_t\}_{t=1}^{\infty}$. It finances expenditure by means of a proportional tax $\tau_{c,t}$ on private consumption, taxes on capital and labor income $\tau_{k,t}$ and $T_t(\cdot)$ and public debt B_{t+1}. The government's budget constraint therefore reads

$$G_t + (1+r_t)B_t = \tau_{c,t}C_t + \int [\tau_k y_r + T_t(y_{\text{tax}})]\Phi_t(\mathrm{d}a \times \mathrm{d}\eta \times \mathrm{d}i \times \mathrm{d}j) + (1+n)B_{t+1}. \tag{3}$$

The consumption tax rate is exogenously given, while the income tax schedule $T_t(\cdot)$ balances the budget. Note that we assume the income tax schedule to be invariant along the transition, i.e. it only closes the intertemporal budget. The temporary budget is balanced by debt, where we assume an initial debt level of 0.

Taxable labor income consists of labor earnings net of employer contributions to the pension system $y_l = y - 0.5\tau_{SS,t}\min\{y, \bar{y}\}$. Capital income is fully taxable $y_r = r(a + Tr_t)$. In the initial equilibrium, henceforth denoted by $t = 0$, the government taxes the sum of labor and capital income $y_{\text{tax}} = y_l + y_r$ according to the schedule $T_0(\cdot)$. There is no additional capital tax, i.e. $\tau_{k,0} = 0$. The policymaker changes the tax schedule once and for all in the reform period $t = 1$. From that moment capital income is taxed at constant rate $\tau_{k,t}$ and labor income according to the schedule $T_t(\cdot)$, i.e. $y_{\text{tax}} = y_l$. The optimal tax structure ($\tau_{k,t}$ and $T_t(\cdot)$) is the one that maximizes aggregate efficiency as defined below.

2.5 Functional forms and calibration

We use the very same calibration as Conesa et al. (2009) in their benchmark scenario. Households are born at age 20 (model age 1), retire at age 65 (model age $j_r = 46$) and may reach a maximum age of 100 years (model age $J = 81$). Population grows at an annual rate of $n = 0.011$, conditional survival probabilities are taken from Bell and Miller (2002). We assume standard Cobb-Douglas preferences

$$u(c, 1-l) = \frac{(c^\gamma(1-l)^{1-\gamma})^{1-\sigma}}{1-\sigma}, \tag{4}$$

where γ is a share parameter and σ determines the risk aversion of the household. We set $\sigma = 4, \beta = 1.001$ and $\gamma = 0.377$ in order for the capital-output ratio to be 2.7 and the share of hours worked in total time endowment to amount to 0.33.

We take Hansen's (1993) age-productivity profile $\{\epsilon_j\}_{j=1}^{j_r}$. Abilities α_1 and α_2 are specified so as to match the cross-sectional variance of household labor income at age 22 reported in Støresletten et al. (2004). We assume the idiosyncratic part of the wage process to be a seven-state discretized version of an AR(1) process with persistence parameter ρ and unconditional variance σ_η^2. Our choice of these two parameters targets the cross-sectional household age-earnings variance profile reported in Støresletten et al. (2004).

Both a capital share parameter of $\alpha = 0.36$ and a depreciation rate of $\delta = 0.08$ guarantee a realistic investment-output ratio. The payroll tax rate $\tau_{SS,t}$ is 12.4 percent and the maximum labor income level $\bar{y}$ amounts to 2.5 times the average income. The social security benefit level is endogenous in the initial equilibrium and balances the pension budget. We keep it constant in the reform periods. Government spending G accounts for 17 percent of GDP and remains constant per capita in all future periods. Consequently, the ratio G/Y would decline, if output increased in consequence of a change in tax policy. We initially abstract from public debt and choose a consumption tax rate $\tau_{c,t}$ of 5 percent. Finally, the tax function is given by

$$T(y_{\text{tax}}) = T(y_{\text{tax}}, \kappa_0, \kappa_1, \kappa_2) = \kappa_0 \, [y_{\text{tax}} - (y_{\text{tax}}^{-\kappa_1} + \kappa_2)^{-1/\kappa_1}], \tag{5}$$

where κ_i are parameters. This functional form proposed by Gouveia and Strauss (1994) is typically employed in the quantitative public finance literature. κ_0 controls the level of the top marginal tax rate and κ_1 determines the progressivity of the tax code. We yet extend out functional choice set in two dimensions. We therefore let

$$T(y_{\text{tax}}) = \begin{cases} \kappa_0 \cdot y_{\text{tax}} + \kappa_2 & \text{for } \kappa_1 = 0 \\ \kappa_0 \, \max[y_{\text{tax}} - \kappa_2, 0] & \text{for } \kappa_1 = \infty, \end{cases} \tag{6}$$

i.e. in the first case we have a proportional tax paired with a lump-sum tax of κ_2 and in the case of $\kappa_1 = \infty$ a flat tax with a deduction of κ_2. In order to approximate the existing U.S. income tax system we set $\kappa_0 = 0.258$ and $\kappa_1 = 0.768$ in the initial equilibrium and adjust κ_2 to balance the budget.

2.6 Welfare and efficiency calculation

The timing of our reform experiment is a follows: Up to period $t = 0$ households are unaware of a tax reform and therefore live in the initial equilibrium. At $t = 1$ there is a sudden unannounced change to a new tax structure. There are two types of cohorts that are affected by the tax reform. We refer to current generations as those who were born during the initial equilibrium $t = 2-J, \dots, 0$ and are surprised by the change in tax structure at some time during their life cycle. Future generations, on the other hand, are those born on the adjustment path or in the new long-run equilibrium $t = 1, 2, \dots, \infty$. The latter consequently are exposed to the new tax system for their whole life.

Given the specific form of the utility function, the welfare consequences of switching from the initial allocation $(c_0, 1 - l_0)$ to a new allocation $(c_t^*, 1 - l_t^*)$ for a specific cohort t can be computed from

$$CEV_t = \left[\frac{W(c_t^*, 1 - l_t^*)}{W(c_0, 1 - l_0)}\right]^{\frac{1}{\gamma(1-\sigma)}} - 1,$$

where $W(c, 1-l)$ is ex ante lifetime utility. CEV_t is the percentage change in consumption at all ages and all states of the world, which makes an individual in the initial allocation as well off as in the new allocation. We can compare all current generations in the reform year $t = 1$ and all future cohorts along the transition path with their respective counterparts in the initial equilibrium, since their individual state variables are identical.

In the standard infinite horizon neoclassical growth model, a typical way to measure the aggregate welfare effects of a tax reform is the following: Let τ be the complete description of a certain tax structure and $C_t(\tau)$ and $L_t(\tau)$ the corresponding paths of aggregate consumption and aggregate labor supply. In addition, let ξ be a fraction that will serve as a consumption supplement. We can then define the indirect utility function

$$V(\xi, \tau) = \sum_{t=1}^{\infty} \beta^{t-1} u\left[(1+\xi)C_t(\tau), 1 - L_t(\tau)\right]. \tag{7}$$

$V(\xi, \tau)$ is the utility the consumer enjoys under a tax system τ with a non-tradeable consumption supplement $\xi C_t(\tau)$, see Lucas (1990). Let now τ denote the present tax system, i.e. that of the initial equilibrium, and τ^* a different one. Then the welfare gain (or loss) from moving to the tax system τ^* is the unique value of ξ that satisfies $V(\xi, \tau) = V(0, \tau^*)$. In Appendix A we show that for our specification of the utility function the solution ξ to this equality can be approximated by

$$\xi \approx \underbrace{\frac{(r-n)}{1+r}}_{\text{annuity factor}} \cdot \underbrace{\frac{1}{C}}_{\text{normalization}} \cdot \underbrace{\sum_{t=1}^{\infty} \left[\frac{1+n}{1+r}\right]^{t-1} \cdot \frac{u[C_t^*, 1-L_t^*] - u[C, 1-L]}{\lambda}}_{\text{sum of discounted wealth-equivalent welfare changes}}, \tag{8}$$

where r, C, L, K and λ denote interest rate, quantities as well as the Lagrange multiplier of the situation without tax changes, i.e. initial equilibrium values. C_t^* and L_t^* are the paths of consumption and labor supply resulting from a change in tax policy from τ to τ^*. This fairly complicated looking approximation has a very simple interpretation. We first compute the present value of wealth equivalent welfare changes for any period t along the reform path τ^* in comparison to a situation with no tax changes. We then convert the derived stock into an annuity that pays out in any period $t \geq 1$ and relate this annuity value to the value of initial consumption C.

In our model, computing an aggregate welfare measure is complicated by the fact that there is no intergenerational connection. Therefore choosing an appropriate discount factor to compute overall utility as in (7) is almost impossible. Yet, we can still make use of the approximation in (8), since our model generates all the necessary data. Our first measure of overall welfare or aggregate efficiency will therefore simply be the discounted sum of wealth equivalent welfare changes of all generations that are affected by a tax reform, i.e.

$$SW = \frac{(r-n)}{1+r} \cdot \frac{1}{C} \cdot \sum_{t=2-J}^{\infty} \left[\frac{1+n}{1+r}\right]^{t-1} \cdot \frac{W[c_t^*, 1-l_t^*] - W[c, 1-l]}{\lambda}, \tag{9}$$

Note that there are three changes to the formula in (8):

(i) In order to stay as close as possible to the work of Conesa et al. (2009) we use ex ante lifetime welfare W as measure of utility of a generation.

(ii) t now denotes the birth date of a generation and (c, l) as well as (c_t^*, l_t^*) denote life time resource allocations before and after a tax reform.

(iii) We have to account for the fact that there are individuals that were born before the reform date $t = 1$, but are still affected by the tax reform in the middle of their life cycle. We refer to these generations $t = 2 - J, \ldots, 0$ as current generations, while all the generations born in $t = 1, \ldots, \infty$ are future generations.

Yet, this first measure of aggregate welfare is only a rough approximation of the efficiency effect of a reform. In order to get more accurate values, we want to tackle the problem numerically. In our second efficiency measure we follow Huang et al. (1997) and numerically compute the actual wealth transfers that would have to be payed to a generation in order to make this generation as well of as in the initial equilibrium, i.e to bring down their welfare change $\frac{W[c_t^*, 1-l_t^*] - W[c, 1-l]}{\lambda}$ to zero. We then again derive an annuity from this stock, but this time we pay out the annuity to any future generation $t \geq 1$. As final result all current generations face a welfare level equal to the one they experienced in the initial equilibrium. All future generations are at the same utility level W^*. We call this level compensated expected utility and use it to calculate the compensated relative consumption change CEV^c. A positive CEV^c indicates a Pareto improvement (after lump-sum compensation), a reform inducing a negative CEV^c is Pareto inferior. Consequently, we may interpret CEV^c as a measure of efficiency.

Compensating transfers induce behavioral reactions. Hence, factor markets and the government budget are not in equilibrium anymore when we calculate compensated welfare changes. Our third approach overcomes this partial equilibrium (p.e.) framework and takes into account all general equilibrium (g.e.) repercussions.[1] Compensating transfers are computed in exactly the same way as in the partial equilibrium framework. Yet, transfers between generations can only be processed through the asset market and therefore trigger price reactions. [2]

Note that all three measures of efficiency are based on the notion of a discounted sum of cohort utilities. Therefore we expect them to be qualitatively identical, but to only differ in effect sizes. We will use the general equilibrium measure as a benchmark in all our analyses, since it is the most common in the literature on models with overlapping generations and transitional dynamics. Yet, we also ran optimal tax calculations with the two other measures and the results did not differ significantly.

3 Simulation results

We now want to study the optimal tax structure in our model. Following Conesa et al. (2009) we allow for a flexible labor income tax code and restrict capital income taxes to be proportional, i.e.

$$T(y_l, \kappa_0, \kappa_1, \kappa_2) + \tau_k y_r.$$

Thus the government optimizes four parameters $(\kappa_0, \kappa_1, \kappa_2, \tau_k)$, where κ_2 is determined by budget balance. Given a specific parameter choice we quantify the macroeconomic, welfare and efficiency effects along the transition path and in the new long-run equilibrium. The optimal tax structure then is the one that maximizes our efficiency measure. We finally test the sensitivity of our results

[1] Note that this implies running the complete model twice: once without and once with compensating transfers.

[2] The computation of compensating transfers in general equilibrium dates back to the work of Auerbach and Kotlikoff (1987, 62f.) and has recently been applied by Nishiyama and Smetters (2005, 2007) as well as Fehr and Habermann (2008) in similar stochastic frameworks.

with respect to the openness of the economy, individual risk aversion and assumptions about initial government debt.

3.1 The optimal tax system

Long-run welfare comparisons Taking long-run welfare as measure of optimality, Conesa et al. (2009) find a capital income tax rate τ_k of 36 percent as well as a labor income tax schedule with marginal rate of 23 percent ($\kappa_0 = 0.23$) and a deduction of about \$7,200 (which corresponds to $\kappa_1 \approx 7$ and $\kappa_2 = 34711$) to be the optimal choice of the policy maker. The first column of Table 1 reports the resulting long-run changes in aggregate variables.[3] Conesa et al. (2009) restrict their parameter choice set to finite values for κ_1. The flat tax case (i.e. $\kappa_1 = \infty$) is only considered in the sensitivity analysis of Table 6 (p. 47) yet without welfare calculations. However, this special case turns out to be the optimal one in terms of long-run welfare maximization. The resulting tax schedule and long-run macro- and welfare effects are shown in the second column of Table 1. A higher tax rate on capital income and a lower marginal tax rate on labor income induce individuals to work more and save less compared to the Conesa et al. (2009) scenario. The long-run gain in equivalent consumption increases from 1.31 to 1.48 percent.

Table 1: Optimal tax schemes: Long-run welfare vs. aggregate efficiency

	Long-run welfare		Aggregate efficiency	
	Conesa et al. (2009)	optimal scheme	base case	optimal scheme
τ_k	0.36	0.43	0.43	0.14
κ_0	0.23	0.20	0.20	0.17
κ_1	7	∞	∞	0
κ_2	34711	12108	12195	712
Average hours worked	-0.66	0.69	0.72	5.84
Total labor supply N	-0.18	1.18	1.19	5.04
Capital stock K	-6.50	-8.16	-8.02	11.14
Government debt to GDP (in %)	0.00	0.00	-0.72	2.98
Output Y	-2.50	-2.29	-2.23	7.20
Aggregate consumption C	-1.45	-0.34	-0.30	7.59
Long run CEV	**1.31**	**1.48**	**1.54**	-0.66
SW			-1.54	0.82
CEV^c (p.e.)			-6.29	2.15
CEV^c (g.e.)			-1.66	**1.07**
Political support (in %)			33.63	72.38

All macro figures are reported as changes in percent of the initial equilibrium values.
Welfare figures are reported as percentage of initial aggregate (SW) or household consumption.

[3] The figures correspond to the ones in Table 2 (p. 36) of Conesa et al. (2009). Slight differences in values arise from a different computational method, see Appendix B.

Transitional dynamics Up to now, transitional dynamics were completely neglected. In the third column of Table 1 we consequently simulate the complete transition path for the tax function maximizing long-run welfare. The optimal marginal tax rates on capital and labor income resulting from this exercise turn out to be identical to those of the previous simulation. The major difference is that we adjust κ_2 only once in the reform year $t = 1$ and keep it constant afterwards. Since public debt then balances the periodic government budget, reported long-run macro and welfare effects slightly differ.[4] The cohort welfare effects of this policy reform are depicted in the left panel of Figure 1.[5] Most importantly, we find long run welfare gains to come along with dramatic welfare losses for initial cohorts who have already accumulated some assets. But where do the different welfare gains and losses come from? As discussed in Erosa and Gervais (2002), since leisure consumption increases over the life-cycle and the government cannot tax leisure directly, from an ex ante perspective optimal consumption taxes should increase with individuals' age. When income and consumption taxes cannot be conditioned on age, a nonzero tax on capital income can (imperfectly) mimic such a system. This explains welfare gains of the future generations in our model, see Conesa et al. (2009). Welfare gains decline slightly along the transition path, since the capital stock decreases and the economy departs further from golden rule capital accumulation. Finally, the welfare losses current generations incur are due to two reasons: (i) The sudden increase in capital income taxes works like a tax on the wealth these generations accumulated in the initial equilibrium. (ii) The older a generation is, the smaller is the increase in leisure consumption over the remainder of the life cycle.[6] Therefore, the need for an age dependent tax structure vanishes as individuals age. The only thing they are left over with then is a severe distortion of their remaining life cycle savings behavior. Summing up across cohorts, these initial losses dominate future welfare gains. Consequently, the present value of all welfare changes is negative and equivalent to an annual reduction in consumption of about 1.5 percent. In partial equilibrium future cohorts experience a compensated welfare loss equivalent to a drop in consumption of more than 6 percent. In general equilibrium this loss reduces to 1.66 percent, which is due to factor price adjustment damping individual reactions and therefore compensating transfers.[7] The left panel of Figure 1 also visualizes the difference between long-run welfare gains and aggregate efficiency. Shifting from labor to capital income taxation induces dramatic welfare losses for older current generations, while younger current and future cohorts benefit from reduced tax burdens. The compensation mechanism eliminates intergenerational redistribution and reveals the aggregate efficiency loss. All initial cohorts, which experience rising burdens from capital income taxes, receive lump-sum transfers which bring them back to the welfare level of the initial equilibrium. Younger and future cohorts have to finance these transfers by means of lump-sum taxes, so that they end up at a lower welfare level. Given the dramatic welfare losses for current cohorts, it is not surprising that the considered reform is quite unpopular. The last row of Table 1 reveals that only one third of the population in the reform year experiences welfare gains and therefore would vote in favor of such a reform.[8]

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4 Alternatively, we could also keep public debt unaltered and adjust κ_2 in every period to balance the budget. That would yield identical long run values in columns two and three. The difference in efficiency numbers are negligibly small.

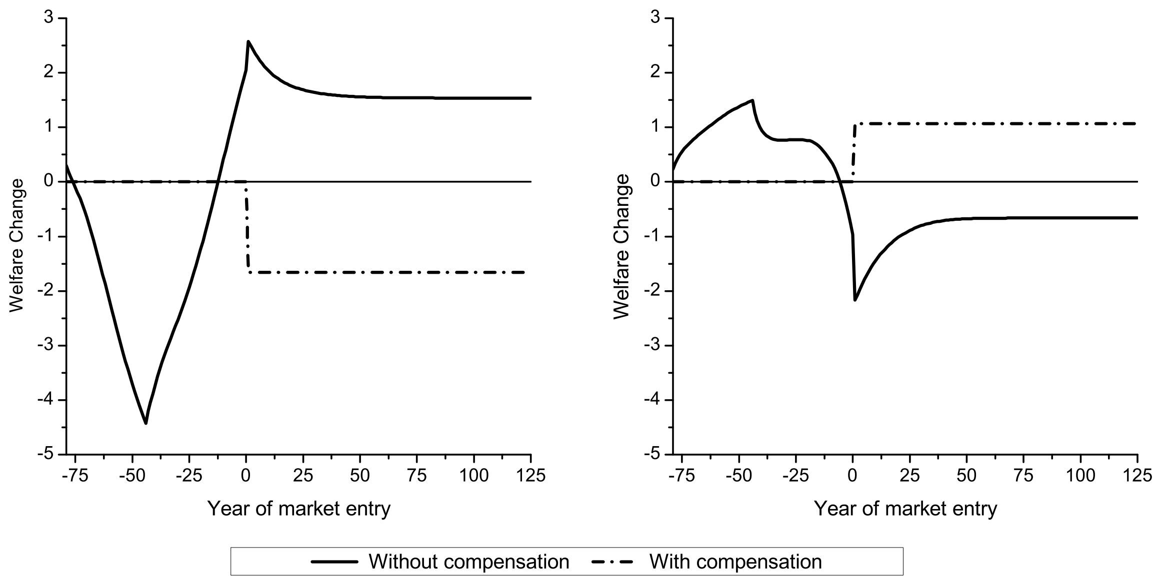
5 Note that we report average welfare changes for current cohorts.

6 Note that when an agent is already retired, his leisure consumption is equal to 1 for the rest of his life and therefore doesn't increase at all.

7 Transfers to current cohorts amount to 33.5 percent of GDP in partial equilibrium and only 17.6 in general equilibrium.

8 Of course, this political economy interpretation has to be taken with care. It is only valid if there is a one time vote with full commitment to the reform ever after.

Figure 1: Intergenerational welfare effects: Base case vs. optimal scheme



Efficiency comparisons But what would an optimal scheme look like that takes aggregate efficiency as measure of optimality? The next column of Table 1 answers this question. It features both lower tax rates on capital and labor and comes along with a lump-sum tax of $ 712. This tax schedule induces individuals to work longer hours and save more. In consequence, long-run labor supply and capital stock increase by more than 5 and 11 percent, respectively. The right panel of Figure 1 shows that current cohorts benefit from this tax structure while tax burdens on young and future generations increase. Long-run welfare therefore declines by an amount equivalent to 0.66 percent of consumption. Aggregating welfare changes across cohorts shows that initial welfare gains dominate future welfare losses. The present value of all welfare changes is now equivalent to an annual increase in consumption of 0.82 percent. Applying our compensation mechanism now yields an aggregate efficiency gain of more than 2 percent of consumption in partial equilibrium and of more than 1 percent in general equilibrium. Note that since the optimal scheme balances efficiency losses from behavioral distortions and benefits arising from loosened liquidity constraints as well as the provision of insurance against labor market risk, it does not completely rely on lump-sum taxation. Lump-sum taxes do not distort individual labor supply and savings, but enforce borrowing constraints at young ages and provides little to no insurance against income fluctuations. Not surprisingly, the optimal tax system receives political support from more than 72 percent of current households.

Our simulations reveal that the optimal capital income tax declines when the weight of transitional cohorts rises in the social welfare function. This is quite intuitive given the intergenerational welfare effects reported in Figure 1. To further support our results, we also derived the optimal tax scheme that maximizes the sum of welfare changes (SW). It turned out that it is very similar to the one reported in the last column of Table 1. If we discounted future cohorts even stronger than we do in SW, the optimal capital income tax rate would decline further.

3.2 Why to tax capital income?

But why is it optimal from an efficiency point of view to tax capital income in the present model? The literature offers two possible explanations. On the one side Cagetti (2001) and Bernheim (2002, p. 1199) point out the fact that the savings motive is important for the interest elasticity of savings. While precautionary savings are fairly inelastic, life-cycle savings for old age are very elastic. Consequently, if the fraction of precautionary savings in total savings is high enough, it will be optimal to tax capital income. On the other side, as already discussed above, Erosa and Gervais (2002) demonstrate that even without precautionary savings it might be optimal to tax capital income in a life cycle model. Since leisure consumption increases over the life-cycle and the government cannot tax leisure directly, optimal consumption taxes should increase with individuals' age. When income and consumption taxes cannot be conditioned on age, a nonzero tax on capital income can (imperfectly) mimic such a system.

Certain income In order to clarify the importance of these two explanations for our model economy, we consider a version without income uncertainty. This is done by setting the variances of both persistent and transitory shocks to zero, i.e. $\alpha_1 = \alpha_2 = 1$ and $\sigma_\eta^2 = 0$. Consequently, there is only one representative individual in each cohort that features the average productivity profile ϵ_j over the life-cycle. We finally adjust κ_2 in order to keep constant income tax revenues as a fraction of GDP. In absence of a precautionary savings motive, the initial capital-output ratio declines and the interest rate increases to 6.05 percent. We now consider two policy scenarios. In the left column of Table 2, we allow the government to raise lump-sum taxes. Not surprisingly, the optimal tax system then is a pure lump-sum tax of $ 7270 per household. Such a tax eliminates all labor supply and savings distortions. The resulting benefits dominate the costs arising from higher liquidity constraints. People work and save much more than under the initial system, so that long-run labor supply, capital stock, output and consumption increase strongly. As a consequence, overall efficiency rises by 12.6 percent of aggregate resources.

In a second step we now would like to examine how the taxation of labor and capital income relate. We therefore compute for an exogenous set of marginal tax rates on labor income κ_0 the optimal tax rate on capital income τ_k, given that $\kappa_1 = 0$ and that a lump-sum tax κ_2 balances the government's budget. The solid line in the left panel of Figure 2 shows the resulting tax rates for values of κ_0 between 0.00 and 0.30. Not surprisingly, when lump-sum tax instruments are available, it is always optimal to not tax capital income. As the interest elasticity of old-age savings is very high, it is more favorable to avoid savings distortions rather than taxing leisure consumption in late life. This decision is independent of the marginal tax rate on labor income. But what if the government was not allowed to raise lump-sum taxes? The answer is given in the second column of Table 2. In this case a combination of positive labor and capital income taxes is the optimal choice of the government. Although savings are very elastic, capital income should be taxed on efficiency grounds, since such a tax indirectly burdens leisure consumption in late life. This is the argument of Erosa and Gervais (2002), which yet is only valid in a certain income world when lump-sum taxation is forbidden.

Uncertain income Now we turn back to the case of uncertain income and again derive optimal capital income tax rates for different values of κ_0 under the assumption that lump-sum taxes κ_2 balance the budget. The result is depicted as dashed line in the left panel of Figure 2. The picture looks quite different in this case meaning that we find the optimal capital income tax rate to now be strictly

Table 2: Optimal tax schemes: Certain vs. uncertain income

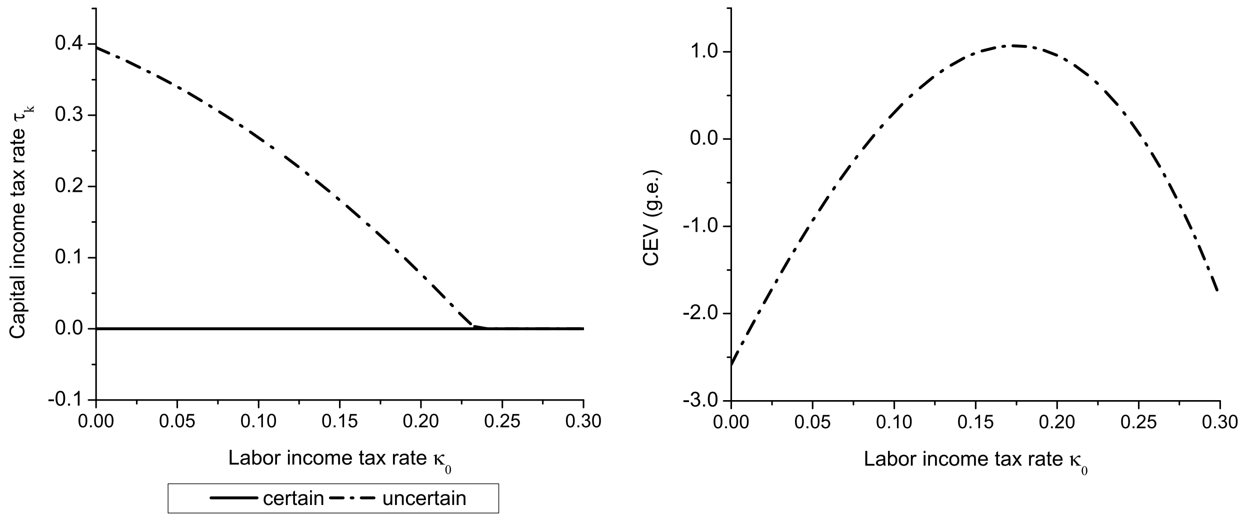
	Certain income		Uncertain income	
Lump-sum taxes	yes	no	yes	no
K_0/Y_0	2.50	2.50	2.72	2.72
r_0	6.05	6.05	4.89	4.89
τ_k	0.00	0.16	0.14	0.17
κ_0	0.00	0.19	0.17	0.19
κ_1	0	0	0	0
κ_2	7270	0	712	0
Average hours worked	17.74	4.04	5.84	3.90
Total labor supply N	17.77	4.07	5.04	3.72
Capital stock K	39.35	7.72	11.14	6.69
Government debt to GDP (in %)	5.19	2.68	2.98	2.14
Output Y	25.13	5.37	7.20	4.78
Aggregate consumption C	26.79	6.00	7.59	5.36
Long run CEV	9.90	2.08	-0.66	-0.57
SW	6.95	2.68	0.82	0.73
CEV^c (p.e.)	13.88	6.50	2.15	2.10
CEV^c (g.e.)	**12.61**	**4.09**	**1.07**	**1.00**
Political support (in %)	72.22	100.00	72.38	74.94

All macro figures are reported as changes in percent of the initial equilibrium values.
Welfare figures are reported as percentage of initial aggregate (SW) or household consumption.

positive for many values of κ_0 although lump-sum taxation is available. The reason for this observation is, as discussed by Cagetti (2001) and Bernheim (2002, p. 1199), that the interest elasticity of precautionary savings is much lower than that of life-cycle savings. Therefore it is optimal to tax interest income in order to burden leisure consumption of the elderly. Furthermore in a model with uncertain income, the taxation of capital income also indirectly fulfills a redistributive role. Since it is those who are lucky in the labor market who tend to save the most, assets (at least imperfectly) mirror the earnings history of an individual. Therefore, a capital income tax redistributes from high to low income individuals. As a consequence, when the marginal tax rate on labor income increases and therefore the labor income tax comprises more redistributive elements, the optimal capital income tax declines. In order to derive the optimal overall tax schedule, the right panel of Figure 2 shows the efficiency numbers that result from the respective tax schedules in the left panel. The efficiency curve obviously peaks for $\kappa_0 = 0.17$ and $\tau_r = 0.14$, i.e. our benchmark case which is reported once more in the third column of Table 2.

The last column of Table 2 complements the discussion of this subsection by assuming that lump-sum taxes are again not available. The optimal (proportional) capital and labor income tax rates then increase to 17 and 19 percent, respectively. This dampens labor supply and capital accumulation during the transition and results in a significantly lower long-run output. Not surprisingly, the elimination of lump-sum taxation slightly reduces the aggregate efficiency gain compared to the optimal tax schedule in the third column. Because of the missing lump-sum tax instrument, the optimal capital income tax rate lies far above the value of roughly 0.10 reported for $\kappa_0 = 0.19$ in the left panel of Figure 2. The reason is again the same as in the case with certainty and no lump-sum taxation.

Figure 2: Optimal tax rate combinations



Summing up the quantitative results of this section, we conclude that the consideration of transitional cohorts in the social welfare function still generates a positive optimal capital income tax, yet at a much lower rate than when only long-run welfare is maximized. The central reason for the positive capital income tax rate is the low elasticity of precautionary savings compared to life-cycle savings.

3.3 Sensitivity analysis

In this section we test the sensitivity of our results with respect to assumptions about the openness of the economy, individual risk aversion and initial government debt. Conesa et al. (2009) already provide an extensive discussion about variations in labor supply elasticities. They state that lower elasticities imply higher taxes on labor and lower taxes on capital income. In the limiting case with fixed labor supply, it is optimal to tax labor income at 100% and distribute the resulting revenue in a lump-sum fashion across individuals. Capital income then stays untaxed. This completely eliminates borrowing constraints and labor income risk without triggering distortions on individual behavior. When we reduced the intertemporal elasticity of substitution, on the other hand, the optimal tax rate on capital income would increase.

The openness of the economy The left column of Table 3 again reports the results for the optimal scheme developed in the previous sections. The next column shows results from simulating a small open economy with the very same initial tax schedule. In a small open economy factor prices remain constant and the national capital market is balanced by cross-border capital flows. Without factor price repercussions behavioral reactions are much stronger. The elasticities of the tax base therefore increase. Consequently, when doing the same reform as in the closed economy setting, i.e. reduce marginal tax rates on both labor and capital income, the resulting efficiency gains are significantly higher.[9] In addition, aggregate efficiency could be even increased by reducing marginal tax rates on

[9] Note that in this case we can compare aggregate efficiency effects since they refer to the same initial equilibrium. However, when capital intensities and therefore interest rates differ initially, it is not possible to compare aggregate efficiency effects.

Table 3: Sensitivity analysis I

	Optimal scheme	Smopec Reference	Optimal	Risk aversion $\nu = 0$	$\nu = 8$
K_0/Y_0	2.72	2.72	2.72	2.65	2.80
r_0	4.89	4.89	4.89	5.28	4.55
τ_k	0.14	0.14	0.06	0.00	0.24
κ_0	0.17	0.17	0.14	0.08	0.22
κ_1	0	0	0	0	5
κ_2	712	700	2145	4429	36415
Average hours worked	5.84	4.09	6.30	15.40	0.24
Total labor supply N	5.04	3.38	4.53	11.82	1.03
Private assets A	11.14	20.88	38.97	31.93	1.17
Government debt to GDP (in %)	2.98	1.04	-0.66	8.00	0.78
Output Y	7.20	3.38	4.53	17.74	1.00
Aggregate consumption C	7.59	7.49	12.16	18.14	1.34
Long run CEV	-0.66	-0.52	-0.88	3.08	0.77
SW	0.82	0.97	1.02	2.64	0.00
CEV^c (p.e.)	2.15	3.00	3.14	5.56	0.16
CEV^c (g.e.)	**1.07**	**1.97**	**2.07**	**4.77**	**0.42**
Political support (in %)	72.38	70.30	56.41	48.03	53.95

All macro figures are reported as changes in percent of the initial equilibrium values.
Welfare figures are reported as percentage of initial aggregate (SW) or household consumption.

capital and labor income further while at the same time increasing lump-sum taxes, see the third column in Table 3.

Individual risk aversion The last two columns on the right side of Table 3 show the optimal income tax regime when we alter the degree of risk aversion. We isolate the elasticity of intertemporal substitution from relative risk aversion by rewriting the preference structure following Epstein and Zin (1991) as

$$v_t(a,\eta,i,j) = \max_{c,a',l} \; u(c,1-l) + \frac{\beta\psi_j}{1-\sigma}\left\{\int \left[(1-\sigma)v_{t+1}(a',\eta',i,j+1)\right]^{\frac{1-\nu}{1-\sigma}} Q(\eta,\mathrm{d}\eta')\right\}^{\frac{1-\sigma}{1-\nu}}$$

where ν denotes the risk aversion parameter. Setting $\nu = 4.0$ we are back at the benchmark calibration.[10] Like in the previous section we adjust κ_2 in order to achieve the same income tax revenues in relation to GDP.[11] When individuals become less risk averse the fraction of precautionary savings in total savings decreases. This causes the interest rate elasticity of total savings to rise, which in turn leads to lower optimal marginal tax rates on capital income. As shown in the fourth column of Table

[10] With Epstein and Zin (1991) preferences altering the risk aversion parameter ν differs from altering the intertemporal elasticity of substitution σ. While the former values different realizations of income *within* a period, the latter values the realization of income *across* periods.

[11] For our sensitivity analysis, we also tried out model versions in which we additionally adjusted the time discount factor β in order to yield the same initial capital to output ratio and interest rates. The differences to the presented results were negligible.

3, with risk-neutral preferences capital income should not be taxed at all. In addition, less risk averse individuals attach less value to insurance provision, which causes the marginal tax rate on labor income to decline. Yet, the optimal κ_0 is still positive, which is due to liquidity constraints binding to strongly under a full lump-sum tax regime. On the other hand, when individuals become more risk averse, the savings elasticity decreases. In consequence, optimal marginal tax rates on capital income are much higher than in the benchmark scenario. The last column of Table 3 reveals a risk aversion parameter of 8 to imply marginal tax rates of 24 percent on capital and 22 on labor income, respectively. Note that this is the only preference combination for which we obtain a tax schedule of the original Gouveia and Strauss (1994) form.[12]

Government debt Up to now government debt played hardly any role. It was only adjusted to balance the annual budget while the intertemporal budget was always in balance. In Table 4 we therefore compare optimal tax systems when the initial equilibrium features different levels of public debt (or assets). We again set κ_2 so as to guarantee identical income tax revenues. Public expenditure also needs to be adjusted, as part of the tax revenue needs to be spend on interest payments on existing debt. A positive amount of public debt crowds out part of the capital stock. This causes interest rates to rise which again has two effects on household behavior. First, the fraction of old-age savings in total savings increases, so that the interest elasticity of savings is higher with a higher level of public debt. Second, wanting to save more, households will less frequently run into the problem of liquidity constraints.[13] As a results, both optimal marginal tax rates on capital and labor income decrease the higher the government is indebted. This result is in stark contrast to the findings of Conesa et al. (2009, p.46), who argue that rising interest rates increase optimal capital income taxes. Of course, the different results are due to the different welfare criteria which are applied to asses economic optimality. Higher public debt increases the burden of future cohorts. Consequently, it is optimal to neutralize (at least partly) this intergenerational redistribution via higher capital income taxes. Taking into account transitional cohorts and maximizing economic efficiency leads to exactly the opposite conclusion.

4 Discussion and conclusion

In this paper we characterized optimal tax schedules when the short-run welfare effects of tax reforms are explicitly taken into account. We therefore developed several aggregate welfare measures that aim at separating the aggregate efficiency consequences of a tax reform from the effects of intergenerational redistribution. We demonstrated that the tax system that maximizes long-run welfare is likely to deteriorate aggregate efficiency, since a sudden increase in capital income taxes induces severe savings distortions on current generations and therefore comes at huge short-run welfare costs. When the welfare effects of transitional cohorts are taken into account, it is still optimal to tax capital income, but the optimal capital income tax rate reduces from 43 to 14 percent in the closed economy and to 6 percent in the small open economy. The driving factor behind the taxation of capital income

[12] A value of $\nu = 8$ seems quite unreasonable for macroeconomists but it is not uncommon in the finance literature, see Cecchetti et al. (2000).

[13] Note that in a model version in which we adapt the time discount factor to yield the same initial capital-output ratio the reasoning about the effects of government debt would be exactly the same. Yet, higher life-cycle savings would result from a higher discount factor.

Table 4: Sensitivity analysis II

	Initial government debt (B_0/Y_0)			
	1.00	0.50	0.00	-0.50
K_0/Y_0	2.55	2.63	2.72	2.84
r_0	5.79	5.37	4.89	4.33
τ_k	0.08	0.11	0.14	0.17
κ_0	0.14	0.16	0.17	0.19
κ_1	0	0	0	0
κ_2	1548	945	712	270
Average hours worked	8.21	6.51	5.84	4.31
Total labor supply N	6.92	5.60	5.04	3.77
Capital stock K	16.28	13.07	11.14	7.87
Government debt to GDP (in %)	103.94	53.92	2.98	-47.05
Output Y	10.20	8.23	7.20	5.23
Aggregate consumption C	9.95	8.32	7.59	5.71
Long run CEV	-1.29	-0.97	-0.66	-0.56
SW	1.31	1.09	0.82	0.51
CEV^c (p.e.)	3.78	3.02	2.15	1.30
CEV^c (g.e.)	**1.95**	**1.15**	**1.07**	**0.56**
Political support (in %)	66.70	72.19	72.38	73.35

All macro figures are reported as changes in percent of the initial equilibrium values.
Welfare figures are reported as percentage of initial aggregate (SW) or household consumption.

is the low interest elasticity of precautionary savings compared to life-cycle savings. When the fraction of the former in total savings decreases, e.g. due to lower risk aversion or higher interest rates, the overall savings elasticity rises and it might be optimal to desist completely from capital income taxation. If households feature higher risk aversion or initial interest rates decline, the optimal capital tax rate increases and the optimal labor tax schedule turns out to be more progressive.

Our results have to be interpreted with care, since they also depend on restrictions of the considered tax system and various other assumptions about the economy and individual behavior. We have assumed that the optimal tax scheme is set once at time zero and then maintained into perpetuity. Of course, a full analysis would search for the best time-indexed sequences of labor and capital income taxes across the transition that maximizes aggregate efficiency. However, this type of analysis is challenging in the overlapping generation economy because it quickly runs up against the curse of dimensionality. Furthermore, increasing uncertainty of the economic environment could result in a more progressive tax system. Uncertainty would for example rise, if unintended bequests were distributed in proportion to ability or realized income, if the pension system was less progressive or the income process more volatile. The progressivity of the optimal tax system will decline with rising labor supply elasticities. This includes considering household production or human capital formation.

A final remark refers to linkages between and within generations due to one- or two-sided altruism. While our approach does not directly incorporate any bequest motive, our compensation mechanism can be interpreted as a system of private transfers neutralizing all intra- and intergenerational redistribution effects. Effectively we are then in a Barro (1974) world, where successive generations are linked by an operative altruistic bequest motive. With this interpretation our approach includes both extremes, the standard overlapping generations model without intergenerational linkage and the infinitely lived agent model, in which cohorts are perfectly linked via bequest motives.

A Efficiency measures

We show how to approximate the standard efficiency measure in the neoclassical growth model. Assume that the population grows year by year at rate n. Households choose a path of consumption $\{C_t\}_{t=0}^{\infty}$ and labor supply $\{L_t\}_{t=0}^{\infty}$ in order to maximize the discounted sum of utilities

$$V = \sum_{t=1}^{\infty} \beta^{t-1} u[C_t, 1 - L_t]$$

subject to the dynamic budget constraint

$$(1 + r_t)K_t + w_t L_t = C_t + (1 + n)K_{t+1}.$$

The Lagrangean of this optimization problem can be written as

$$\mathcal{L} = \sum_{t=1}^{\infty} \beta^{t-1} u[C_t, 1 - L_t] + \sum_{t=1}^{\infty} \lambda_t \cdot \frac{(1+n)^{t-1}}{\prod_{s=2}^{t}(1+r_s)} \cdot \left[(1 + r_t)K_t + w_t L_t - C_t - (1 + n)K_{t+1}\right],$$

where λ_t is the Lagrange multiplier for the budget constraint in period t prices. Let $R_t = \frac{(1+n)^{t-1}}{\prod_{s=2}^{t}(1+r_s)}$ be the relevant discount factor for period t. First order conditions of the optimization problem with respect to consumption and capital then read

$$\frac{\partial \mathcal{L}}{\partial C_t} = \beta^{t-1} \cdot u_{C_t}(C_t, 1 - L_t) - \lambda_t R_t = 0 \tag{10}$$

$$\frac{\partial \mathcal{L}}{\partial K_{t+1}} = -\lambda_t R_t (1 + n) + \lambda_{t+1} R_{t+1} (1 + r_{t+1}) = 0 \tag{11}$$

Note that equation (11) immediately gives us $\lambda_t = \lambda_{t+1} = \lambda$ for all $t \geq 1$. Now lets imagine we are in a situation where the tax system stays at its initial equilibrium values described by τ, i.e. there is no tax reform. Consequently, the economic system will remain in steady state, i.e. per capita variables will be constant over time. Using $C_t = C$, $L_t = L$, $r_t = r$ and equation (10) we obtain $\beta^{t-1} = \frac{(1+n)^{t-1}}{(1+r)^{t-1}}$. The utility function becomes

$$V = u[C, 1 - L] \cdot \sum_{t=1}^{\infty} \left[\frac{1+n}{1+r}\right]^{t-1} = \frac{1+r}{r-n} \cdot u[C, 1 - L].$$

We can then write[14]

$$V(\xi, \tau) = \frac{1+r}{r-n} \cdot u[(1 + \xi)C, 1 - L] = \frac{1+r}{r-n} \cdot \left[u[C, 1 - L] + \int_{C}^{(1+\xi)C} u_{C_t}(C_t, 1 - L_t)\, dC_t\right].$$

For small changes in ξ we can approximate $u_{C_t}(C_t, 1 - L_t) = \lambda$, see equation (10) for $t = 1$. This yields

$$V(\xi, \tau) \approx \frac{1+r}{r-n} \cdot \left[u[C, 1 - L] + \lambda \xi C\right] = V(0, \tau) + \frac{1+r}{r-n} \cdot \lambda \xi C.$$

The approximate solution to the equation $V(\xi, \tau) = V(0, \tau^*)$ then can be calculated as

$$\xi = \frac{V(0, \tau^*) - V(0, \tau)}{\lambda C} \cdot \frac{r-n}{1+r} = \frac{r-n}{1+r} \cdot \frac{1}{C} \cdot \sum_{t=1}^{\infty} \beta^{t-1} \frac{u[C_t^*, 1 - L_t^*] - u[C, 1 - L]}{\lambda}.$$

[14] Obviously we assume the existence of derivatives and definite integrals, which is certainly the case for standard choices of utility functions.

Again substituting $\beta^{t-1} = \frac{(1+n)^{t-1}}{(1+r)^{t-1}}$ we can finally write

$$\xi = \underbrace{\frac{r-n}{1+r}}_{\text{annuity factor}} \cdot \underbrace{\frac{1}{C}}_{\text{normalization}} \cdot \underbrace{\sum_{t=1}^{\infty} \left[\frac{1+n}{1+r}\right]^{t-1} \cdot \frac{u[C_t^*, 1-L_t^*] - u[C, 1-L]}{\lambda}}_{\text{sum of discounted wealth-equivalent welfare changes}} .$$

We can therefore compute the consumption equivalent welfare change of a tax reform as follows:

(i) Compute the present value of wealth equivalent instantaneous welfare changes between the situation with a tax reform and a situation without.

(ii) Convert the resulting stock into an annuity that pays out in any period $t \geq 1$.

(iii) Relate the resulting annuity value to the value of consumption in the case of the present tax system, i.e. in the initial equilibrium.

B Computational appendix

We use two distinct solution algorithms: one to solve the household problem and one to solve for macroeconomic quantities and prices.

B.1 Solving the household problem

We first have to discretize continuous elements of the state space (a, η, i, j), respectively the asset dimension. We therefore choose $\hat{\mathcal{A}} = \{\hat{a}^1, \ldots, \hat{a}^{n_A}\}$. We then solve the household problem by backward induction, iterating on the following steps:

1. Compute household decisions at maximum age J for any $(\hat{a}, \eta, i, J)$. Since households are not allowed to work anymore and they die for sure in the next period, they consume all remaining resources.

2. Find the solution to the household optimization problem for all possible $(\hat{a}, \eta, i, j)$ recursively using a line search method à la Powell, see Press et al. (2001, 406ff.). This algorithm requires a continuous function to optimize. We therefore use an interpolated version of $v_{t+1}(a, \eta, i, j+1)$. Having computed the data $v_{t+1}(\hat{a}, \eta, i, j+1)$ at any discrete asset grid point in the last iteration step, we can find a piecewise polynomial function $sp_{t+1,j+1}$ satisfying the interpolation conditions
$$sp_{t+1,j+1}(\hat{a}^k, \eta, i, j+1) = \int v_{t+1}(\hat{a}^k, \eta', i, j+1) Q(\eta, \mathrm{d}\eta') \tag{12}$$
for all $k = 1, \ldots, n_A$. We use the multidimensional spline interpolation algorithm described in Habermann and Kindermann (2007).

We choose $n_A = 25$. We also tried higher values, but the results didn't change.

B.2 The macroeconomic computational algorithm

We solve for quantities and prices using a Gauss-Seidel procedure in line with Auerbach and Kotlikoff (1987). Starting with a guess for quantities and government policy, we compute prices, optimal household decisions, and value functions. Next we obtain the distribution of households on the state space and new macroeconomic quantities. We then update the initial guesses. These steps are iterated until the initial guesses and the resulting values for quantities, prices and public policy have sufficiently converged.

B.3 Computational efficiency

Our algorithm turns out to be quite efficient. It differs from the one used in Conesa et al. (2009) in that we first apply a more efficient interpolation routine and second allow any choice dimension (consumption, leisure, and assets) to indeed be continuous. Minor differences in simulation results are the consequence. Solving for a long-run equilibrium in the original model of Conesa et al. (2009) takes about 10 to 15 minutes, depending on the calibration.[15] Our simulation approach obtains the

[15] We simulate our models on a regular PC with a Intel® Core™ i7-870 Processor with 2.93 GHz and 8M Cache.

same results within 4 to 6 seconds! Computing a complete transition path with 320 transition periods takes about 40 minutes time.

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