Courtship as a Waiting Game

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In most times and places, women on average marry older men. We propose a partial explanation for this difference and for why it is diminishing. In a society in which the economic roles of males are more varied than the roles of females, the relative desirability of females as marriage partners may become evident at an earlier age than is the case for males. We study an equilibrium model in which the males who regard their prospects as unusually good choose to wait until their economic success is revealed before choosing a bride. In equilibrium, the most desirable young females choose successful older males. Young males who believe that time will not treat them kindly will offer to marry at a young age. Although they are aware that young males available for marriage are no bargain, the less desirable young females will be offered no better option than the lottery presented by marrying a young male. We show the existence of equilibrium for models of this type and explore the properties of equilibrium.

In most times and places, men, on average, are older than their wives. A recent United Nations (1990) study reports the average age of marriage for each sex for more than 90 countries over the time in-

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terval between 1950 and 1985.[1] In every country and in every time period reported, the mean age at marriage of males exceeded that of females. The smallest difference in mean ages was 1 year (Ireland) and the largest difference was 10.9 years (Mali). In 1985 in the United States, the difference was 1.9 years, in western Europe about 2.5 years, and in southern and eastern Europe about 3.5 years. In Japan the difference was 3.7 years, in India nearly 5 years, and in the Middle East about 4 years. In the Caribbean the age gap is about 5 years, in Central America about 4 years, and in South America between 2 and 3 years. In African countries, this gap ranges between 5 and 10 years. In most countries, the age difference between the sexes at marriage has diminished substantially between 1950 and 1985, but nowhere has it disappeared altogether.[2]

This paper proposes a partial explanation for the difference in age at marriage of males and females, for why this difference is diminishing over time, and for why it tends to be greater in traditional societies than in modern societies. We suggest that this difference is a result of the different economic roles of males and females and a corresponding difference between the sexes in the rate at which evidence accumulates about one's "quality" as a possible marriage partner.

In societies in which male roles as economic providers are relatively varied and specialized, information about an individual male's economic capabilities may be revealed only gradually after he has spent time in the work force. In contrast, for a female whose anticipated tasks will be childbearing, child care, and traditional household labor, it may be that once she has reached physical maturity, the passage of time adds little information about her capabilities for these tasks.

We propose a model in which males who expect to prosper will delay marriage until the evidence of their success allows them to attract more desirable females. The most desirable females, on the other hand, have little to gain by postponing marriage since the relevant information about their quality is available at an earlier age. In the long-run stationary equilibrium of this model, males with poor prospects marry at an early age, whereas those who expect success will marry later in life. All females marry relatively early in life. The more desirable females marry successful older males and the less desirable females marry the young males who do not expect to prosper.

[1] The average computed is the "singulate mean age at marriage." This statistic estimates the average number of years spent in the single state by those who marry before age 50 and is computed from census statistics on the proportion of the population that have never married in each age group. See Hajnal (1953) for details.

[2] In 72 of the 91 countries listed in App. B, the age gap decreased and in 14 countries the gap increased. Exceptions to this pattern are Japan, Germany, and several countries in southern and eastern Europe.

This model predicts that males who marry young will tend to have lower earnings in later life. There is evidence, at least for the United States, that this is the case. According to the U.S. Census Bureau (1980), 35 percent of males aged 45–54 who married before age 20 had annual incomes below $10,000. For those who first married between ages 21 and 29, only 17.5 percent had incomes below $10,000. The median income of persons who married before 18 was $14,500, the median income of those who married between 18 and 20 was $16,800, and the median income of those who married between 22 and 29 was $19,000.[3]

The formal model presented in this paper is starkly oversimplified. We confine the analysis to two possible ages of marriage for each sex. We assume much more dramatic differences between the sexes than is justified by reality. Furthermore, the model lacks a realistic treatment of search costs. While our model lacks sophistication in many directions, it is unusual in its explicit treatment of the dynamics of an assignment equilibrium taking place in real time. Though we make no claim that this model is detailed enough to "explain" the observed distributions of age at marriage and income after marriage, we believe that we have identified an important influence on marriage patterns and have taken a useful first step in untangling the logic of a dynamic marital "lemons" model. We hope that this model will be useful as a building block for more realistic and detailed theories.

I. Preferences, Information, and the Distribution of Quality

We consider a population of constant size, in which people are born, marry, and die. In every year, equal numbers of males and females reach maturity. People can choose to marry in either the first or the second year of maturity. Those who marry in the first year are said to marry at age 1 and those who marry in the second year of maturity are said to marry at age 2. Marriages are monogamous, and there is no divorce or remarriage.

Some people are more desirable marriage partners than others, but it is assumed that members of each sex agree in their rankings of the opposite sex. It is further assumed that all members of each sex have identical von Neumann–Morgenstern utility functions over lotteries in which their marriage partners are randomly selected from the opposite sex. Let x_i be the von Neumann–Morgenstern utility

[3] Bergstrom and Schoeni (1992) examined U.S. census data in detail and showed that average male incomes are an increasing function of age at first marriage until approximately age 30. But it is interesting to note that males who marry for the first time after age 30 earn less than those who marry in their midtwenties.

that males assign to the prospect of marrying female i and let y_i be the von Neumann–Morgenstern utility that females assign to the prospect of marrying male i. We shall call x_i or y_i the "quality" of individual i. The quality of females is distributed over an interval $[L_g, U_g]$ with a cumulative distribution function $F_g(x)$, and the quality of males is distributed over an interval $[L_b, U_b]$ with a cumulative distribution function $F_b(y)$. Other things being equal, everyone would prefer marrying at age 1 to marrying at age 2. The utility cost of delaying marriage from age 1 to age 2 is c_b for males and c_g for females. Marrying even the least desirable member of the opposite sex is preferred to the prospect of remaining single.[4]

The quality of each female is known to all persons when she reaches age 1. The quality of a male does not become public information until he reaches age 2. At age 1, each male knows what his own quality will be at age 2.[5] To the females, his prospects are indistinguishable from those of his contemporaries, except insofar as his choice of when to marry acts as a signal.

II. Marriage Market Equilibrium

We model the marriage market as a game of incomplete information. Players have only two available strategies: to marry at age 1 or to marry at age 2. The quality of males of age 2 and of females of all ages is common knowledge. The quality of a male of age 1 is known only to himself. Members of each generation make simultaneous choices about when to marry without observing the choices made by their contemporaries. Thus each individual believes that his or her choice of strategy will not alter the choices made by contemporaries. In equilibrium, although knowledge of the quality of specific age 1 males is private information, the *distribution* of quality among young males who choose to marry and among young males who choose to wait will be common knowledge.

The payoffs to each strategy are determined by a matching rule applied to the set of people whose choose to marry in any time period. Females who marry in any period are matched to males of corresponding *expected* quality who choose to marry in the same period. If the quality of all persons in the marriage market were public information, then this matching would be entirely straightforward. The most desirable male would be matched to the most desirable female, the

[4] This assumption involves no loss of generality since the population referred to consists only of those persons for whom being married is better than being single.

[5] This model extends without formal alteration to the case in which young males are not certain about how well they will turn out but have some private information about their prospects. Then y_i is interpreted as i's *expectation* of his quality in period 2.

second most desirable male to the second most desirable female, and so on until the supply of persons of at least one sex is exhausted. If the number of available persons of one sex exceeds that of the other, then some people from the lower tail of the quality distribution will be left unmatched. Unmatched persons of age 1 may reappear in the marriage market in the next period.

The actual matching rule is complicated by the fact that males of age 1 are indistinguishable to females and hence have equal expected quality. Applying the principle of matching by corresponding rank leads to the following assignment. At time period t, the best unmarried male of age 2 will be matched to the best female who chooses to marry at time t, the second-best unmarried male will marry the second-best unmarried female, and so on until the supply of males whose quality exceeds the average of available age 1 males is exhausted.

The assignment of partners for the remainder of the population follows directly from the principal of matching by corresponding rank and from the fact that females cannot distinguish between males who choose to marry at age 1.

Let $N_m(t)$ be the number of age 1 males who choose to marry at time t. Let $N_f(t)$ be the number of females who choose to marry at time t and who are not matched to a male who is better than a random draw from the available age 1 males. There are three possible cases.

1. If $N_m(t) = N_f(t)$, then each of the males who choose to marry at age 1 will be randomly assigned a partner from the set of $N_f(t)$ females who want to marry in this period and are not already taken by an age 2 male.

2. If $N_m(t) < N_f(t)$, then the best $N_m(t)$ of the $N_f(t)$ available females will be randomly matched to the males of age 1. The remaining $N_f(t) - N_m(t)$ females will be matched in order of corresponding quality with any remaining males of age 2 who have lower quality than the average available male of age 1. Females left over at the end of this process will be left unmatched. Those who are of age 1 may reenter the marriage market in the next period at age 2.

3. If $N_m(t) > N_f(t)$, then a random draw of $N_f(t)$ males from the set of available males of age 1 will be paired with the $N_f(t)$ females who are available and have not been matched with a male of higher quality. Males who chose to marry at age 1 but did not receive partners in the random assignment will be able to reenter the marriage market in the next period at age 2.

With these matching rules, the assignment of partners within the set of people who choose to marry in any given year has the *core* or *stable marriage assignment* property (Gale and Shapley 1962; Shapley and Shubik 1972). That is to say, no two people of opposite sexes *who* *marry in the same year* would both get higher expected utility from marrying each other than they do from their actual choices.[6]

Equilibrium must determine when each person chooses to marry as well as how the people who choose to marry at a given time are matched up. In equilibrium, each person's choice of whether to marry at age 1 or age 2 must maximize his or her expected utility, given the choices of all other individuals.[7] Optimal strategies for individuals depend nontrivially on the actions of others because these choices determine the quality distribution in the marriage pool in each year and thus determine the payoffs from marrying at age 1 or age 2. In equilibrium, no person who marries at age 1 would have a higher expected payoff from waiting to marry at age 2, and no person who marries at age 2 would have a higher expected payoff by marrying at age 1.

By restricting strategic choices to a decision of whether to marry at age 1 or age 2, we have arbitrarily excluded strategies that may be preferred by some males. Since males who choose to marry young are matched randomly to females who are willing to marry young males and since the quality of young females is common knowledge, one might expect some males to try a strategy of the following form: Go into the marriage market while young. If you are lucky enough to draw one of the better females who is willing to marry a young man, marry her. If you draw a female from the lower end of the distribution, don't marry but wait until you are older. Indeed there is nothing in this model to prevent this strategy from being preferred by some males to accepting a random draw. A thorough treatment of strategies of this type must await a model with a more detailed search theory and with more than two possible ages of marriage.

III. Long-Run Stationary Equilibrium

Since we have assumed that the number of persons born in each year is constant and that quality distributions and preferences are the same in each generation, we can expect to find a long-run stationary equi-

[6] Because we have assumed that persons of each sex agree in their rankings of the opposite sex, the only assignment in the *core* from an ex ante standpoint is the assignment that matches persons in order of expected quality. For further references and a masterful survey of the general problem of stable marriage assignments, see Roth and Sotomayor (1990).

[7] This is a Bayesian equilibrium of an agreeably simple nature. In this model, we do not have to wonder what inferences are to be drawn about a player's type if he or she deviates from equilibrium behavior. By the time that a deviation is observed by another player, the deviator will have reached age 2. The type of every age 2 person is common knowledge, so there is no mystery about how to regard someone who has in the past deviated from equilibrium strategies.

librium in which each generation behaves in exactly the same way as all preceding generations.

Analysis of equilibrium is much simplified by the fact that in *any* equilibrium, whether it is stationary or not, the following proposition is true.

PROPOSITION 1. At any time t, the set of males who choose to wait until age 2 to marry will be an "upper tail" of the quality distribution, that is, a set of the form $\{y|y \geq y_t\}$ for some $y_t \in [L_b, U_b]$.

Proof. Consider two males, born at the same time, of quality y' and y $(y' > y)$. If these males both marry at age 1, then they face the same lottery and their expected payoff will be the same. If they wait until age 2 to marry, then the male of quality y' will be matched to a female whose quality is at least as great as the quality of the female matched to the male of quality y. From this it follows that if it is worthwhile for a male of quality y to wait until age 2 to marry, any male of higher quality than y will find it worthwhile to wait. Q.E.D.

It turns out that in long-run equilibrium, all females marry at age 1. There is a threshold level of quality, y^*, such that in each time period, males of higher quality than y^* marry at age 2 and males of lower quality marry at age 1. The highest-quality male from a generation will marry at age 2 the highest-quality female from the next younger generation. The second-highest-quality male will marry at age 2 the second-highest-quality female of age 1, and so on until the threshold quality y^* is reached. Males of quality lower than y^* will choose to marry at age 1 and will receive a random assignment from the set of age 1 females who did not have sufficiently high quality to be matched with the available males of age 2.

Let us define a function g such that $x = g(y)$ means that a female of quality x has the same ordinal rank among females that a male of quality y has among males. Thus $g(z)$ is the (unique) solution to the equation $F_b(z) = F_g(g(z))$. We further define $\mu_b(y)$ to be the quality of the "average male who is no better than a male of quality y" and define a similar notation $\mu_g(y)$ for females. Formally,

$$\mu_b(y) = \int_{L_b}^{y} \frac{z dF_b(z)}{F_b(y)}$$

and

$$\mu_g(y) = \int_{L_g}^{y} \frac{z dF_g(z)}{F_g(y)}.$$

In long-run equilibrium, a male of threshold quality y^* will be just indifferent between marrying at age 1 and marrying at age 2. If he marries at age 2, he will be matched to a female of quality $g(y^*)$ and his utility will be $g(y^*) - c_b$, where c_b is the utility cost of waiting. If he marries at age 1, he will be indistinguishable from the other males who marry at age 1 and will be randomly assigned to one of the females who cannot marry an older male of quality $y > y^*$. The average quality of females in this pool is $\mu_g(g(y^*))$. Therefore, his expected utility if he marries at age 1 is $\mu_g(g(y^*))$ and he will be indifferent between marrying at age 1 and marrying at age 2 if

$$\mu_g(g(y^*)) = g(y^*) - c_b. \quad (1)$$

Long-run stationary equilibrium is fully characterized by the following result.

PROPOSITION 2. If $y^* \in [L_b, U_b]$ satisfies equation (1), then there is a long-run stationary equilibrium such that, in every generation, each male of quality $y \geq y^*$ marries at age 2 a female of age 1 whose quality is $g(y)$, and each male of quality $y < y^*$ marries at age 1 a female randomly selected from the set of females in his own generation of quality $x < g(y^*)$. Conversely, every long-run stationary equilibrium is of this type.

Proof. The assertion that the proposed arrangement is an equilibrium will be demonstrated if we can show that no individual can gain by deviating from the proposed equilibrium strategy. Consider a male of quality $y > y^*$. If he chooses to marry at age 2, he will be matched to a female of quality $g(y)$ and his payoff will be $g(y) - c_b$. If he chooses to marry at age 1, he will have an expected payoff of $\mu_g(g(y^*))$.[8] Since g is an increasing function of y, it follows from equation (1) that he cannot gain by marrying at age 1 rather than at age 2.

Consider a male of quality $y < y^*$. If he marries at age 1, he will have a random draw from the set of females of quality less than $g(y^*)$ and his expected payoff will be $\mu_g(g(y^*))$. If he waits until age 2 to marry, then his quality will be common knowledge. All the males from his own generation of quality $y \geq y^*$ will be in the marriage pool at this time and will be matched to all the age 1 females of quality $x \geq g(y^*)$. Therefore, his payoff from marrying at age 2 will be smaller than $g(y^*) - c_b$. From equation (1), it follows that he cannot gain by marrying at age 2 rather than at age 1.

Consider any female. If she deviates from the strategy of marrying at age 1, the expected quality of her partner will be no higher than

[8] Since we have assumed that people in the same generation choose their age of marriage simultaneously, his choice to marry at age 1 will not change the set of females who choose to marry at age 1, nor will it change the set of unmarried age 2 males. Therefore, the pool of females who are available to marry age 1 males does not change in response to his decision to marry at age 1. It follows that the expected payoff from marrying at age 1 remains $\mu_g(g(y^*))$ whether or not he chooses to marry at age 1.

the expected quality she can get at age 1.[9] Since waiting is costly, she would not gain from choosing to marry at age 2 rather than at age 1.

We have shown that if y^* satisfies equation (1), no person can gain by deviating from the proposed equilibrium strategies. All that remains is to show that every long-run stationary equilibrium is of the type described in this proposition. From proposition 1, it follows that in any equilibrium, the set of males divides into an upper-quality interval who marry at age 2 and a lower-quality interval who marry at age 1. If equilibrium is to be stationary, then the threshold quality at which these groups divide must be some constant y^*. If the pool of available males is the same in every period, then (since waiting is costly) it can never be worthwhile for females to choose to marry at age 2 rather than at age 1. Therefore, in equilibrium all females must marry at age 1. Finally, it is straightforward to verify that males better than y^* will choose marriage at age 2 and males worse than y^* will choose marriage at age 1 only if equation (1) is satisfied. Q.E.D.

IV. Existence and Uniqueness of Long-Run Equilibrium

The questions of existence and uniqueness of long-run equilibrium reduce to the question of whether equation (1) has a solution and whether that solution is unique. Let us define the difference between a male or female's own quality, z, and the quality of the average male or female who is no better than z. Let $\delta_b(z) = z - \mu_b(z)$ and $\delta_g(z) = z - \mu_g(z)$. Then equation (1) is equivalent to

$$\delta_g(g(y^*)) = c_b. \tag{2}$$

The following two assumptions will be sufficient for the existence and uniqueness, respectively, of a solution to equations (1) and (2).

ASSUMPTION 1. The distribution function for the quality of each sex is continuous, and the difference between the quality of the most desirable female and the average quality of females exceeds the cost, c_b, to a male of waiting to marry at age 2.

ASSUMPTION 2. The function $\delta_g(x)$ (which is the difference between x and the average quality of females worse than x) is a monotone increasing function of x.

[9] There is a slight complication. If she decided to delay marriage, then when her age is 1, the number of males in the marriage market would exceed the number of females by one. Therefore, a randomly selected male who chose to marry at age 1 would not find a mate. He would reappear in the marriage market in the next year. But the addition of a randomly selected male from the set of males of quality $y < y^*$ will not improve the expected quality assignment for any female who waits until the next period to marry.

PROPOSITION 3. If assumption 1 holds, then there exists at least one long-run stationary state equilibrium in which y^* solves equation (1).

Proof. By assumption 1, $\delta(U_g) > c_b$. From the definition of the function $\delta(\,)$, it follows that $\delta(L_g) = 0 < c_b$. The function $\delta(x)$ inherits continuity from the distribution function for x. Therefore, from the intermediate value theorem, there must be at least one solution, x^*, to the equation $\delta(x^*) = c_b$. Let $y^* = g^{-1}(x^*)$. Then $\delta(g(y^*)) = c_b$. Therefore, there exists a solution to equations (1) and (2). From proposition 2, it follows that there exists a long-run stationary equilibrium. Q.E.D.

PROPOSITION 4. If assumption 2 holds, then any long-run stationary equilibrium is unique.

Proof. From assumption 2 and the monotonicity of g, $\delta_g(g(y)) - c_b$ must be a monotonic increasing function of y, and hence there can be only one y^* for which $\delta(g(y^*)) = c_b$. From proposition 2 it follows that every long-run stationary equilibrium must satisfy this equation. Q.E.D.

An example.—Suppose that the quality of females is uniformly distributed on an interval $[0, a]$ and the quality of males is uniformly distributed on the interval $[0, b]$. Then the function that maps males to females of corresponding quality rank is $g(y) = (a/b)y$. For the uniform distribution, the average quality of females worse than x is just $x/2$. Thus we have $\mu_g(x) = x/2$ and $\delta_g(x) = x - \mu_g(x) = x/2$. We see that $\delta_g(x)$ is an increasing function of x, so that assumption 3 is satisfied. In fact we can readily solve for the unique equilibrium. The equilibrium condition $\delta_g(g(y^*)) = c_b$ will be satisfied if $(a/2b)y^* = c_b$ or, equivalently, if $y^* = 2bc_b/a$. Therefore, if $0 < 2c_b < a$, there will exist a unique solution for y^* in the interval $(0, b)$. In long-run equilibrium, all males of quality $y < y^* = 2bc_b/a$ will choose to marry at age 1. Males who marry at age 1 will get a random draw from the population of females of age 1 whose quality is lower than $g(y^*) = 2c_b$. The expected payoff of a draw from this pool will then be c_b. If a male of quality y^* marries at age 2, he will be paired with a female of quality $g(y^*) = 2c_b$, but he has to bear the cost of waiting until age 2. His utility payoff from waiting is $2c_b - c_b = c_b$, which is the same as the payoff from marrying at age 1. Females of quality $x > 2c_b$ will marry age 2 males of quality bx/a. Females of quality $x < 2c_b$ will get a random draw from the population of males who choose to marry at age 1.

We have shown that the monotonicity of the function $\delta_g(z)$ is sufficient for the uniqueness of equilibrium, and as our example shows, if the quality of females is uniformly distributed, then $\delta_g(x)$ is strictly monotone increasing. It would be useful to know more generally what probability distributions have this property. As it happens, the class of distributions that have this property is quite large, and many of its members can be identified by an easily checked sufficient condition.

It turns out that a necessary and sufficient condition for $\delta_g(x)$ to be an increasing function of x is that the log of the integral of the cumulative density function be a concave function.[10] This fact is not as useful as one might hope because it is rarely possible to find a closed-form expression for the cumulative density function, let alone its integral. Therefore, it is not easy to verify whether a random variable has this property. But we are rescued by the remarkable fact that "log concavity begets log concavity" (under integration). This result seems to have been discovered by Prékopa (1973) and has since appeared in several places in the literature on operations research, statistics, and economics (see, e.g., Flinn and Heckman 1983; Goldberger 1983; Caplin and Nalebuff 1988; Dierker 1989). This result, which is proved in Appendix A, follows.

LEMMA 1. If $f(x)$ is a differentiable, log concave[11] function on the real interval $[a, b]$, then the function $F(x) = \int_a^x f(t)dt$ is also log concave on $[a, b]$, and so in turn will be the function $G(x) = \int_a^x F(t)dt$.

In a recent study, Bagnoli and Bergstrom (1989) examine the log concavity of density functions, cumulative density functions, and their integrals for numerous common probability distributions. All the following probability distributions have log concave densities and hence monotone increasing $\delta_g(x)$ functions: uniform, normal, logistic, extreme value, chi-squared, chi, exponential, and Laplace. Therefore, according to their theorem 2, if the distribution of female quality belongs to any of these families, equilibrium will be unique. The following probability distributions have log concave density functions for some but not all parameter values: Weibull, power function, gamma, and beta.

Log concavity of the density function is a sufficient but not a necessary condition for $\delta_g(x)$ to be monotone increasing. Bagnoli and Bergstrom (1989) show that although the lognormal distribution and the Pareto distribution do not have log concave density functions, they do have log concave cumulative density functions and monotone increasing $\delta(x)$.[12]

[10] To see this, integrate the expression for $\delta_g(x)$ by parts.

[11] A function f is said to be log concave if $\log f$ is a concave function.

[12] In fact, we are aware of no probability distributions famous enough to be "named" for which $\delta(x)$ is not monotone increasing. However, as Bagnoli and Bergstrom showed, the probability distributions defined as "mirror images" of the Pareto distribution and the lognormal distribution have, respectively, monotone increasing and nonmonotonic $\delta(x)$.

V. The Trajectory to Long-Run Equilibrium

If the population starts out in long-run stationary equilibrium, it will remain there. But if initially the population is not in long-run stationary equilibrium, it will *not* immediately jump to a stationary equilibrium.[13] Somehow the system must move gradually toward equilibrium. During the process of adjustment to long-run equilibrium, some people are going to have to be left without partners.

When the system does not start out in long-run equilibrium, the dynamics are complicated by the fact that, in some time periods, females will choose to delay their date of marriage because the supplies of available males may be more favorable to them in the second period of their lives than in the first. A complete general characterization of the behavior of the system outside of long-run equilibrium appears to be very difficult. Here we settle for a pair of general results, one for each sex, and an example.

Proposition 1, which we proved earlier, states that along any equilibrium path the set of males who choose to wait until age 2 to marry is an upper tail of the distribution of males. The behavior of females is much more complicated and is not fully described here, but we do have one rather interesting general result.

PROPOSITION 5. In equilibrium, no female will ever marry a male of age 2 whose quality rank is higher than her own.

Proof. If in period t some female marries a male of higher quality rank than her own, then there will have to be some young female of higher quality who does not marry a male of quality rank as high as her own in period t. This second female must, therefore, have voluntarily postponed marriage. But if she is willing to postpone her marriage, she must get a male whose quality exceeds that of her quality match by at least c_b. This means that a third female of yet higher quality must be displaced one generation later. The process would have to continue, with females of ever higher quality in later generations being displaced. Eventually there would be no male sufficiently good to compensate the best displaced female for waiting until age 2. Q.E.D.

We conclude with an example that is simple enough that we can work out an exact solution for the pattern of marriage that starts out from a position off of the long-run equilibrium path and moves gradually toward long-run equilibrium.

[13] This problem is echoed by occasional informal arguments to the effect that perhaps the reason that women marry older men is that somehow people got started doing things this way and now it cannot be stopped because there are so many unmarried older men around who compete the women away from younger men.

An example.—For each sex, "quality" is uniformly distributed on the interval [0, 1]. Equal numbers of males and females are born in each period. In the initial period, there are no unmarried persons of age 2 available from either sex. The utility cost of marrying at age 2 rather than at age 1 is $c < 1/2$ for members of either sex. A person's desirability to members of the opposite sex neither increases nor decreases between ages 1 and 2.

In long-run equilibrium, males of quality lower than $2c$ marry at age 1, males of quality higher than $2c$ marry at age 2, and all females marry at age 1. But this population will not go all the way to long-run equilibrium in a single step. If it did so, then no age 1 males of quality $y > 2c$ would marry, and since there are no males of age 2 available, any female who marries in the first period would have to accept a random young male whose expected quality would be c. But by waiting until the next period when some high-quality age 2 males become available, females of the highest quality could get spouses of nearly quality 1. Since, by assumption, $1 - 2c > 0$, it must be that $1 - c > c$, so some of the best females will be better off waiting to marry at age 2.

For this example, the pattern of ages at marriage converges to long-run equilibrium in a simple but rather surprising way. The proportion of males who choose to marry at age 2 goes immediately to the equilibrium level and stays there. But females divide into four groups. In each period after the first, some females of age 1 at the top of the quality distribution marry males of age 2. Some of intermediate quality wait until age 2 to marry, at which time they marry males of age 2. The females of age 1 just below these marry males of age 1. Finally, at the bottom of the quality distribution of females are those who are left without partners.

Let X_t^1 denote the set of females born in year t who at age 1 marry males of age 2, let X_t^2 be the set of females born in year t who at age 2 marry males of age 2, let X_t^3 be the set of females born in year t who at age 2 marry males of age 1, let X_t^4 be the set of females born in year t who at age 1 marry males of age 1, and let X_t^5 be the set of females born in year t who are left without mates. If initially there are no unmarried persons of age 2, each of these sets is an interval. These intervals take the following form: $X_t^1 = (x_t^1, 1)$, $X_t^2 = (x_t^2, x_t^1)$, X_t^3 is empty, $X_t^4 = (x_t^4, x_t^2)$, and $X_t^5 = (0, x_t^4)$. Specifically, it turns out that

$$x_t^1 = 2c + (1 - 2c)(1/2)^{t-1},$$

$$x_t^2 = 2c + (1 - 2c)(1/2)^t,$$

$$x_t^4 = (1 - 2c)(1/2)^t.$$

This means that x_t^1 starts out at 1 in the first period. In the second period, x_t^1 moves halfway from 1 to the equilibrium value $2c$ and in each subsequent period again moves halfway from its previous location to $2c$. Notice also that, for all t, $x_t^2 = x_{t-1}^1$ and that the length of the interval X_t^2 of females who marry at age 2 is halved in every period and is being squeezed asymptotically to $2c$. The interval set X_t^5 of females who are left unmatched is being halved in every period. In the limit, the behavior of females approaches the long-run equilibrium in which all females of quality $x > 2c$ belong to X_t^1 and all females of quality $x < 2c$ belong to X_t^4.

VI. Remarks and Possible Extensions

Becker (1974) suggests a reason to expect that high-wage males might marry earlier rather than later. He argues that high-wage males have more to gain from marriage than low-wage males because they will enjoy greater returns to specialization (by marrying low-wage females who will specialize in doing household work). Since there is more to be gained from being married, they will spend less time searching and hence marry earlier. Keeley (1977) investigates this relation empirically using a sample of households from the 1967 Survey of Economic Opportunity. Although he finds a positive relation between age at first marriage and income if one does not include years of schooling as an explanatory variable, he finds a *negative* relation between age at first marriage and income when years of schooling is included. Using 1982 census data, Bergstrom and Schoeni (1992) find that controlling for education does reduce the positive relation between age at first marriage and income in later life, but even in this case, expected male income in later life increases with age at first marriage up to an age in the midtwenties and then decreases for older ages.

As a test of our model, it seems inappropriate to "control for education" in exploring the relation between age at first marriage and economic success. To do so seems to beg the question of why it is that people who get more years of education tend to marry later. It is hard to see why the benefits from marriage are likely to be smaller for those who are attending universities than for persons of the same age who are working for wages.[14] One of the most convincing ways that a young man can demonstrate to potential mates that he is able and diligent is to finish a college degree.

[14] Those who have observed fraternities at large universities will find it hard to believe that this environment is as well suited to scholarship as married life.

Of course we would not be so narrow-minded to claim that our model is a full explanation of when people marry or that the considerations suggested by Becker and Keeley can be neglected. To confront the data more convincingly, one would like to have a much more elaborate model than we have presented. The model should be enriched to incorporate search costs, to allow more varied roles for females, to allow the gradual accretion of evidence about members of each sex as time passes, and to take into account the role of nonhuman wealth. There is much interesting work to be done.

Appendix A

Proof of Lemma 1 (Log Concavity Begets Log Concavity)[15]

By elementary calculus, $F(x)$ will be log concave if $0 \geq F'(x)/F(x) = f'(x)F(x) - f(x)^2$. If f is log concave, then also by elementary calculus it must be that, for $x \leq t$, $f'(x)/f(x) \geq f'(t)/f(t)$. Therefore, for all $x \in [a, b]$,

$$\frac{f'(x)}{f(x)}F(x) = \frac{f'(x)}{f(x)}\int_a^x f(t)dt \leq \int_a^x \frac{f'(t)}{f(t)}f(t)dt.$$

But

$$\int_a^x \frac{f'(t)}{f(t)}f(t)dt = \int_a^x f'(t)dt = f(x) - f(a).$$

Therefore,

$$\frac{f'(x)}{f(x)}F(x) \leq f(x) - f(a) \leq f(x),$$

and hence $0 \geq f'(x)F(x) - f(x)^2$. Therefore, $F(x)$ is log concave. Since $F(x)$ is log concave, the same argument can be applied to show that $G(x)$ inherits log concavity from $F(x)$. Q.E.D.

[15] The idea for this proof is borrowed from Dierker's (1989) proof of the same proposition.

Appendix B

TABLE B1

Mean Age at First Marriage

Country	Males: Mean Age at First Marriage	Males: Average Annual Change, 1950–85	Females: Mean Age at First Marriage	Females: Average Annual Change, 1950–85	Difference between the Sexes: In Age at First Marriage	Difference between the Sexes: Average Annual Change
	Asia and the Middle East					
Brunei	26.1	.01	25.0	.20	1.1	–.19
Hong Kong	29.2	.02	26.6	.19	2.6	–.17
Indonesia	24.8	.07	21.1	.13	3.7	–.06
Japan	29.5	.08	25.8	.04	3.7	.04
Korea	27.8	.11	24.7	.14	3.1	–.03
Malaysia	26.6	.07	23.5	.15	3.1	–.08
Nepal	21.5	.07	17.9	.07	3.6	.0
Philippines	25.3	.01	22.4	.01	2.9	.0
Singapore	28.4	.10	26.2	.26	2.2	–.16
Thailand	24.7	.01	22.7	.05	2.0	–.04
Bangladesh	23.9	–.01	16.7	.04	7.2	–.05
India	23.4	.09	18.7	.11	4.7	–.02
Pakistan	24.9	.09	19.8	.10	5.1	–.01
Sri Lanka	27.9	.03	24.4	.12	3.5	–.09
Algeria	25.3	–.02	21.0	.03	4.3	–.05
Cyprus	26.3	.11	24.2	.09	2.1	.02
Egypt	26.9	.05	21.4	.08	5.5	–.03
Iraq	25.2	–.06	20.8	.01	4.4	–.07
Iran	24.2	–.07	19.7	.12	4.5	–.19
Israel	26.1	.02	23.5	.10	2.6	–.08
Jordan	26.8	.10	22.8	.12	4.0	–.02
Kuwait	25.2	.01	22.4	.18	2.8	–.17
Morocco	27.2	.09	22.3	.17	4.9	–.08
Syria	25.7	.02	21.5	.09	4.2	–.07
Tunisia	27.8	.02	24.3	.18	3.5	–.16
Turkey	23.6	.06	20.7	.07	2.9	–.01
	North America, Oceania, and Europe					
United States	25.2	.05	23.3	.08	1.9	–.03
Canada	25.2	.0	23.1	.02	2.1	–.02
Australia	25.7	.01	23.5	.09	2.2	–.08
New Zealand	24.9	–.03	22.8	.02	2.1	–.05
Denmark	28.4	.06	25.6	.13	2.8	–.07
Finland	27.1	.04	24.6	.06	2.5	–.02
Norway	26.3	–.05	24.0	.03	2.3	–.08
Sweden	30.0	.10	27.6	.19	2.4	–.09
Ireland	24.4	–.23	23.4	–.11	1.0	–.12
England	25.4	–.02	23.1	.03	2.3	–.05
Austria	27.0	–.02	23.5	–.03	3.5	.01
Belgium	24.8	–.05	22.4	–.03	2.4	–.02
France	26.4	.0	24.5	.05	1.9	–.05
West Germany	27.9	.01	23.6	–.03	4.3	.04
Luxembourg	26.2	–.08	23.1	–.05	3.1	–.03
Netherlands	26.2	–.04	23.2	–.05	3.0	.01
Switzerland	27.9	–.01	25.0	.01	2.9	–.02
Greece	27.6	–.07	22.5	–.11	5.1	.04
Italy	27.1	–.05	23.2	–.05	3.9	.0
Portugal	24.7	–.08	22.1	–.08	2.6	.0
Spain	26.0	–.10	23.1	–.11	2.9	–.01

TABLE B1 (*Continued*)

Country	Males: Mean Age at First Marriage	Males: Average Annual Change, 1950–85	Females: Mean Age at First Marriage	Females: Average Annual Change, 1950–85	Difference between the Sexes: In Age at First Marriage	Difference between the Sexes: Average Annual Change
Bulgaria	24.5	.03	20.8	−.01	3.7	.04
Czechoslovakia	24.7	−.08	21.7	−.04	3.0	−.04
East Germany	25.4	−.01	21.7	−.07	3.7	.06
Hungary	24.8	−.06	21.0	−.05	3.8	−.01
Poland	25.9	.02	22.8	.04	3.1	−.02
Romania	24.9	.04	21.1	.08	3.8	−.04
USSR	24.2	.0	21.8	.07	2.4	−.07
Yugoslavia	26.1	.07	22.2	.0	3.9	.07
	Caribbean, Central America, and South America					
Cuba	23.5	−.09	19.9	−.08	3.6	−.01
Dominican Republic	26.1	.02	19.7	.05	6.4	−.03
Haiti	27.3	−.04	23.8	.06	3.5	−.10
Trinidad	27.9	.05	22.3	.12	5.6	−.07
Costa Rica	25.1	−.03	22.2	.01	2.9	−.04
El Salvador	24.7	−.03	19.4	−.01	5.3	−.02
Guatemala	23.5	−.02	20.5	.06	3.0	−.08
Honduras	24.4	−.05	20.0	.16	4.4	−.21
Mexico	24.1	−.02	20.6	−.03	3.5	.01
Nicaragua	24.6	−.08	20.2	.01	4.4	−.09
Panama	25.0	.01	21.3	.10	3.7	−.09
Argentina	25.3	−.07	22.9	−.01	2.4	−.06
Bolivia	24.5	.0	22.1	−.02	2.4	.02
Brazil	25.3	−.09	22.6	−.04	2.7	−.05
Chile	25.7	−.04	23.6	.0	2.1	−.04
Colombia	25.9	−.04	22.6	.03	3.3	−.07
Ecuador	24.3	−.04	21.1	.0	3.2	−.04
Paraguay	26.0	−.02	21.8	.03	4.2	−.05
Peru	25.7	.01	22.7	.05	3.0	−.04
Uruguay	25.4	−.13	22.4	−.03	3.0	−.10
Venezuela	24.8	−.05	21.2	.10	2.6	−.15
	Africa					
Benin	24.9	.0	18.3	.07	6.6	−.07
Central African Republic	23.3	.04	18.4	.07	4.9	−.03
Congo	27.0	.13	21.9	.18	5.1	−.05
Ghana	26.9	.06	19.4	.15	7.5	−.09
Kenya	25.5	.08	20.3	.11	5.2	−.03
Mali	27.3	.05	16.4	.01	10.9	.04
Liberia	26.6	.03	19.4	.12	7.2	−.09
Mauritius	27.5	.06	23.8	.15	4.7	−.09
Mozambique	22.7	−.04	17.6	−.06	5.1	.02
Réunion	28.1	.03	25.8	.10	2.3	−.07
Senegal	28.3	.02	18.3	.05	10.0	−.03
South Africa	27.8	.02	25.7	.10	2.1	−.08
Togo	26.5	.07	17.6	−.07	8.9	.0
Tanzania	24.9	.07	19.1	.11	5.8	−.04
Zambia	25.1	.06	19.4	.11	5.7	−.05

SOURCE.—United Nations (1990), tables 7, 18, 34, and 44.

NOTE.—The time trends for some countries are not the same as the rest: Bangladesh, 1974–81; Iraq and Romania, 1966–77; Iran, 1957–77; Bulgaria, 1956–75; and USSR, 1979–85. The time intervals for Africa vary from country to country depending on the available data.

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