An n-person Rubinstein bargaining game*

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June 19, 2005

Abstract

When Herrero (1985) extends Rubinstein's (1982) alternating-offers bargaining model to the case of three or more players any agreement can be supported as a subgame perfect equilibrium (SPE) outcome, given a sufficiently large discount factor. We show that this is not the case when players demand shares for themselves instead of proposing agreements to each other. Although it is possible to rule out agreements, the majority remains to be SPE outcomes.

Keywords: Multilateral bargaining; n-person bargaining; subgame perfect equilibria.

JEL classification: C72; C78.

1 Introduction

Shaked (see e.g. Sutton, 1986) showed that Rubinstein's result (1982) on the uniqueness of perfect equilibrium outcome did not hold for bargaining situations with three players. If fact, any allocation is an subgame perfect equilibrium (SPE) outcome if the common discount factor δ is sufficiently large ($\delta \geq 1/2$). Herrero (1985) obtains the same result when she generalize Shaked's model to n-person bargaining (for $\delta \geq 1/(n-1)$). Haller (1986) shows that the indeterminacy prevails irrespective of δ when the players are required to respond simultaneously instead of sequentially. It is generally held that the indeterminacy cannot be removed by any obvious reorganization of

*Financial support from The Jan Wallander and Tom Hedelius Foundation is gratefully acknowledged.

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the sequence of moves (see Krishna and Serrano, 1996; or Sutton 1986). It is possible to overcome the problem, e.g. by restricting the players to use stationary strategies or changing the extensive form of the game. Most attention is directed to the latter approach through the introduction of exit opportunity. Jun (1987), Chae and Yang (1988, 1994), Yang (1992) and Krishna and Serrano (1996) are the main contributions in this area. Other papers are Binmore (1986), Asheim (1992), Merlo and Wilson (1995), Calvó-Armengo (1999), Vannetelbosch (1999) and Chatterjee and Sabourian (2000).

In this paper we show, using a different extensive form than Herrero (1985), that Shaked's and Herrero's result is not necessarily true for a generalized Rubinstein model. By letting the players demand shares for themselves we can rule out agreements as SPE outcomes even when $\delta \geq 1/(n-1)$. This entails no discrepancy from the Rubinstein model, who identifies a partition with a number s by interpreting s as the proportion of the pie that Player 1 receives. Clearly, the demand; "I want s", is also a proposed partition; "I get s and you get $1-s$". A similar implication is not present in multilateral bargaining. Unfortunately, we also show that most agreements can be supported as SPE outcomes in our model.

2 The model

The n-person bargaining game $G(t; P_i, ..., P_j)$ over a pie of size 1, starting in period t $(= 1, 2, ..., \infty)$ and with identical players P_1, P_2, ..., P_n, where $n \geq 3$, is defined as follows:

An agreement is a partition x, where x_i denotes P_i's share of the pie. To reach an agreement the players must be unanimous regarding an element of the set of feasible agreements X, where $X = \left\{x \in \mathbb{R}^n_+ \mid \sum_1^n x_i = 1\right\}$. If agreement is reached in period t, P_i's utility is $u_i = \delta^{t-1} x_i$, where $\delta \in (0,1)$.

The bargaining order is clockwise. In each period, the first $n-1$ players in turn make demands which the last player then either accepts or rejects. In period 1 the game is $G(1; P_1, P_2, ..., P_n)$. If the demands are compatible and the last player (P_n) accepts, the game ends with an agreement. If the demands exceed the size of the pie or the last player (P_n) rejects, the play moves to the next period where the game will be $G(2; P_2, P_3, ..., P_1)$. Thus, the players leapfrog until an agreement is reached.

It is easy to see that this change has an impact on the set of possible SPE outcomes. For example, the players will never reach the immediate agreement $(0, ..., 1) \in X$ which assigns the entire pie to P_n. P_n ends up with the entire pie if and only if all the other players in turn demands nothing.

Clearly, P_n accepts all demands in the first period that leave him with δ or a bigger share of the pie. Hence, if all players before P_{n-1} demands nothing of the pie, then it can never be optimal for P_{n-1} to do likewise - it would be strictly better for him to demand $1 - \delta$.

3 The result

Let X_- denote the set of agreements in which P_n is assigned δ or less of the pie, i.e. $X_- = \{x \in X \mid x_n \leq \delta\}$.

Proposition 1 *If $\delta \geq 1/(n-1)$, then for any partition $x^* \in X_-$ there exists a subgame perfect equilibrium in which the outcome is the immediate agreement x^*.*

Proof. Fix a partition $x^* \in X_-$. In state x^*: P_i always demands x_i^* and, in addition, any slack left by the players before him, and he always accepts demands that leave him with δx_i^* or more of the pie.

Besides state x^*, we use:

e^m : Player P_i who occupies position m $(m = 2, ..., n)$ in the *first round* in this state, always demands 1 and only accepts demands that leave him with δ or more of the pie; the other players always demand 0 and accept any demands.

The transition rules between states are as follows:

- If the player in position $m-1$ $(m = 2, ..., n-1)$ demands more than the share specified in the current state, excluding any slack left by the players before him, then the state immediately shifts to e^m. (Notice that the player in position $m-1$ can pick up any slack left by the players in positions 1 to $m-2$ without triggering a transition to state e^m.)
- If the player in position $m-1$ $(m = 2, ..., n-1)$ demands more than specified in the current state (thus causing a shift to state e^m) and the player in position m demands less than 1, then the state immediately shifts to e^{m+1}. (Thus, the player next in line punishes the player in position m for omitting to punish the first deviation.)
- If the player in position n rejects the demands although they comply with the current state, then the state immediately shifts to e^2.

Every transition occurs immediately after the event that triggers it, i.e. before the next player makes his move. To see that these strategies form an SPE, we need to study the players' rules for making and accepting demands. Consider the rule for P_i who makes the first demand in state $y \in \{x^*, e^2, ..., e^n\}$. If he demands more than y_i, the state shifts to e^2 and the player in position 2 demands the whole pie. This prevents an agreement from being reached in this period. The game moves to the next period, still in state e^2, which ensures that the deviating player receives nothing. If he demands less than y_i, then the player in position 2 picks up the slack and an agreement is reached. Hence, demanding less than y_i is non-profitable. Consider the rule for P_j who makes the k^{th} demand in state y. It is optimal for P_j to prevent the agreement if the player before him, say P_s, demands more than y_s. The state has shifted to e^k, so the payoff for preventing the agreement is worth δ. By deviating (making a lower demand than 1) the state shifts to e^{k+1} and he gets nothing. If P_s demands $\theta_s \leq y_s$ then it is optimal for P_j to pick up the slack. The state shifts to e^{k+1} if he demands more than y_j and the slack; thus P_j gets nothing by deviating. Consider the rule for the player accepting the demands in state y. If the demands are in accordance with the state, then it is optimal for him to accept. If he rejects, then the state shifts to e^2 and he gets nothing. If the player in position $n-1$ deviates by demanding more than specified and any slack or by omitting to punishing the player before him, the state shifts to e^n which makes it optimal for him to reject.

Since no player gains anything by a one-shot deviation, $x^* \in X_-$ is an SPE outcome in period 1. ■

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