Comments

A Superior Solution to Captain MacWhirr's Problem: An Illustration of Information Problems and Entitlement Structures

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You couldn't tell one man's dollars from another's, he said, and if you asked each man how much money he brought on board he was afraid they would lie, and he would find himself a long way short. [JOSEPH CONRAD, Typhoon]

In his novel Typhoon, Joseph Conrad posed a classic problem of information revelation. At first glance, the problem appears to be insoluble because of the apparent incentive for individuals to generate wrong information. After further consideration, however, it turns out that an appropriate change in the entitlement structure of the problem allows it to be solved by generating an incentive for each individual to reveal correct information. In this sense, the problem is akin to the class of free-rider problems associated with public goods that can be solved by using the demand-revealing process (Tideman and Tullock 1976).

Conrad's problem is stated in Section I, along with the arbitrary solution imposed by the fictional character Captain MacWhirr. A superior solution, which actually generates correct information, is presented in Section II. Section III contains a brief discussion of the similarities between the superior solution and the demand-revealing process for public goods, followed by conclusions.

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I. The Problem and Captain MacWhirr's Solution

In *Typhoon*, 200 Chinese workers are being transported to their home province on a ship commanded by Captain MacWhirr, after working for 7 years in various tropical colonies. Each worker's accumulated savings of silver dollars are stored in his own camphorwood chest. As the title of the story suggests, all is not smooth sailing during the trip and the ship runs into a typhoon.

The problem that we are interested in arises when, during the course of the storm, all the wooden chests get smashed and the silver dollars are all scattered between decks. A riot ensues among the Chinese workers as they try to retrieve their money. In order to stop the conflict, the captain sends the first mate and some men to pick up all of the money, which they do. When the storm is over, the captain, being a fair man, wants to return the money to its rightful owners, but he has a problem. He does not know how much money each man had.

The problem here is to obtain information about how much money each man actually had, assuming only that the captain knows the total amount of money and that each man knows the amount he had but not necessarily the amount that anybody else had or even the total amount of money. The captain realizes that he cannot just ask each man how much money he had and then give him that much because some of them might overstate the amount. In this event, the sum of the claims would exceed the total amount of money, thus making a distribution on this basis impossible. The captain concludes that it is impossible to guarantee correct information and therefore imposes an arbitrary distribution that he thinks is fair. According to him, the fair assumption is to assume that all the men had the same amount of money and, therefore, he gives an equal share of the total to each man.

It is obvious that this distribution is not necessarily fair at all unless all the men really did have the same amount of money, which is not known to be the case. The captain's solution to the problem is not a solution to the information problem but rather a surrender. Such a surrender, however, is not necessary, as will now be demonstrated.

II. A Superior Solution

If only the individual knows how much money he had, the captain is right to think that this information can only be obtained by getting the individual to reveal the correct amount. He is wrong, on the other hand, to think that there is necessarily an incentive for the individual to overstate the amount. The mistake here is the implicit assumption that the amount stated by an individual is also the amount to which he
is entitled. Under such an entitlement structure, each individual has a clear monetary incentive to overstate his share. The way to solve the problem, then, is to cut this direct link between amount revealed and entitlement in such a way as to obtain correct information while at the same time entitling individuals to the amounts they actually had.

In Captain MacWhirr's problem, there is a fairly straightforward entitlement structure which generates an incentive for individuals to reveal correct information. Instead of letting the amount claimed be the entitlement, let the maximum amount to which an individual is entitled be the difference between the total amount of money and the amount claimed by everyone else. If an individual claims less than the maximum amount, then he gets what he claims, but if he claims more he is penalized and actually receives less than the maximum amount to which he is entitled. As a result, no individual has an incentive to overstate his share (or to understate it) and he reveals the amount he actually had. This also means that each individual is thereby entitled to the amount he actually had. A more formal presentation of the problem will make this point clearer.

For the following analysis, let \( r_i \) = the amount received by the \( i \)th individual; \( c_i \) = the amount claimed by the \( i \)th individual; \( a_i \) = the amount the \( i \)th individual actually had; and \( A \) = the total amount of money. Under an entitlement structure like the one sketched out above, the amount received by the \( i \)th individual can be written as

\[
r_i = c_i - \max \left[ (1 + \alpha)(c_i - (A - \sum_{j \neq i} c_j), 0 \right] \quad j = 1, \ldots, n, \tag{1}
\]

where \( A - \sum_{j \neq i} c_j \) is the difference between the total amount of money and the amount claimed by everyone else and is, hence, the \( i \)th individual's maximum entitlement. It follows that \( \alpha < 0 \) if individuals are to be penalized for claiming amounts greater than the maximum entitlement.\(^1\)

The individual's problem is to choose \( c_i \) so as to maximize \( r_i \). This, however, calls for an assumption about \( \sum_{j \neq i} c_j \). The reasonable assumption here is that \( \sum_{j \neq i} c_j = \sum_{j \neq i} a_j \), as will be seen by the solution to the individual's problem. If we substitute \( \sum_{j \neq i} a_j \) for \( \sum_{j \neq i} c_j \) and note that it follows by definition that \( A - \sum_{j \neq i} a_j = a_i \), we can rewrite equation (1) as

\[
r_i = c_i - \max \left[ (1 + \alpha)(c_i - a_i), 0 \right]. \tag{2}
\]

\(^1\) If \( c_i < A - \sum_{j \neq i} c_j \), then \( r_i = c_i \) because \( (1 + \alpha)(c_i - (A - \sum_{j \neq i} c_j)) < 0 \). Also note that it is assumed that the amount of money that individuals are penalized is kept by the captain.
From equation (2), it is easy to see that \( r_i = a_i \) if \( c_i = a_i \) but that \( r_i < a_i \) if \( c_i < a_i \), and as a result, the individual honesty reveals and receives the amount of money he actually had. Furthermore, this is true for all \( i \), which means that the assumption of \( \sum_{j \neq i} c_j = \sum_{j \neq i} a_j \) is validated and we have a Nash equilibrium solution for our problem.²

This solution is obviously superior to the captain’s solution because it returns to all individuals the amounts of money they actually had, regardless of what the actual amounts were. This depends on obtaining correct information, which is accomplished by eliminating the direct link between individuals’ claims and their entitlements. It is that direct link which gives rise to the incentive for wrong information.

III. Comparison with the Demand-revealing Process and Conclusion

It was stated in the introduction that the superior solution to Captain MacWhirr’s problem has a certain similarity with the demand-revealing process for determining preferences for public goods. We are now in a position to see what this similarity consists of. In brief, the similarity is that both are solutions to information problems that are obtained by separating the information revealed by an individual from the individual’s entitlement.

The classic public goods problem is that the free-rider phenomenon generates an incentive for individuals to undervalue their valuations of a public good if their share of the financing depends on that valuation. That is, if individuals feel that their own share of total financing is negligible and, hence, that they are quantity takers from the total collective decision, then they have an incentive to undervalue their demand for the good if their entitlement (obligation) to share in the financing is determined by the demand they reveal. The twist in the demand-revealing process that causes individuals to accurately reveal their demands is that the entitlement structure is changed from the one that is usually implicitly assumed. Instead of obligating the individual to share only in the financing of the stated quantity desired at a given tax price, the individual is obligated, rather, to share in the financing, at a given tax price, of the quantity chosen by everyone else.

This simple change in entitlement basically eliminates the possibility of a free ride. Then, by using suitably designed penalties for deviating

² This is not the case with other assumptions about \( \sum_{j \neq i} c_j \) and, hence, the justification for the assumption used. That is, if it is initially assumed that \( \sum_{j \neq i} c_j > \sum_{j \neq i} a_j \), this means that \( r_j \) is maximized with \( c_j < a_j \). But, if each person knows that everyone else has the same incentive to act in a similar fashion, this would lead to the conclusion that \( \sum_{j \neq i} c_j < \sum_{j \neq i} a_j \), which contradicts the initial assumption. In the same way, the initial assumption of \( \sum_{j \neq i} c_j < \sum_{j \neq i} a_j \) also leads to a contradiction.
from the basic entitlement given by everyone else, the individual has the incentive to reveal his true demand for the good. Because everyone thus has incentive to reveal his true demand, the total demand for the good is honestly revealed and the appropriate quantity can be chosen. The basic feature of this process is strikingly similar to the basic feature of the superior solution to Captain MacWhirr's problem.

The point here is not that there is necessarily a large class of economics problems that correspond to the fictional problem posed by Conrad and susceptible to solution by the method developed above in Section II. Rather, the fictional problem nicely illustrates that the apparent incompatibility between economic incentives and revealing true information does not mean that the information problem is insoluble. On the contrary, it indicates that a suitable entitlement structure restores the compatibility between incentives and true information, thus allowing the information problem to be solved.

Reference