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Demand Competition and Policy Compromise in Legislative Bargaining

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I propose a new majoritarian bargaining model in which more than one implicit proposal can be on the table at the same time. Institutional differences from system to system affect the order of play, the equilibrium majorities, and the policy outcome. The ex post distribution of payoffs within a winning coalition is, however, invariant to fundamental institutional differences, and it is always proportional to the distribution of relative ex ante bargaining power. The bargaining process is modeled as a sequential demand game, in which players are called to propose a policy and to specify their desired share of the private benefits related to being in office. The order of play is endogenous, and the distribution of payoffs within an equilibrium majority usually does not depend on who is the proposal maker. The role of the head of state and the advantages to center parties are also studied.

In any parliamentary democracy the major decisions are made within the parliament. The executive power is delegated to the ministers who form the cabinet, but even the set of executive decisions is indirectly limited by the relative bargaining power of the various parties supporting the government in parliament. The formation of the government itself is an outcome of bargaining among the elected members of parliament. This article provides a new noncooperative model of such a formation process.1

When parties bargain about what government coalition to form, they are generally motivated by the desire to be in office as well as by the desire to affect policy outcomes. The literature on coalition formation in parliaments initially focused on the office-seeking behavior of legislators (I will often call this the private benefits case). In that context, that is, in the absence of policy preferences or cleavages, Riker (1962) was the first to indicate that any winning coalition must have the same total value as any other (simple game), and hence all equilibrium winning coalitions must be minimal.

The most common reference point in the related noncooperative theory of coalition formation with pure office-seeking motivations is the "closed rule" model of Baron and Ferejohn (1989). In that model a randomly selected first mover proposes a government coalition and a precise way of sharing the private benefits associated with being in office within such a coalition. If everyone accepts, then the government coalition is formed; if someone rejects, then another party is randomly selected to make another proposal, and so on.2 The formation of minimal winning coalitions is predicted. This is by no means the only noncooperative model proposed within Riker’s private benefits framework, but it is the most used, and hence I adopt it as a benchmark for comparison.3

Many authors have explored the opposite extreme scenario, in which the parties care exclusively about policy outcome. Some of these authors focus on the prediction that majority coalitions should be formed by parties close to each other in the policy space (connectedness), as long as the latter is unidimensional (see Axelrod 1970; de Swaan 1973). Within the same policy-bargaining category, other authors focus on the role of the median party, which can effectively be a policy dictator when the policy space is unidimensional. Baron (1991) makes this point very clearly: When parties play an alternating offer game exclusively in the policy space and the latter is unidimensional, the Median Voter’s Theorem obtains.4 Laver and Shepsle (1990, 1996) introduced an important government formation model in which parties bargain over a "grid" of policies. Their policy space is intrinsically multidimensional, and hence their results cannot easily be compared with those of this article, in which the policy space is unidimensional.

Very few models consider both office-seeking motivations and policy preferences and parties that bargain explicitly on both dimensions. In the noncooperative games literature the most important contribution, in which these interactions are explicitly studied, is Austen-Smith and Banks (1988).5 In their model, as in Baron (1991), the government is formed by a large and a small party, and the policy outcome is closest to the

1 In presidential-congressional systems the executive can be supported by different majorities on different issues, but in a typical parliamentary system the majority coalition supporting the government is not allowed to vary. The rules of the game in any parliamentary democracy also allow the majority coalition to decide on the termination of a government or even on the dissolution of parliament (see Lupia and Strom 1995). I focus exclusively on the formation process, leaving the dynamic extension to future research.

2 The open rule allows a randomly selected second mover to decide whether to agree with the first mover’s proposal and let it be voted upon or to propose an amendment.

3 Harrington (1990) independently introduced a legislative bargaining model very similar to Baron and Ferejohn’s.

4 Austen-Smith and Banks (1990) and Baron (1993) also have only policy in the utility function.

5 A related cooperative model is described by Sened (1996), who extends the applicability of the ‘‘structurally stable core’’ (introduced by Schofield 1986) to the case in which the utility function has both components.
one preferred by the large party. In my model, in contrast, the relative size does not matter in three-party situations, and the policy outcome is not necessarily closer to the one preferred by the large party. Interestingly enough, Lupia and Strøm (1995) identify a wide range of real circumstances under which the size and the bargaining power of parties are not correlated.

Other frameworks that explicitly consider both office-seeking motivations and policy preferences are those of Crombez (1996) and Diermeier and Merlo (1998). In their models the probability of observing minority governments or surplus majorities is related to the size and the policy location of the largest party. My article does not directly deal with those issues, but it indicates that the size of the largest party may not be the relevant issue.

Most attempts to model legislative bargaining as a noncooperative game follow the logic of the alternating offer model used in Rubinstein (1982), and they all assume an exogenous order of play. The order of play is crucial for the determination of which majority coalition prevails; hence, an exogenous order limits the predictive power in terms of which coalition is likely to be winning. I propose an alternative model of majoritarian bargaining in which players demand a compensation for their participation in a coalition and in which the order of play is endogenous, in that the head of state chooses the first mover, and the latter chooses the order of response. These two features together have a number of analytical implications that make the model applicable to a variety of situations.

Other demand bargaining models exist (e.g., Selten 1992; Winter 1994a, 1994b), but they have an exogenous order of play, and the problems studied are of the cake-splitting nature. In my framework the order of play is endogenous, and each party's strategy at each bargaining round contains a demand that may specify a policy proposal and the amount of private benefits (ministerial and nonministerial payoffs) a party would want if selected in a majority coalition. Who else should be in the majority coalition and how much each member should receive are items that do not need to be specified. The various demands actually compete, because at some point in the order of play some party has the choice of which coalition to form, given the demands made by the previous movers.

Browne and Franklin (1973) and Browne and Fendreis (1980) studied the actual distribution of private benefits associated with being in office, and they showed that in parliamentary democracies the prediction of a proportional payoff division within the majority coalition, first conjectured by Gamson (1961), is supported by the data. Schofield and Laver (1985) confirmed the empirical validity of the proportionality norm, at least as far as ministerial payoffs are concerned. They also emphasized that in multipolar party systems the consideration of bargaining power can improve the prediction with respect to the basic proportionality norm. Most important, even in their empirical findings there is no evidence that the first proposer will obtain a share of the ministerial payoffs higher than any other party of the same size in a majority coalition.

All the models based on Baron and Ferejohn (1989) yield a disproportionate payoff share for the proposal maker regardless of the distribution of seats, and hence they are not consistent with the basic empirical findings highlighted above. The ex post payoff distribution within an equilibrium majority predicted by those models is proportional neither to the distribution of seats nor to the ex ante distribution of bargaining power. Harrington (1990) noted the same feature in these bargaining models based on alternating offers. He showed that the excessive power of the proposal maker declines as the voting rule tends to the unanimity rule, but even the lower bound of the share of the proposal maker is always larger than the share of any other party. This disproportionate share for the proposal maker is not justified in simple cake-splitting problems.

In contrast, the demand bargaining game introduced here performs very well with respect to the evidence mentioned above. The subgame perfect equilibrium payoff distribution of this game is proportional to the ex ante distribution of bargaining power and approximately proportional to the distribution of seats in the winning coalition, consistent with Gamson's Law. Moreover, consistent with the other findings of Browne and Franklin (1973) and Browne and Fendreis (1980), my model shows that when the number of parties needed in a majority coalition is small, the smaller parties receive more than their relative share of seats in the coalition, and the larger parties receive less ("relative weakness effect"). In the extreme case in which only three parties play the bargaining game, the distribution of ministerial and nonministerial payoffs tends to be an equal split, and this is robust to the introduction of policy preferences. Schofield and Laver (1985) and Laver and Schofield (1990) introduced a different notion of bargaining power, and they also found evidence that the bargaining power correction often helps explain payoff division. Yet, they did not find that the relative weakness effect is strong enough to lead to an equal split of ministerial payoffs in three-party games.

The positive analysis of this article applies to all kinds of parliamentary systems and premier-presiden-

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7 It will be possible to identify some differences between the case in which the head of state has discretion over the choice of the first mover and the case in which the first mover is, by a fixed rule, the party that received the most votes.

8 The ex ante bargaining power of a party may differ slightly from its share of the total number of seats, depending on the number of alternative "minimal" majority coalitions available to the various parties. The term "ex post payoff distribution" refers to the equilibrium payoff distribution given some order of play, whereas "ex ante payoff distribution" refers to the expected payoff distribution before the order of play is known.

9 The agenda setter may well have access to greater opportunities when she gets to choose the order in which multiple issues will be voted upon.
tial systems, as long as the government needs support by a majority in parliament.\textsuperscript{10} When parties have policy preferences, the equilibrium policy outcome may vary according to institutional differences in the amount of power given to the head of state. In some systems s/he is an active player and is free to choose the candidate prime minister, who will then start the bargaining game. In other systems the election results determine the order of play almost automatically, leaving the head of state little choice. Both possibilities are included in my model, and center parties are shown to have an advantage when the head of state has discretionary power.\textsuperscript{11} Since the head of state does not have access to the private benefits available to party members in parliament, s/he is assumed to care only about the equilibrium policy outcome. Especially when the head of state is elected directly, a safe assumption would be that s/he would like the equilibrium policy outcome to be as close as possible to the median voter’s preferred outcome.\textsuperscript{12}

If parties care about the policy outcome, then the head of state always prefers the median party as first mover, and in the three-party case the equilibrium majority is either center-left or center-right. In contrast with Austen-Smith and Banks (1988), the relative shares of the total number of seats do not matter at all in the three-party game. Which majority coalition forms may depend on the parties’ ideological positions but not on the relative seats. When the head of state has discretionary choice over the first proposal maker, the median party’s preferred policy is always the equilibrium policy outcome, and the proportional payoff sharing result obtained in the pure private benefits case still holds.

The model is described in the next section. The case in which parties only care about private benefits will be analyzed first. The model will then be extended to the case in which ideology matters.

\section*{THE MODEL}

Consider a legislature in which \(n\) parties are represented. Denote by \(w_i\) the fraction of the total number of seats (and hence votes) held by party \(i\) (where \(\sum_{i=1}^{n} w_i = 1\)).\textsuperscript{13} Since the players who actually participate in the bargaining process are the parties, not the individuals of parliament, the implicit assumption is that party members are loyal to the party line. As a consequence of the loyalty assumption, the players have heterogeneous endowments, that is, the fraction of seats assigned to them through an election. The main example of legislative bargaining to keep in mind is the one that leads to the formation of a majority coalition in support of, or creating, a government.\textsuperscript{14} In this and in most other decision problems in the parliament, simple majority is sufficient.

Denoting by \(q\) the quota for simple majority, a coalition \(S\) is winning if and only if \(\sum_{i \in S} w_i \geq q\). Denote by \(\Omega(w)\) the set of winning coalitions for a given vector of weights \(w\), and denote by \(\Omega^m(w)\) the set of minimal winning coalitions (henceforth MWC):

\[\Omega^m(w) = \left\{ S: \sum_{i \in S} w_i \geq q \quad \text{and} \quad \sum_{i \in T} w_i < q \forall T \subset S \right\}.\]

It is also useful to denote by \(M'(w)\) the set of MWCs containing \(i\), and by \(m'(w)\) its cardinality.

\textbf{Assumption 1:} \(w_i < q \forall i\).

When assumption 1 is violated, there is no coalition formation problem, and the majority party gets all the private benefits and chooses its preferred policy outcome. Assumption 1 will be kept throughout, since that is the interesting case.

Let us normalize to unity the total amount of private benefits (ministerial and nonministerial payoffs) to be distributed among the elected parties. Besides caring about its share of private benefits, each party may have an ideological policy position. Assume that the policy space is the segment \([0, 1]\), and let us denote the position of party \(i\) by \(\theta_i \in [0, 1]\). The utility function below encompasses the possibility that parties may care both about private benefits and about policy outcomes. The (quasilinear) utility function is

\[u_i = x_i + 1 - \beta|\theta^* - \theta_i|,\]

where \(x_i\) denotes the share of private benefits accruing to party \(i\), \(\theta^*\) is the equilibrium policy outcome, and \(\beta > 0\) measures the intensity of the ideological component of parties’ preferences.

Suppose that the head of state acts to maximize the utility of the median voter.\textsuperscript{15} The only choice variable the head of state can have in this model is the choice of a candidate prime minister, which determines the party that moves first in the bargaining game. As shown in the next sections, the results on payoff division do not depend on this discretionary choice, and hence they apply also to the case in which the head of state applies some fixed constitutional rule instead of actually “choosing” the first mover. The first mover of the sequential demand game is then either the party se-

\textsuperscript{10} Following the definition given by Shugart and Carey (1992), a premier-presidential system differs from a presidential system in that the cabinet is responsible to parliament. The relevance of the majority coalition in parliament supporting the government is similar to that in parliamentary systems.

\textsuperscript{11} See Article 8 of the French Constitution for a clear case of discretionary power to choose the candidate prime minister. The same discretionary power is also used in Italy, even though the head of state is not directly elected.

\textsuperscript{12} Even when the head of state is a monarch or is chosen by parliament, there usually are constitutional requirements of impartiality and/or a general need of popular support, both of which lead to balanced positions.

\textsuperscript{13} There is no need to specify the total number of seats in the parliament. Only relative bargaining power matters.

\textsuperscript{14} As shown later in the section on equal weights, the results of this article extend to all cases of legislative bargaining, even when the loyalty assumption does not hold.

\textsuperscript{15} The model would obviously work no matter what the utility function of the head of state. As mentioned in the introduction, however, there are many reasons to believe that normally the policy preferences of the head of state should be close to those of the median voter.
lected to move first by the head of state or the party with the most votes in the case of a fixed rule.

If the appointed first mover manages to form a majority coalition in parliament, then the prime minister is in charge of choosing the cabinet that will implement the policy platform agreed upon within the majority coalition. If instead the first round of demands by the \( n \) parties ends without any agreement on a majority coalition, then the head of state chooses another potential prime minister. If the second round fails as well, then a third one may begin, and so on, up to \( T \) times. If after \( T \) rounds all attempts have failed to form a majority coalition, then the head of state forms a caretaker government until the next elections.

Let us assume that a caretaker government does not distribute any private benefits to the parties, and the policy outcome is the one desired by the median voter: \( \theta^\ast(c.t.) = \theta_m \). There is a discount factor \( \delta \in (0, 1) \), so that the utility for party \( i \) associated with the agreement reached at round \( t = 1, \ldots, T \) is:

\[
u^i(t) = \delta^{t-1} \{ x^i(t) + 1 - \beta (\theta^\ast(t) - \theta_m) \},\]

where \( x^i(t) \) is the share of private benefits going to \( i \) given the agreement reached at round \( t \), and \( \theta^\ast(t) \) is the equilibrium policy outcome chosen by the majority coalition formed at round \( t \). If the caretaker stage is reached, then the assumptions above imply that party \( i \) obtains \( u^i(c.t.) = \delta^T \{ 1 - \beta |\theta_m - \theta^*| \} \).

Let us now formalize the above sequence of events by describing the extensive form of the game, which will be denoted \( \Gamma(n + 1, w, \theta) \). First, the head of state chooses the potential prime minister from among \( N \) parties. Second, that party chooses the order of play \( \rho_1 : N \rightarrow \{ 1, 2, \ldots, n \} \), where \( \rho_1(i) = 1 \), that is, party \( i \) is the first to move. Third, the \( n \) parties then make sequential demands, playing in the order \( \rho_1 \) determined at the previous node. Each party \( j \)'s demand is a pair \((x_j(1), y_j(1))\); the first element denotes the share of private benefits requested for participation in a majority coalition, and the second denotes the proposed policy outcome. If \( \exists S \subseteq \Omega(w) \) such that \( \sum_{j \in S} x_j(1) \leq 1 \) and \( y_j(1) \) are the same for every \( j \in S \), then such a coalition is the majority coalition supporting the government, which will implement \( y_j(1) \) and distribute the private benefits according to the demands made.

In order to clarify how such a coalition may or may not form at this round, let us be more specific about the sequential game. For any order \( \rho_1 \), denote by \( p(\rho_1, w) \) the number such that

\[
\sum_{j : p(\rho_1, w) \leq p(\rho_1, w)} w_j < q
\]

that is, the position in the order \( \rho_1 \) before which forming a winning coalition is not possible, no matter what the demands are.

(a) For all \( j \) such that \( p(\rho_1, w) = p(\rho_1, w) \), a demand \((x_j(1), y_j(1))\) is the only action that player \( j \) can take.

(b) If \( \exists S \subseteq \{ j : p(\rho_1, w) \leq p(\rho_1, w) \} \) such that \( \sum_{j \in S} x_j(1) \leq 1 \) and such that everyone in that subset has proposed the same \( y_j(1) \), then the player in position \( p(\rho_1, w) \) can choose whether to form a coalition (demanding \( x \leq 1 \) and \( \sum_{j \in S} x_j(1) \) and \( y_j(1) \)) or just make a demand and let the next player move. If, instead, \( \sum_{j \in S} x_j(1) > 1 \forall S \subseteq \{ j : p(\rho_1, w) \leq p(\rho_1, w) \} \), or if policy demands differ, then making a demand is the only option.

(c) For any node \( l \) reached by the game, player \( \rho_1^{-1}(l) \) has the same set of possibilities as those just described for \( \rho_1^{-1}(p) \).

Fourth, if no majority coalition \( S \) is formed before the end of round \( 1 \), then it is the turn of the head of state again, who chooses another potential prime minister from some party \( k \). This new appointed party chooses another order \( \rho_2 \) (with party \( k \) moving first), and another subgame of sequential demands identical to the one above begins.

Fifth, the game ends at the round \( t \) in which a majority coalition \( S \subseteq \Omega(w) \) forms, with \( \sum_{j \in S} x_j(t) \leq 1 \) and the same \( y_j(t) \forall j \in S \); if that does not happen at any of the \( T \) rounds, the game ends at the caretaking stage.

The strategies are as follows. The head of state simply establishes a first-moving party for each round reached. The strategy of any party \( i \) prescribes at any round \( t \) reached by the game a pair \((x_i(t), y_i(t))\), as a function of the pairs demanded by the parties who played before in round \( t \) given \( \rho_t \); moreover, in the round in which it moves first, party \( i \) also has to choose \( p \). Finally, at any round \( t \) in which \( \exists S \subseteq \{ j : p(j) < p(i) \} \), such that \( \sum_{j \in S} x_j(t) \leq 1 \) and \( y_j(t) \) are the same \( \forall j \in S \), and such that \( S \cup i \) belongs to \( \Omega(w) \), party \( i \) has the additional option of choosing one such coalition and closing the game, demanding a compatible share and the same \( y \).

A more formal definition of parties' strategies would be as follows.

1. At every \( t \), \( \forall i \) such that \( 1 < p(i) < p(\rho_t, w) \) and \( \forall i \) such that \( p(i) = l \geq p(\rho_t, w) \), but \( \sum_{j \in S \subseteq \Omega(w)} x_j(1) > 1 \) or \( y_j \neq y_k \) for some \( j, k \in S \), the action can only be a demand:

\[
a^i_t(p, X_{A_{\rho}(i)-1}) \rightarrow [0, 1]^2,
\]

where \( X_{A_{\rho}(i)-1} \) denotes the vector of demands previously expressed in the same round.

2. At the round \( t \) in which party \( i \) moves first, the action is

\[
a^i_t(p_1^{-1}(1), \ldots, p_1^{-1}(1)) \rightarrow \{ p : p(i) = 1 \} \times [0, 1]^2.
\]

3. At every round \( t \), \( \forall i \) such that \( p(i) = l > p(\rho_t, w) \), such that \( \sum_{j \in S \subseteq \Omega(w)} x_j(1) \leq 1 \) for some winning coalition \( S \subseteq \{ j : p(j) \leq l \} \), the action is

\[
a^i_t(p, X_{A_{\rho}(i)}) \rightarrow \Sigma(i) \times [0, 1]^2,
\]

where \( \Sigma(i) = \{ S \subseteq \{ j : p(j) \leq l \} : S \in \Omega(n) \} \cup \emptyset \), where choosing the null set means that the game goes on.
A strategy for party $i$ is then summarized by the triplet $a_1^i, a_2^i, a_3^i$, since at each round one of the three mappings has to apply.

I will consider the subgame perfect equilibrium of $\Gamma(n + 1, w, \theta)$, starting from the benchmark case of $\beta = 0$. Note that the order of play, which is usually treated as an exogenous protocol, is endogenous here.

**EQUILIBRIUM OUTCOMES IN THE PURE PRIVATE BENEFITS CASE**

Let us first analyze the case in which parties do not care about the policy outcome, only about their share of transferable benefits ($\beta = 0, u_i = x_i$). I will first explain the properties of this demand bargaining model for the simplest case of three parties; I will then consider all the other possible distributions of seats among $n$ parties.

**The Three-Party Case**

Since the parties do not care about which policy outcome is chosen, the head of state has no reason to prefer a particular party in office. $S/\theta$ could just as well randomize the assignment of first mover as follow the standard rule and select the party with the most votes. In any case, whoever is appointed by the head of state is indifferent between the two possible orders of play available. The expected payoff for the first mover is $1/2$ whatever order of response it chooses, and the majority coalition is formed by the first two movers of the first round.

**PROPOSITION 1.** *In every three-party majoritarian bargaining game $\Gamma(3 + 1, w, \theta)$ in which parties care only about their share of transferable benefits ($\beta = 0$), the unique subgame perfect equilibrium payoff distribution within the prevailing MWC is $(1/2, 1/2)$.*

See Appendix A for the formal proof.

**Remark 1.** Proposition 1 holds for every fraction of seats held by the three parties, as long as $w_i < q\forall i$.

In any situation in which no party has the absolute majority, any pair of parties can form a MWC, and hence the prediction that the equilibrium distribution will be $(1/2, 1/2)$ independently of the relative shares is perfectly consistent with the fact that the relative bargaining power of each party is the same in those situations. Most important, the first mover does not have any advantage in terms of payoff share with respect to the other member of the majority coalition, in contrast to the disproportionate share obtained by the first mover in Baron and Ferejohn (1989). Note that in their “closed rule” model the first mover obtains $1/5$ even if it is the smallest party.

**Homogeneous Weights**

Let us now analyze the equilibrium properties of the model when the number of parties is $n >= 4$. Contrary to the three-party case, the payoff distribution becomes sensitive to the distribution of seats, in a roughly proportional way. Before making this statement precise with the next proposition, it is necessary to introduce the concept of an equivalent homogeneous representation.

A game characterized by a vector of weights $w$ and by the simple majority quota $q$ is said to admit an equivalent homogeneous representation if and only if there exists a vector $w^h \in (0, 1]^n$ with $\Sigma_{i=1}^n w^h_i = 1$ and a quota $q^h \in (1/2, 1)$, such that

$$\Omega^*(w) = \Omega^*(w^h) = \left\{ S \subset N; \sum_{i \in S} w^h_i = q^h, \sum_{j \in T} w^h_j < q^h \forall T \subset S \right\}$$

and such that $\sum_{i \in S} w^h_i = q^h \forall S \in \Omega^*(w^h)$. For example, a game with $n = 4$ and $w = (1/7, 1/7, 1/7, 1/7)$ is not homogeneous, since some MWCs have $1/7$ of the votes and one has $3/7$; the (unique) equivalent homogeneous representation of this game is $w^h = (1/7, 1/5, 1/5, 1/5, 1/5), q^h = 0.6$.

The original vector $w$ gives only an approximate idea of the relative bargaining power, not a precise one. In fact, in the example just mentioned, one can notice that the party with $3/7$ of the votes can belong to as many MWCs as each of the parties with $1/7$ of the votes, and hence there is no reason the relative bargaining power should differ. Indeed, in the equivalent homogeneous representation they have the same weight. The share $w^h_i$ of the equivalent homogeneous representation seems a compelling measure of the relative bargaining power of party $i$, because the renormalization generating it makes it possible to assign the same weight to all the parties who belong to the same number of MWCs.

**PROPOSITION 2.** *Consider any game $\Gamma(n + 1, w, \theta)$ in which $n >= 4$ and $\beta = 0$. If there exists a unique equivalent homogeneous representation $(w^h, q^h)$, then there is a unique equilibrium payoff distribution for every $p_i$:

$$x_i^* = \frac{w^h_i}{q^h_i} \forall i \in S^*(p_i),$$

where $S^*(p_i) = \{ i : p_i(i) = p_i(w, \theta) \}$. If there is a dummy player (i.e., a player $i$ such that $M'(w) = \emptyset$), then she receives 0 private benefits, independently of whether she joins the majority coalition.*

See Appendix B for the formal proof.

The payoff distribution within the majority coalition is proportional to the adjusted weights’ vector $w^h/q^h$, and when the actual vector of seat shares $w$ is homogeneous, the payoff distribution is exactly proportional to the relative seat shares. In contrast, Harrington (1990) shows that alternating offer bargaining games

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16 It would be easy to show that existence of an equivalent homogeneous representation is not a problem for $n < 5$, while for $n >= 5$ some counterexamples can be found, and that is why proposition 3 is needed for a complete characterization. The uniqueness of an equivalent homogeneous representation can be established when there is some symmetry (as in the three-player case) and when the actual types are not too many (as in the Apex game below) but it cannot be established in general.
have the property that the share of the proposal maker is always larger than that of anyone else, and proportionality cannot be obtained. This property then affects the results of any model based on a “closed form” alternating offer bargaining game following Baron and Ferejohn (1989). As argued before, in a simple cake-splitting problem there is no reason the first proposer should have such a disproportionate share, and, indeed, the empirical evidence goes against this prediction.17

The Apex Game Example

Example 1. The apex game, in which one party has \( \frac{3}{4} \) of the seats and four other parties have \( \frac{1}{4} \) each, is homogeneous.18 The subgame perfect equilibrium payoff distribution is \((\frac{3}{4}, \frac{1}{4})\), with \( \frac{1}{4} \) to the large party if it is either selected as first mover or is selected as second by one of the small parties, or \((\frac{1}{2}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4})\) if a small party is the first mover and it chooses not to have the large party in the majority coalition. The equilibrium order of play can have the large party moving first, second, or last.19

The above are the only two possible payoff distributions, whatever \( \delta \) and \( T \). To see this, suppose \( T = 2 \). If round 2 is reached and the large party is the first mover, then the continuation equilibrium payoff distribution is \((\frac{1}{4}, \frac{1}{4})\) because, if the large party demands more, the second mover can still demand \( \frac{3}{4} \), and the following movers would find it optimal to do the same and coordinate on the policy proposal of the second mover. By the same token, if the first mover at round 2 is one of the small parties, then the maximum demand it can make is \( \frac{1}{4} \). Hence, whatever the strategy of the head of state, the upper bound on the continuation payoff of the large party if no agreement is reached at round 1 is \( \delta \frac{1}{4} \), and the upper bound for any small party is \( \delta \frac{1}{4} \). This implies that at round 1 the optimal behavior of the first two movers in \( \rho_1 \) is the same as it would be at round 2. If the large party is the first mover, then it cannot demand more than \( \frac{1}{4} \), otherwise the second mover, counting on the fact that the continuation payoff of the following movers is less than \( \frac{1}{4} \), can demand \( \frac{1}{4} \) and implicitly “convince” the others to form the majority coalition of small parties. The same is true when one of the small parties is the first mover at round 1. When \( T > 2 \), the same logic applies.

In sharp contrast, the prediction for the apex game provided by the finite closed rule model of Baron and Ferejohn (1989), the proposer would get the whole cake. If the large party is the proposer in the second-to-last round (prob. \( \frac{1}{2} \)), then it obtains \( 1 - \delta \frac{1}{2} \). If instead a small party is the first proposer (prob. \( \frac{1}{2} \)), then the large party is always selected as first responder (since everyone has the same reservation payoff at that stage). The expected payoff of the large party before such a round is less than \( 9\% \) (equal to it when \( \delta = 1 \)). Thus, if \( T = 3 \) and the first mover at round 1 is a small party, then the large party is never given more than \( 9\% \); if the first proposer is the large party, then the small party never obtains more than \( 1\% \). The share of the proposal maker remains very high even when many bargaining rounds are allowed.

Equal Weights

This article deals mainly with the extreme loyalty case, that is, individual members of parliament vote in the same way if they belong to the same party. This assumption captures the reality of a parliamentary system better than other assumptions, especially when closed lists are used. The following remark points out, however, that even without this assumption the model still yields an easy prediction that conforms to the empirical evidence better than the previous models.20

Remark 2. If \( w = (1/n, \ldots, 1/n) \), which corresponds to the extreme case of “free” individual voting by each legislator, the game is obviously homogeneous. In equilibrium the first \( q \) players demand and obtain \( 1/q \) (where \( q = (n + 1)/2 \) if \( n \) is an odd number).22 This \( w \) represents well the situation in which each legislator is responsible only to her district, not to a party.

The finite version of Baron and Ferejohn’s model in this equal weights case has a unique symmetric equilibrium in which every member of the proposed MWC, apart from the proposer, receives \( \delta(1/n) \).23 The de-

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17 See Browne and Franklin 1973 and Browne and Frandreis 1980. When the number of parties needed in a majority coalition is large, each party belonging to such a coalition obtains in its data a share of the total amount of transferable ministerial and nonministerial payoffs that is more or less proportional to its share of the total number of seats held by the majority coalition, irrespective of who is prime minister. They also find evidence of the so-called relative weakness effect, which means that the party of smaller size (fewer seats) in a government coalition tends to have a more than proportionate share of ministerial payoffs when the number of parties in the majority coalition is small. When only three parties can belong to a MWC, this effect is strong and tends to determine an equal split, despite significant differences in terms of the number of seats. Laver and Schofield (1990) do not support the equal split prediction, but the relative weakness effect pushes payoff distribution in that direction.

18 This simple weighted majority game is often used to compare the predictive power of different solution concepts. See, for example, Bennett and Van Damme 1991, Davis and Maschler 1965.

19 If a small party moves first, then it is never in its interest to choose an order in which the large party is third or fourth. With such orders it can be shown that the first mover would be left out of the continuation equilibrium majority.

20 These probabilities of recognition are equal recognition probabilities, but even if one considers different recognition rules, for example, proportional to seat shares, the main property of disproportionate payoff share to the first mover remains.

21 If the members of parliament represent localities, then loyalty to party leaders is weaker, and free individual voting is more common.

22 The proof of this statement follows directly from proposition 2. As explained in the previous section, this result does not depend on \( \delta \) or on \( T \).

23 To see this, consider the \( n = 3 \), \( T = 3 \) case. In the last round the proposer receives 1 but is selected with probability \( \frac{1}{3} \), so in round 2 a responder accepts anything greater or equal to \( \delta \frac{1}{3} \). Restricting attention to symmetric equilibria, the acceptance threshold in round
mand bargaining model introduced here, in contrast, predicts $1/q$ for every member, including the first mover, in line with Gamson’s Law.

Also notice that the finite version of Baron and Ferejohn’s model has a continuum of subgame perfect equilibria. In fact, all players have the same acceptance threshold in the second-to-last period, and this implies that the proposer in such a round is indifferent among the other players (when they all have the same weight). This in turn implies that it is costless to punish and reward players by making different choices of players at different histories. If $T = 3$ and if players are included in the MWC of the second-to-last round with different probabilities, then different players have different acceptance thresholds at round 1. This makes it possible to support a continuum of divisions as equilibrium outcomes even in stationary strategies. Hence, the symmetry requirement is necessary to have a unique prediction in Baron and Ferejohn’s model, whereas in this model no refinement of subgame perfection is needed. See also Norman (1997) on this multiplicity issue.

**Nonhomogeneous Weights**

To complete the analysis of the pure private benefits case, consider what happens when the distribution of seats is such that there does not exist a unique equivalent homogeneous representation.

Let $N' = N \setminus \{i : M'(w') \subset M'(w) \setminus \{i\} \}$ be the set of parties with alternatives to any proposal. Denote by $w'$ the modified weights’ vector, obtained as follows:

$$w'_j = w'_{j'} \quad \text{if } j \in N'$$

$$0 \quad \text{if } j \in \NN$$

where $(w^{h'}, q')$ denote the unique homogeneous representation obtained by restricting attention to the set $N'$. In other words, $w'$ is obtained from $w$ giving the votes of the parties in $\NN$ to the parties they are “hostage” of, in a way to make $(w', q')$ homogeneous.\footnote{1}

**Proposition 3.** Consider any game $\Gamma(n + 1, w, \theta)$ in which $n \geq 4$ and $\theta = 0$. If there does not exist a unique equivalent homogeneous representation $(w^h, q^h)$, then:

1. If the selected first mover is some party $j \in N'$, party $j$ always chooses an order of play $p_j$ such that the first $p(p_j, w')$ parties form a majority coalition $S'(p_j)$

   $$w'_i = q'_i \quad \forall i \in S'(p_j).$$

   (2) If the selected first mover is some party $i \in \NN'$ (belonging to a nonempty set of MWCs), then $i$ can have a positive payoff share, depending on $\delta$ and on the rule used by the head of state to select the first mover of the subsequent round.

   See Appendix C for the formal proof.

   The following example gives a complete account of the forces yielding this result.

**Example 2.** Consider a five-player game in which one party has 30% of the votes, three parties have 20% each, and one has the remaining 10%. This distribution of seats does not determine a homogeneous representation, because some MWCs have 60% of the votes and others 70%. Moreover, there is no way to find an equivalent homogeneous representation vector $w^h$ that leads to the same set of MWCs as with the original $w$. The party $i$ with 10% of the votes belongs only to MWCs containing the largest party $j$ $(M'(w) \subset M'(w))$, and this makes direct renormalization impossible. Applying the definitions given above, $w' = (2/5, 1/5, 1/5, 1/5, q') = (3/5, 1/5, 1/5, 1/5, 1/5, q')$, where $q'$ is the weight of the largest party $j$, and $1/5$ is the weight of each one of those with 20% of the seats.

If the first mover is party $j$, it chooses an order of play in which party $i$ moves second or third, and the equilibrium demands are $1/3$ for $j$, $1/3$ for the intermediate party who moves among the first three of $p$, and $0$ for party $i$. In fact, if party $i$ moves second, after the large party has asked $1/3$, it does not have a profitable deviation: Asking $\epsilon > 0$, the third mover can keep the proposed equilibrium demand of $1/3$, since it is compatible with the next two players asking for the same and forming the MWC with all 20% parties. There is no deviation for the third mover either, because if it asks for $1/3 + \epsilon$, then the MWC with the first three movers is not feasible, and the next mover will simply demand $1/3$ and go with $i$ and $j$, rather than be forced to demand less than $1/3$ to form the MWC with the third mover (same reasoning for the last mover’s reaction). An order of play that the large party will never choose is the one in which $i$ plays last, because in that case the maximum demand party $j$ could make would be $1/3$. When the first mover is one of the 20% parties, it is able to demand and obtain $1/3$, either by having the other two parties of the same size second and third or by letting $i$ and $j$ play second and third. In summary, the equilibrium payoffs when the first mover is not the small and powerless party are as follows.

1. \(x_i = 1/3\) if \(j\) is in the equilibrium majority coalition (0 otherwise), and
2. \(1/3\) for any 20% party who happens to be in the equilibrium majority coalition.

Neither a head of state who applies a fixed selection rule nor one who uses discretionary power is likely, in
realities, to choose a candidate prime minister from a small and powerless party that is totally hostage to some other party. Therefore, the almost proportional payoff distribution indicated above can be considered the only relevant one. For completeness, however, consider the case in which the first mover is the small party $i$. Party $i$'s equilibrium choice of $p_i$ includes party $j$ as second or third, and its equilibrium demand is $x_i \in [0, 1/3]$. The lower bound of 0 would be reached if $\delta = 1$ and if the strategy of the head of state is to choose party $j$ as first mover next time; $x_i = 1/3$ if $\delta$ is very low and/or if party $j$ is never selected to be first mover (or is selected with low probability). The equilibrium demand of party $j$ is $\frac{1}{3} - x_i$, and the equilibrium demand of the other party is $1/3$. So, if the small party is the first mover in the first round, then the equilibrium shares of $i$ and $j$ depend on $\delta$ and on the strategy of the head of state in future selections.

As shown by example 2, a distinguishing feature of weighted majority games that do not admit an equivalent homogeneous representation is that the first mover is not indifferent among all possible orders of play. Whether the determination of who moves first comes from a fixed protocol, a random assignment, or a discretionary decision by the head of state, the order of play is a relevant choice variable for the appointed first mover. Gamson's law is approximated by the equilibrium payoff distribution, and it is exactly obtained when $w$ is part of a homogeneous representation, as in the apex game example.

**EQUILIBRIUM OUTCOMES WHEN POLICY PREFERENCES MATTER**

Let us now extend the analysis to the situations in which $\beta$ is greater than 0, that is, when parties have policy preferences. In these situations it is possible to pin down a precise equilibrium policy outcome that is sensitive to the parties' ideological positions and to the choice of the prime minister. The results for the pure private benefits case do not depend on how the head of state comes to power or on her preferences. Institutional differences regarding the amount of discretion left to the head of state in choosing the first mover do not affect the proportionality results at all. In contrast, when $\beta > 0$ the head of state is typically not indifferent among all possible candidate first movers. In this context the institutional differences mentioned above may acquire considerable importance. Consider, for example, a country in which the largest party is extremist—say, far Left. Then the head of state who has discretion over the identity of the first mover chooses a member of some center party. This way the equilibrium policy outcome is closer to the center of the policy space than it would be if the head of state were forced to let the largest party move first (recall the assumption that the head of state has policy preferences more or less in line with those of the median voter).

Since the equilibrium policy outcome depends on the identity of the first mover, the head of state has a clear preference ordering about who should be the first mover. The center parties always belong to the equilibrium majority, in contrast with the prediction of the last-stage game of Austen-Smith and Banks (1988). Moreover, the relative size of parties has no bearing on the equilibrium majority, which depends on relative ideological positions only. When the head of state has discretionary power, the equilibrium policy outcome is always the median party's preferred policy, and the equilibrium distribution of private benefits is always proportional to the ex ante bargaining power, which confirms that the results obtained in the previous section are robust to the introduction of policy preferences.

**PROPOSITION 4.** Consider a three-party bargaining game $\Gamma(3 + 1, w, 0)$. Suppose, without loss of generality, that $0 \leq \theta_1 < \theta_2 < \theta_3 \leq 1$. Suppose that the center party's position is the closest to that of the median voter. The head of state with discretionary power weakly prefers to have the median party move first. In any case, the equilibrium policy outcome is $\theta^* = \theta_2$, and the private benefits associated with being in office are equally split between the first two movers.

See Appendix D for the formal proof. Some results of proposition 4 also hold for $n > 3$. In particular, (1) the head of state with discretionary power would still, under general conditions, choose one of the center parties to move first; (2) the equilibrium policy outcome would be in a neighborhood of the median voter's preferred policy; and (3) the distribution of private benefits would be the proportional one obtained in the pure private benefits case.

**CONCLUSION**

The predictions of the bargaining models in the literature give an unrealistic power to the first proposer, overemphasizing the dependence of the results on the exogenous order of play. Obviously, in many instances agenda-setting power is crucial, but in a simple cake-splitting problem the value of moving first should not be that high. The model introduced here shows that it is possible to obtain equilibrium outcomes that approximate the proportional payoff distribution conjectured by Gamson, thus reducing the effect of the order of play on the distribution of transferable benefits in the winning coalition and making the order of play itself endogenous. The description of the bargaining process as a sequence of demands elicits some implicit "competition" among the various proposals in any given

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26 If the head of state does not get to choose, then it is possible that the first mover is an extreme party. In that situation the policy outcome is not necessarily $\theta_2$, and one cannot rule out a coalition comprised of Left and Right parties. Preliminary analysis of a demand bargaining game with three parties and a more general utility function seems to indicate that when $\beta$ is large enough, the order of play does not matter anymore.
round of negotiation. This logic applies not only when politicians care exclusively about being in office (pure private benefits case) but also when parties have policy preferences. If those preferences are not too strong, then institutional and constitutional differences may significantly affect the equilibrium policy outcomes and the equilibrium majorities; if ideological preferences are strong, then the median party’s policy is always implemented.

The result that with demand bargaining the ex post equilibrium distribution of payoffs is proportional to the ex ante distribution of bargaining power (within the majority coalition) holds for every parliamentary system, regardless of whether the head of state is directly elected. Institutional differences in the way the proposal maker is chosen also do not have any implication for this result. When parties have policy preferences, however, the amount of discretionary power left to the head of state makes a major difference.

In parliamentary systems there is a high level of legislative cohesion, which means that the majority coalition usually votes the same on all issues. An interesting question for future research is what happens if the institutional system is such that separate policy decisions on separate issues do not have to be made with the support of the same coalition in parliament. The model presented here cannot answer this question because of the unidimensional policy space. When this model is extended to multidimensional policy spaces, I expect to obtain an explanation of why parties outside a government coalition often receive positive payoffs: Every party should receive an equilibrium compensation that is still a function of its relative ex ante bargaining power, but the latter should depend also on the expected number of issues on which the government will need its support. Moreover, in the case of pure legislative cohesion, it is reasonable to expect the formation of minimal winning coalitions. Yet, when parties have preferences in a multidimensional policy space and legislative cohesion is lower, one would expect some larger equilibrium coalitions to form.

APPENDIX A: PROOF OF PROPOSITION 1

Let us first show that at round \( T = 1 \), for any \( R = p_{T}(I) = 1 \), \( p_{T}(J) = 2 \), \( p_{T}(K) = 3 \). The equilibrium profile is as follows:

\[
a_{i} = \{ x_{i} ; y_{i} \},
\]

\[
a_{j} = \begin{cases} (1 - x_{j}) ; y_{j} \; ; \; \{ i, j \} & \text{if } x_{j} \leq \frac{1}{2} \\ \{ (1/2 + \epsilon) ; y_{j} ; \emptyset \} & \text{if } x_{j} = \frac{1}{2} + \epsilon > \frac{1}{2} \\ \{ (1 - x_{j}) ; y_{j} \; ; \; \{ i, k \} & \text{if } x_{j} \leq x_{i} \\ \{ (1 - x_{j}) ; y_{j} = y_{j} \; ; \; \{ i, k \} & \text{otherwise.} \end{cases}
\]

To see that equation 2 represents an equilibrium strategy profile for the subgame starting at round \( T \), one can easily follow the backward induction logic. Player \( k = p_{T}^{-1}(3) \), if that node is reached, chooses to go with the player who the least, and obviously there is no deviation from this rule that would make her better off.\(^\text{27}\) Now consider player \( j = p_{T}^{-1}(2) \). Given the strategy of \( k \) just discussed, if \( j \) tries to demand more than \( (1 - x_{j}) \) when \( x_{j} \leq \frac{1}{2} \), she will simply not be chosen by \( k \); and when the first mover asks more than \( \frac{1}{2} \), \( x_{j} \) is the maximum share that player \( j \) can demand subject to the constraint imposed by the reaction of \( k \); so there is no profitable deviation here either. Finally, for player \( i, x_{i} = \frac{1}{2} \) is the maximum demand that can be accepted by \( j \); demands higher than that would entail the formation of the MWC \( \{ j, k \} \). As far as the policy proposals are concerned, they are written in the strategy profile just for completeness and serve only as coordination devices, but \( y_{j} \) and \( y_{j} \) can be whatever.\(^\text{28}\)

Having shown that equation 2 is a subgame perfect equilibrium profile at round \( T \) for any \( p_{T} \), it is now easy to show that the equilibrium payoff distribution will be an equal split between the first two movers at round \( 1 \). In fact, at round \( t = 1, 2, \ldots, T - 1 \) the action profile is the same as equation 2, with the only difference that \( \sigma_{k} \) is defined as:

\[
\sigma_{k} = \begin{cases} (1 - x_{j}) ; y_{j} = y_{j} \; ; \; \{ i, j \} & \text{if } x_{j} \leq \min \{ x_{i}, (1 - \delta x_{j}(t + 1)) \} \\
\{ (1 - x_{j}) ; y_{j} = y_{j} \; ; \; \{ i, k \} & \text{if } x_{j} \leq \min \{ x_{j}, (1 - \delta x_{j}(t + 1)) \} \\
\{ x_{j} ; y_{j} ; \emptyset \} & \text{otherwise.}
\end{cases}
\]

where the expected continuation payoff for the third mover at round \( T \) for any \( p_{T} \) is at most \( \frac{1}{2} \) (when it is the next first mover, or when the strategy of the next first mover has it as second mover). In any case, the outside option of the third mover of every round is always between 0 and \( \frac{1}{2} \). For any value in this range, the arguments above about why the behavior of the first two movers should be that described in equation 2 extend to \( t = 1, 2, \ldots, T - 1 \). There is no opportunity for strategic delay or anything like that, and hence the first two movers at the first round form the majority coalition and split the pie equally. This also confirms that the first mover appointed by the head of state is indifferent between the two possible orders. \( \tau \). D.

APPENDIX B: PROOF OF PROPOSITION 2

Let us show why only players who can belong to some MWC \( (M(w) \neq \emptyset) \) can receive positive payoffs in equilibrium. Consider, as an example, a four-player game in which three parties have \( \frac{3}{4} \) of the seats each, and one has the remaining \( \frac{1}{4} \). This vector \( w \) is actually homogeneous, since all MWCs have four votes, but the player with \( \frac{1}{2} \) of the votes does not belong to any MWC. Therefore, even if the latter player moves first, there will be no demand greater than 0 that she can make with hope of being included in the prevailing coalition. This reasoning obviously extends to any other situation of this kind, at every round. It follows that the strategy of player \( i \), such that \( M(w) = \emptyset \), is irrelevant and can be ignored henceforth.

As shown in the formal description of parties’ strategies, each party \( i \) has to choose three mappings \( (a_{i}^{1}, a_{i}^{2}, a_{i}^{3}) \). Let \( \sigma \) denote a generic strategy profile:

\[ \sigma = (P; (a_{i}^{1}, a_{i}^{2}, a_{i}^{3}))_{i=1, \ldots, n} \]

\(^\text{27}\) Note also that even if \( k \) could not ask more than \( 0 \), still it would not be profitable to go to the caretaking stage, where everyone gets 0 private benefits.

\(^\text{28}\) If the tie-breaking rule for \( k \) is kept as in equation 2, then the behavior of player \( j \) (and in turn that of player \( i \)) is the unique rational action. If the tie-breaking rule is the opposite, that is, if player \( i \) is chosen by \( k \) when the first two movers make the same demand, then it can be easily shown that no equilibrium will exist.
where \( P \) denotes the head of state's choice of first mover at each round. In order to show that the equilibrium payoff distribution is the proportional one stated in this proposition, I propose a strategy profile that yields such a result and show that it is indeed an equilibrium profile. I will also argue that even though there are degrees of freedom about the strategy profile, these do not involve the equilibrium payoff distribution.

The strategy of the head of state \((P = P_1, \ldots, P_n)\) remains indeterminate, since s/he cares only about the policy outcome, which is not affected by the order of play in any way when parties care only about private benefits. Whatever \( P \) is, denoting by \( v_i^{-1}(\sigma) \) the expected payoff for player \( i \) at time \( t \) if the game gets to the subgame starting at round \( t + 1 \), the rest of the equilibrium strategy profile is uniquely characterized as follows.

1. The action \( a^P_i \) always includes a demand \( x_i = w_i^q/q^h \), a demand of some policy outcome \( y_i \), and an order of play \( \rho \in \{ p: \{ i \in P : p(i) = p(\rho, w) \} \in \Omega^m(w) \} \).

2. The candidate for equilibrium \( a^P \) at every round and for every \( \rho \) is as follows (I drop the \( t \)-subscript of \( \rho \) for notational convenience):

\[
a^P_{\rho_{t-1}} = \left\{ \left( \frac{w^P_{t-1}/q^h}{\rho^t}; y_{\rho_{t-1}} \right), 2 \leq l < p(\rho, w) \right\}.
\]

3. The candidate \( a^P \) (i.e., \( \forall i: p(\rho, w) \leq l \leq p(\rho, w) \)) is as follows.

(a) Let

\[
S^*(x, l) = \arg \min_{S \in \Omega(\rho, w) \cap \Omega^m(w)} \sum_{i \in S} x_i.
\]

if \( 1 - \sum_{i \in S^*(x, l)} x_i \geq \frac{w_i^q/q^h}{\rho^t} \), then

\[
a^P_{\rho_{t-1}} = \left\{ \left( 1 - \sum_{i \in S^*(x, l)} x_i \right); y(S^*(x, l)); \{ p(l) \cup S^*(x, l) \} \right\}.
\]

(b) If (off the equilibrium path)

\[
1 - t \sum_{i \in S^*(x, l)} x_i < \frac{w^P_{\rho_{t-1}}/q^h}{\rho^t},
\]

then

\[
a^P_{\rho_{t-1}} = \left\{ x_{\rho_{t-1}}^*; y(T(x, l), l) \right\},
\]

where

\[
x_{\rho_{t-1}}^* = \max \ S.T. \rho^{-1}(l) \in T(x, l), l \leq l' \leq n,
\]

and

\[
T(x, l) = \arg \min_{S \in \Omega(\rho, w) \cap \Omega^m(w)} \sum_{i \in S} x_i.
\]

In words, \( x_{\rho_{t-1}}^* \) is the maximum demand \( \rho^{-1}(l) \) can make without risking exclusion from the prevailing MWC and obviously depends on the vector of demands. This is true if \( x_{\rho_{t-1}}^* \geq \sum_{i=1}^n l_i(\sigma) \); otherwise, the optimal response is to let the game go on to the next round (asking an uneasible share).

There are no profitable deviations from this \( \sigma \). Take equation 3 as given, and use backward induction to see that the rest of the profile is the only set of optimal responses. Then I will show (by example) that equation 3 is the only equilibrium choice of orders of play. Any \( \sigma \) with the elements described above is such that \( v_i^{-1}(\sigma) \in [0, w_i^q/q^h] \), \( \forall i, \) and if \( \delta \in (0, 1) \), accepting the proportional share is always strictly better than the highest possible reservation payoff of getting to the next round of negotiation. The best response described in equation 6 is optimal by definition. Given that best response, it is clear that if \( \delta \) observes all the previous movers make a proportional demand, she should do so as well, since otherwise the best response of the subsequent players will be to eliminate her from the majority coalition. They can always eliminate her because the existence of a homogeneous representation guarantees that all the MWCs have the option of sharing the total benefits proportionally; any party can be replaced without changing the payoff shares of the others. For this reason the game can only have subgame perfect equilibrium strategy profiles that entail the proportional payoff distribution.

The reason the rational choice of the order of play by a first mover must be some \( p: \{ i : p(i) = p(\rho, w) \} \in \Omega^m(w) \) can be made clear by means of an example. Consider a four-player game with distribution of seats \( (\frac{1}{5}, \frac{2}{5}, \frac{1}{5}, \frac{1}{5}) \); the equivalent homogeneous representation here is \( (\frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5}) \). Can any order of play be fine here for any first mover? The answer is “no,” because if the first mover is one of the small parties, then it will never choose the order of play in which the large party moves third. When the large party is third in \( p \), it chooses the second mover and the first is left out, since whatever the first mover's demand, the second will underbid just enough to be selected by the third mover. If the large party is placed either second or fourth in the order, condition 3 is satisfied, and the rest of the argument above leading to proportional sharing among the first movers applies. Note that the endogeneity of the order of play is crucial in order to obtain uniqueness of the subgame perfect equilibrium payoff distribution. If, instead, a fixed order of play is imposed, then a refinement will be needed.

**APPENDIX C: PROOF OF PROPOSITION 3**

Consider the case in which the first mover is a party \( j \in N' \). The candidate strategy profile fully described in the proof of proposition 2 extends to this case as far as \( a^P_i \) and \( a^P \) are concerned, with the caveat that \( (w^P_i, q^h) \) substitutes \( (w_i^q, q^h) \). In addition, the candidate equilibrium is such that the first mover chooses an order \( \rho \) such that \( \pi_{\epsilon}(\rho, p(\rho, w)) = q^f \) and such that the parties in \( N\backslash N' \) belonging to a nonempty set of MWCs are the last among the first \( p(\rho) \). Note that, given such a \( \rho \), there is no continuation equilibrium profile in which a party \( i \in N\backslash N' \) obtains \( \epsilon > 0 \). To see this, consider a profile \( \sigma \) in which party \( i \) demands \( \epsilon \) and someone closes a majority coalition that contains \( i \); this \( \sigma \) cannot be a subgame perfect equilibrium because, in order to let \( i \) obtain \( \epsilon > 0 \), some other party \( j \) (that has alternative MWCs) must be demanding less than \( w_i^q/q^h \); then player \( j \) could do better to demand \( \epsilon \) more, counting on the fact that \( i \) does not have alternatives. Having shown this, it is clear that placing any such player(s) among the first \( p(\rho, w) \) movers never hurts, and it may be strictly preferred by the first mover. In this way the proportional demands \( w_i^q/q^h \) become feasible for every other player moving before them, and no one can profitably demand more, since the exploitable parties are already included in the first group, and since if someone asks more than a proportional share, then a subsequent player will close a majority coalition without her. Given the minimal size, no one can do better than this.

Now consider the case in which at some round \( t \) the

29. See Bennett and Van Damme 1991 for this issue of multiplicity of subgame perfect equilibrium payoffs when the order of play is exogenously given.

30. There can be other equilibrium profiles in which the order of play...
strategy of the head of state prefers to have as first mover a party i such that \( M'(w) \subseteq M'(w) \) for some j. If i chooses a \( p_0 \) with party j not in the first \( p(p, w) \) positions, i cannot be part of the majority coalition in the continuation equilibrium, because there is no MWC to which i belongs that does not contain j. Hence, only demanding 0 i could ever be chosen in a majority coalition that is not a MWC and that does not contain j. Therefore it is better for i to select a \( p_j \) with j in one of the first positions: In fact, with j in one of the first positions i can always demand a positive share in equilibrium. The lower bound 0 could only be reached if \( \delta \) is equal to 1 and if \( p_{0,1}^{-1}(1) = j \). In that case the reservation payoff of j is \( w/q' \) (see part 1 of this proof). If, however, \( \delta < 1 \) and/or if the probability that j will be selected as first mover next period is not 1, then i can demand a difference between \( w/q' \) and which is the simple one \( (x, y, j) \) such that \( y, j < \theta_j \), because by doing so 3 has a profitable deviation to demand \( (x, y, j) \), \( y, j < \theta_j \), which would be strictly preferred by player 2 moving last. Given this, now show that \( (x, y, j) = (0, 1) \) is an equilibrium. If 1 makes this demand, 3 does not have any profitable deviation: The linearity of the utility function implies that if 3 wants to demand more than \( \| \), then she has to compensate 2 accordingly on the policy dimension (and vice versa), without any profit. Could 1 do better? The answer is negative because of the usual argument of proposition 1.

**Q.E.D.**

**APPENDIX D: PROOF OF PROPOSITION 4**

Let us call the three parties 1, 2, 3, with \( 0 \leq \theta_1 < \theta_2 < \theta_3 \leq 1 \). Assume that the head of state chooses the same party as first mover in each round. The consequence is that with \( \delta \in (0, 1) \) the possibility of pushing the game to the next round is never used in equilibrium. At every round the relative bargaining power remains the same. For this reason we can focus on one round and drop the time subscript. Suppose that player 2 is called to move first and chooses 1 as second mover. Consider the subgame in which 2 has already moved (demanding \( x_2, y_2 \)), and 1 has to decide what to do. The outside-option payoff for player 3 is \( 2 - x_2 - \beta(\theta_3 - y_2) \). Thus, the best 1 can do if she decides not to go with 2 is to choose \( (x_1, y_1) \), such that

\[
2 - x_1 - \beta(\theta_1 - y_1) = 2 - x_2 - \beta(\theta_3 - y_2),
\]

that is,

\[
\max_{x_1, y_1} 1 + x_1 - \beta(y_1 - \theta_1) \text{ S.T. } x_1 = x_2 - \beta y_2 + \beta y_1. \tag{7}
\]

There is an infinite number of solutions to equation 7 (one of which is the simple one \( (x_1, x_2, y_1, y_2) \)). All the solutions yield the same outside-option payoff for player 1: \( u_1 = 1 + x_2 - \beta y_2 \). Such a payoff for player 1 is the constraint for player 2 at the initial node. So player 2 maximizes \( 1 + x_2 - \beta(\theta_3 - y_2) \) subject to \( 2 - x_2 - \beta y_2 = u_1 \), which implies \( x_2^* = \frac{1}{2} \). Since any \( y_2 \) would work in terms of inducing 1 to accept, \( y_2^* = \theta_3 \). (The case in which 2 chooses 3 as second mover is completely symmetric to the case just studied; the solution is again \( y_2^* = \theta_3 \) and an equal split of private benefits between the first two movers.) So, given that the head of state wants to have the policy outcome as close as possible to the median voter’s preferred outcome, and given that the center party has the closest position to that, the head of state can achieve the unconstrained optimum by choosing the center party as first mover.

The head of state may choose an extreme party as first mover only if in the continuation equilibrium such a player demands and obtains \( \theta_1 \). In all such cases the equal split result also extends. Consider first the order of play \( p = 1, 2, 3 \). Suppose that 1 chooses \( \theta_1 \), as a policy demand. We need to show that the only compatible demand of private benefits is \( x_1 = \frac{1}{2} \). If this is the demand of player 1, then player 2 does not have a profitable deviation: If 2 demands an \( \epsilon \) more than \( \frac{1}{2} \) in the deviation, she can convince 3 only by increasing \( y_2 \) (with respect to \( \theta_3 \)) of \( \epsilon/\beta \) at least, and hence there is no reason for 2 to try that (this is obviously due to the linearity of the utility function). Given that 2 does not have any profitable deviation, the only thing left to check is that 1 is doing the best possible with such an equal split demand. But this is established with the same logic of proposition 1: If 1 demands \( \frac{1}{2} + \epsilon \) (with \( \epsilon > 0 \)), then 2 can profitably underbid and go with 3, since \( \frac{1}{2} + \epsilon - \theta > \frac{1}{2} + \epsilon, \forall \theta < 2\epsilon \).

What if \( p = 1, 3, 2 \)? Player 1 can never demand \( (x_1, y_1) \) such that \( y_1 < \theta_2 \), because by doing so 3 has a profitable deviation to demand \( (x_1, y_3, y_3 = \theta_2) \), which would be strictly preferred by player 2 moving last. Given this, now show that \( (x_1^*, y_1^* = \theta_3) \) is an equilibrium. If 1 makes this demand, 3 does not have any profitable deviation: The linearity of the utility function implies that if 3 wants to demand more than \( \frac{1}{2} \), then she has to compensate 2 accordingly on the policy dimension (and vice versa), without any profit. Could 1 do better? The answer is negative because of the usual argument of proposition 1.

**Q.E.D.**

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