Bargaining and Agenda Formation in Legislatures

By DAVID P. BARON AND JOHN FEREJOHN*

A legislature is a collection of members choosing among a set of alternatives according to some voting rule. Researchers following Kenneth Arrow (1963) have established that except in special cases, no voting equilibrium exists in this setting, that is, there is no alternative that defeats every other alternative. Moreover, when no voting equilibrium exists, any alternative y may arise as the outcome of the proposal-making and voting process: that is, starting at any alternative x it is possible to construct a sequence of alternatives such that each one defeats its predecessor and leads from x to y. (See Richard McKelvey, 1976.)

Traditionally, these results have been given two interpretations. William Riker (1982), for example, has argued that they imply that the outcomes of voting processes are entirely unpredictable and that, as a result, nothing can be said about the likely outcomes except to remark on their arbitrariness. Others have argued that the absence of a majority rule equilibrium allows a member in the position of an "agenda setter" to guide the voting process to attain any alternative he prefers.

When an agenda is exogenously imposed, however, members will take account of its structure and vote in a sophisticated manner, which, as demonstrated by Robin Farquharson (1969), and Nicholas Miller (1980) results in a unique equilibrium outcomes. Even though "admissible" agendas—agendas that can be formed under the rules of the legislature—always produce unique outcomes, the set of admissible agendas, and therefore of outcomes, is very large. In this sense, classical agenda theory does not possess sharp predictive power for the behavior of voting institutions.

In real legislatures, agendas are not imposed by an external party but are built instead by members of the voting body itself. Agendas are thus viewed as endogenously formed by the members of a voting body who make substantive and procedural motions under prespecified rules. Thus, one would expect the outcome of the voting process to reflect the institutional structure of the proposal generation process: the rules that specify which members are able to put motions on the floor, which amendments if any can be made, and how and when motions may be brought to a vote.

From this point of view, the structure of rules governing agenda formation are crucially important. The purpose of the present paper is to develop a noncooperative theory of majoritarian legislatures that permits the examination of these rules. In particular we focus on two commonly observed rules for proposal generation: the closed rule, in which proposals may not be amended, and the open rule, which permits amendments to motions.

We show that majority voting procedures with endogenous agendas generally produce "essentially unique" outcomes. Moreover, these outcomes are *ex post* asymmetric under either an open rule or closed rules, and that an advantage falls to whomever is recognized first. Finally, we show that, in contrast to models of bilateral bargaining under complete information, proposals are not necessarily accepted immediately.

I. The Model of a Legislature

A. *Legislative Structure*

For analytical convenience we assume that the task before the legislature is to divide a dollar among its members according to

*Graduate School of Business, and Department of Political Science and Hoover Institution, respectively, Stanford University, Stanford, CA 94305. This research has been supported by NSF grant nos. SES-8310597 and IST-8606157.

majority rule. Each member is assumed to have selfish, risk-neutral preferences, representable by a von Neumann-Morgenstern utility function. Preferences and the legislative rules are assumed to be common knowledge, so the model involves full information. Although this model is stylized, the division problem is one in which the members have deeply conflicting preferences, and in which there is no majority rule equilibrium in the standard social choice framework. To simplify the presentation, the legislature will be assumed to have three members.

The legislature is governed by a sequential recognition rule in which members compete for recognition to make a proposal. It will turn out that recognition is valuable in equilibrium and so every member will attempt to be recognized. Therefore, the legislature will have to resort to some rule for deciding who shall have the floor, and since side-payments are prohibited, we assume that the legislature adopts a random recognition rule.

Thus at the beginning of a session, member i has a probability p_i of being recognized, and if recognized by the chair, he may propose a bill that specifies how the dollar is to be divided. This is then the motion on the floor, and the resolution of that motion is governed by the rules of the legislature. Under a *closed rule* the motion is voted on immediately, and if approved the legislative adjourns. If the motion fails, the next session begins with a member being recognized to propose another bill. Under an *open rule*, after a bill has been proposed, another member j is recognized (with probability $p_j/\Sigma_{k \neq i} p_k$,) and he or she may either offer an amendment or move the previous question.[1] If the previous question is moved, the legislature votes on whether or not to accept the proposed division of the dollar. If an amendment is offered by member j, it becomes the question on the floor, and another member l may be recognized (with probability $p_l/\Sigma_{k \neq j} p_k$). If the previous question is moved at this point, the amendment is put to a vote and, if it wins, the bill as amended becomes the new motion on the floor. If the amendment fails, the original bill becomes the question on the floor. The process continues with members proposing amendments or moving the previous question and forcing a vote until the bill itself, as amended, is put to a vote. If the bill is agreed to, the process stops; otherwise, another member is recognized to propose a new bill. Whenever the previous question is moved, voting takes place sequentially and openly. That is, there is a fixed order in which each member must announce his vote on the question before the body and every other member may observe each vote as it is cast. Members thus have full information about previous votes.[2]

Unlike the two-member bargaining problem studied by Ariel Rubinstein (1982), agents need not exhibit impatience to be motivated to approve a bill. If a member fails to vote for a bill, that member runs the risk that others will exclude him or her from the division of the dollar in the next session. In order to increase the generality of our results, however, we assume that agents are impatient and have a common discount factor $\delta \leq 1$. At times, the limit of the division as δ converges to one from below will be studied, which corresponds to the case in which the period between offers (which we call a session) is very short.

In contrast to models in which the agenda is exogenous and hence all members have perfect information about the sequence of votes to be taken, members in the legislature considered here must form expectations about future proposals, motions, and votes. The process of proposal generation and voting yields an extensive-form game with an infinite game tree. A strategy in this game is a prescription of what motion to make at each point at which the member is recognized, and a prescription of how to vote whenever a vote is required.

[1]See Ferejohn, Morris Fiorina, and McKelvey (1986) for a similar specification.

[2]If voting were by secret ballot, the set of subgame perfect equilibria would be very large, and a stronger equilibrium concept such as that of sophisticated equilibrium would be required. See Farquharson and Hervé Moulin (1979).

The game is generated as follows: The chair randomly recognizes a member to make a proposal. This member then may propose a division of the dollar, which will be called a *bill*. The chair then recognizes another member who may either propose an alternative division or require the members to vote on the motion on the floor. If the second member proposes an alternative division, it is regarded as an *amendment* to the bill. The chair then recognizes another member, who is distinct from the second member but not necessarily from the first, who may either propose another division (an amendment to the amendment) or require the members to vote on the previous motion; the amendment. The chair then recognizes another member and so on. The game terminates when a bill, as amended, passes.

A *history*, h_t, of the game up until session t is a specification of who had a move at each previous session and the move selected by each member at every time he had a move to make. If H_t denotes the set of histories, then a *strategy* for member i is a sequence of functions s_t mapping H_t into his or her available actions at t. An important feature of this formulation is that at any time that an agent is to take an action, he or she knows which history has occurred, so the game is one of perfect information.

Members are not able to make binding commitments to vote in a particular manner or to offer a particular proposal. Thus, an equilibrium strategy must be "self-enforcing" in the sense that the member would wish to execute it at each point in the game tree at which he or she has an opportunity to act. Therefore, attention is restricted to subgame perfect equilibria. An equilibrium collection of strategies is subgame perfect if the restriction of those strategies to any subgame constitutes an equilibrium in that subgame. Because votes and proposals occur sequentially and openly, subgame perfect equilibria correspond to dominant solvable solutions (see Moulin). For any particular subgame perfect equilibrium, the *continuation value* of a subgame is defined as the vector of values to the members resulting from the play of that subgame perfect equilibrium strategies.

B. *An Illustration: A Closed Rule with a Finite Number of Sessions*

To illustrate the basic structure of the model, consider the simple case of a three-member legislature governed by majority rule, with equal probabilities of recognition, and in which there is a closed rule that prohibits amendments once a bill has been offered. The agenda will be assumed to be a finite with at most two proposals made prior to adjournment. With a closed rule, the member recognized in the first session may offer a bill specifying how the dollar is to be divided, which must then be voted up or down. If this bill receives a majority vote, the dollar is divided according to the bill and the legislature adjourns. If it is defeated, the second session commences and a member is selected at random to offer a bill. If there is no agreement in the second session, the legislature dissolves, and each member receives zero.

In this case, the subgame perfect equilibria are easily characterized. Note that, if the first offer is rejected and a second (and last) session were to take place, whoever is recognized will propose to take the whole dollar and this proposal will be accepted by the other members who stand to get zero in any case. Thus, if each member has an equal likelihood of gaining recognition in the second session, the continuation value of the game prior to anyone gaining recognition in the second stage must be equal to $1/3$ for each member. Thus, in the first session whoever is recognized can propose to offer $\delta/3$ to one other player and keep $1-\delta/3$ for himself. This proposal will be accepted and the legislature will adjourn immediately.

The important features of this equilibrium are: A) *ex post*, 1) the allocation reflects the majoritarian distribution of power in that only a minimal majority of members receives a positive payoff, and 2) the member recognized in the first session has agenda power and thus receives the largest allocation; B) *ex ante* the symmetry of the legislature is reflected in the fact that every member attaches the same value to the game. Finally, as in the bargaining models of Rubinstein and Kenneth Binmore (1986), the initial offer is accepted and the legislature adjourns after only one bill has been proposed and voted on. This last feature of the equilibrium is not due to impatience but results from the probability that the member will not be recognized in the last session.

The majoritarian equilibrium exhibited here involves the division of the dollar among a minimal majority of the legislature. This minimal majority is not a coalition in the sense that that term is used in cooperative game theory and in the social choice literature. The members of the majority in the legislature considered here act noncooperatively and each finds it in his own interests to act as specified in the equilibrium.

II. A Folk Theorem under a Closed Rule

While the case considered in the previous section has a unique subgame perfect equilibrium, this result does not extend to legislatures that have no limitation on the number of sessions. When the number of sessions is unlimited, any division of the dollar may be supported as a subgame perfect equilibrium if the members are not too impatient. In view of the uniqueness results for two-member bargaining, it is perhaps surprising that multimember bargaining yields complete indeterminacy.[3]

Remark: For the sake of simplicity, this result is stated for a majority-rule legislature but it should be clear from the nature of the argument that such a theorem holds for any voting rule as long as the legislature contains at least three members.

PROPOSITION 1: *For an n member, majority-rule legislature with an infinite number of sessions and a closed rule, if* $\delta > (n+1)/2(n-1)$ *and* $n > 4$, *then any division of the dollar may be supported as a subgame perfect equilibrium. In every equilibrium the first offer is accepted.*[4]

[3] This observation has been made by Maria Herrero (1985) who considered bargaining under a unanimity rule and a fixed order of recognition.

[4] The construction in this proof is essentially the same as that found in Herrero, and is presented in our earlier paper (1986).

The idea of the proof is simple: In order to support an arbitrary division x, a strategy configuration is constructed in such a way that any member recognized is certain to be punished if he deviates by failing to propose the prescribed division x, when required to do so, or by failing to punish someone who deviated before him.

The punishment scheme required for the folk theorem must be "infinitely nested" to allow any alternative to be supported as a subgame perfect equilibrium. Thus, the members of the legislature are able to base their choice at any stage of the game on the whole history of play to that point. If such history dependence is not allowed, the set of alternatives that may be supported is reduced. One way to achieve this restriction is to require members to choose among stationary strategies.

Definition: A strategy is *stationary* if it dictates that a member take the same action in identical subgames.

For example, if, in two different sessions, a member is recognized and there are no motions on the floor, he must make the same proposal in both sessions. Clearly, the strategies required for the folk theorem fail to satisfy this restriction, since what a member is required to propose depends on the history of play leading to the subgame.

III. A Closed Rule and Stationary Strategies in an Infinite Session Legislature

We now provide a general treatment of a legislature with an infinite number of periods in which proposals are considered under a closed rule. A proposal made in a session is then either approved by a majority and the legislature adjourns or it is rejected, and the next session commences with a member recognized at random by the chair. Without sacrifice of generality, we restrict our investigation to the three-member case. Member i has a probability p_i of being recognized and each member employs a common discount factor $\delta \leq 1$. For convenience attention is restricted to the symmetric case in which p_i is equal to $1/3$. The following proposition indicates that a majoritarian outcome results with stationary strategies.

PROPOSITION 2: *For all* $0 < \delta \leq 1$, *a strategy configuration is a stationary subgame perfect equilibrium in an infinite-session, majority-rule legislature governed by a closed rule if and only if it has the following form*:

1. *Each member recognized proposes to receive* $1 - \delta/3$ *of the dollar and offers one other member* $\delta/3$ *of the dollar.*
2. *Each member votes for any proposal in which he receives at least* $\delta/3$.
3. *The strategy configurations are balanced in the sense that each member receives an offer of* $\delta/3$ *from one other member in each session, for example, from the member on his left.*

Proposition 2 provides a rationale for the restriction to stationary strategies when the agenda is governed by a closed rule. The division of the dollar specified in the proposition is the limit of the divisions chosen in finite session games as the number of sessions increases.

Proposition 2 may be thought of as the natural extension of the Rubinstein model to a three-member legislature. As in two-member bargaining theory, the first member recognized proposes a bill that is sufficiently attractive to a majority that it is immediately accepted and the legislature adjourns after one motion. As in the model with finitely many sessions, however, this is due to majority rule rather than impatience and thus holds for all $\delta < 1$.

Unlike the bilateral case Rubinstein studied, division under majority rule is *ex post* asymmetric even if $\delta = 1$. Since a majority can exclude a minority and divide the dollar among themselves, the recognized member has an incentive to exclude as large a minority as possible. A minimal winning majority will thus be formed with the dollar divided among the members of that majority. To determine how the dollar will be divided, consider the case in which $\delta = 1$. Equal division among all members involves $1/3$ for each, so the one-third share of the excluded member is to be allocated between the member recognized and the other member of the winning majority. Under majority rule that member recognized is able to capture the entire share of the excluded member. With impatience that member also captures a premium of $(1 - \delta)/3$ due to the impatience of his partner.

The condition in item 3 of Proposition 2 is a balance property of the stationary strategy equilibrium. If the strategies were not balanced so that each member receives exactly one offer to be in the majority, then the disadvantaged member would be a preferred "partner" in the previous session and would receive offers from each of the other members if they were recognized. This would then bid up his continuation value until he is no longer a preferred partner. Competition to be a preferred partner would then equalize the continuation values.

Remark: In the present case, the requirement that members restrict themselves to stationary strategies implies the balance property of the equilibrium outcome in Proposition 2. For example, if the continuation values of the game after the first session resulting from some subgame perfect equilibrium are $v_i = 1/3$, $i = 1,2,3$, then proposals x^i,

$$\big(x^1 = (1 - \delta/3, \delta/3, 0),$$

$$x^2 = (0, 1 - \delta/3, \delta/3),$$

$$x^3 = (0, \delta/3, 1 - \delta/3)\big),$$

in session one and thereafter the strategies yielding the continuation value form an equilibrium. This equilibrium also has the properties that the first bill proposed is accepted and that the member recognized receives $1 - \delta/3$, one other member receives $\delta/3$, and the other receives zero. Thus, in one sense, the equilibrium is *ex post* majoritarian.

Remark: While Proposition 2 is established in the special case of a three-member, majority-rule legislature, extensions to the n member, majority-rule legislature are immediate: The initial proposer must offer the continuation value δ/n to $(n-1)/2$ members (assuming that n is odd) and keep $1 - \delta(n-1)/2n$ for himself, and, as before, each member must receive the same number of offers in a stationary equilibrium. As n increases, the share of the member recognized approaches $1-\delta/2$, so the value of recognition is decreasing in the size of the legislature. For the case of k member majority rule in which k members are required to adopt a proposal, the member recognized must again offer δ/n to $k-1$ members and keep $1-\delta(k-1)/n$ for himself. Note that as k goes to n, the share retained by the initial proposer goes to $1-\delta(n-1)/n$, which for $\delta=1$ equals $1/n$. This is the unanimity allocation under stationary strategies.[5] As k goes to one, the member recognized receives the entire dollar. This may be thought of as awarding a property right to the member recognized.

IV. Amendments: A Simple Open Rule

With an open rule, motions on the floor may be subject to amendment. An amendment is itself an allocation of the dollar and may be viewed as a substitute for the motion on the floor. The simplest open rule is one in which no more than one amendment may be on the floor at any time. That is, once an amendment is offered it must be disposed of by bringing it to an immediate vote. If the amendment fails, the prior motion remains on the floor, and the next session commences. If the amendment obtains a majority, the amendment becomes the motion on the floor, and the next session commences. At this point another amendment may be offered or the previous question on the bill may be moved.[6] This rule is properly termed an open rule because there is no limit to the number of amendments that may be offered before the bill itself is brought to a vote. In this setting, discounting is assumed to occur whenever a new amendment is moved.

The legislature operating under this open rule may thus be described as follows. Each member has a 1/3 probability of gaining initial recognition. A member recognized may offer a bill. Then, each of the other two members has a 1/2 probability of being recognized for the purpose of making an amendment or moving the previous question. If the previous question is moved, the bill is voted and if approved, the dollar is divided. If the bill is defeated, one of the two members who did not move the question is recognized with equal probability. If an amendment is offered, it must be voted against the bill before another amendment or moving the previous question is in order. The winner then becomes the motion on the floor, and each member other than the amender has a probability of 1/2 of recognition for the purpose of offering an amendment or moving the previous question. The process continues in this fashion until the previous question is moved on a bill and the bill passes.

In this open rule procedure the power of the member recognized is limited because no member will ever be in a position to introduce a bill or an amendment that cannot itself be subjected to an amendment. That is, after an amendment is moved and voted on, another member will be recognized who may introduce an amendment, if that is in order, or may move the question on the bill, which allows yet another member to offer an amendment. Thus, whoever proposes a bill or an amendment must take account of the fact that other members will subsequently be recognized. Another difference between an open rule and a closed rule is that a vote takes place only after a member has been recognized. To simplify the mathematics, a member who is indifferent between two proposals will be assumed to vote for the one proposed last.[7] A stationary equilibrium in a legislature operating under a simple open rule is characterized in the following proposition.

PROPOSITION 3: *In a legislature with a simple open rule, equal probabilities of recognition, and δ near one, a stationary subgame perfect equilibrium strategy configuration has the following form*:

1. *The member recognized first offers* $1-y_1$ *to another member and proposes to keep* y_1 *for himself, where* $y_1=(1-\delta/4)/(1+\delta/4)$.

[5] B. Dutta and Louis Gevers (1981) obtained this result.

[6] In the terminology of Ferejohn et al., such amendment rules exhibit a "depth" limitation. In legislative parlance, no second-degree amendments may be offered.

[7] Otherwise, the set of amendments that defeat the proposal on the floor is open, and the equilibrium concept has to be weakened to that of an ε equilibrium.

2. *If the next member recognized, has been offered at least* $1-y_1$, *she moves the previous question and the question is approved; otherwise she proposes to offer* $1-y_1$ *to the member that did not make the previous motion and to keep* y_1 *for herself.*[8]

The following are the important features of the stationary strategy equilibria for this open rule when δ is near one. First, *ex post*, the dollar is divided among a minimum winning coalition. Second, and unlike the usual case in bargaining models with complete information, the initial offer is not necessarily accepted. Rather, with some probability, an amendment is offered and the legislature continues to the next session. When impatience is low, the probability is thus positive that the legislature will not reach an agreement in any finite number of sessions. Thus, if $\delta < 1$, the sum of the continuation values is less than one. Third, the power of the member initially recognized is greater the greater is the impatience of the members. Fourth, while amendments may be offered in equilibrium, the initial proposer is still advantaged. Thus, as in the case of the closed rule, recognition is valuable. Fifth, recognition is not as valuable as under a closed rule. For example, for $\delta = 1$, $y_1 = 3/5$ and so the value of the game to the first member recognized is 2/5.

V. Conclusions

Our purpose in this paper has been to introduce a game-theoretic model that permits the study of agenda formation in legislatures. To do this we made a number of simplifying assumptions that deserve close examination in future work. The principal result of this paper is that, in a complete information setting, legislative outcomes with endogenous agenda formation are quite determinate. This determinacy follows from the bargaining structure of the agenda formation process and is due largely to the ability of members to make proposals. The actual payoffs depend also on the voting rule as well as on the rules of agenda formation. These findings stand in sharp contrast to the results of agenda voting models that do not allow for endogenous agenda formation.

[8] The proof is provided in our earlier paper.

REFERENCES

Arrow, Kenneth, *Social Choice and Individual Values*, New Haven: Yale University Press, 1963.

Baron, David and Ferejohn, John, "Bargaining in Legislatures," unpublished, Stanford University, 1986.

Binmore, Kenneth, "Bargaining and Coalitions," in Alvin Roth, *Game-Theoretic Models of Bargaining*, Cambridge: Cambridge University Press, 1986.

Dutta, B. and Gevers, Louis, "On Majority Rules and Perfect Equilibrium Allocation of a Shrinking Cake," unpublished, Namur University, 1981.

Farquharson, Robin, *Theory of Voting*, New Haven: Yale University Press, 1969.

Ferejohn, John, Fiorina, Morris and McKelvey, Richard, "Sophisticated Voting and Agenda Independence in the Distributive Politics Setting," *American Journal of Political Science*, forthcoming 1987.

Herrero, Maria, "A Strategic Bargaining Approach to Market Institutions," unpublished doctoral dissertation, London University, 1985.

McKelvey, Richard, "Intransitivities in Multidimensional Voting Models and Some Implications for Agenda Control," *Journal of Economic Theory*, June 1976, *12*, 472–82.

Miller, Nicholas, "A New Solution Set for Tournaments and Majority Voting," *American Journal of Political Science*, February 1980, *24*, 68–96.

Moulin, Hervé, "Dominance Solvable Voting Schemes," *Econometrica*, November 1979, *47*, 1337–51.

Riker, William, *Liberalism Against Populism*, San Francisco: Freeman, 1982.

Rubinstein, Ariel, "Perfect Equilibrium in a Bargaining Model," *Econometrica*, January 1982, *50*, 97–109.

LINKED CITATIONS

Bargaining and Agenda Formation in Legislatures
David P. Baron; John Ferejohn
The American Economic Review, Vol. 77, No. 2, Papers and Proceedings of the Ninety-Ninth Annual Meeting of the American Economic Association. (May, 1987), pp. 303-309.
Stable URL:
http://links.jstor.org/sici?sici=0002-8282%28198705%2977%3A2%3C303%3ABAAFIL%3E2.0.CO%3B2-J

[Footnotes]

[2] **Dominance Solvable Voting Schemes**
Hervé Moulin
Econometrica, Vol. 47, No. 6. (Nov., 1979), pp. 1337-1351.
Stable URL:
http://links.jstor.org/sici?sici=0012-9682%28197911%2947%3A6%3C1337%3ADSVS%3E2.0.CO%3B2-S

References

A New Solution Set for Tournaments and Majority Voting: Further Graph- Theoretical Approaches to the Theory of Voting
Nicholas R. Miller
American Journal of Political Science, Vol. 24, No. 1. (Feb., 1980), pp. 68-96.
Stable URL:
http://links.jstor.org/sici?sici=0092-5853%28198002%2924%3A1%3C68%3AANSSFT%3E2.0.CO%3B2-P

Dominance Solvable Voting Schemes
Hervé Moulin
Econometrica, Vol. 47, No. 6. (Nov., 1979), pp. 1337-1351.
Stable URL:
http://links.jstor.org/sici?sici=0012-9682%28197911%2947%3A6%3C1337%3ADSVS%3E2.0.CO%3B2-S

NOTE: *The reference numbering from the original has been maintained in this citation list.*

LINKED CITATIONS

Perfect Equilibrium in a Bargaining Model
Ariel Rubinstein
Econometrica, Vol. 50, No. 1. (Jan., 1982), pp. 97-109.
Stable URL:
http://links.jstor.org/sici?sici=0012-9682%28198201%2950%3A1%3C97%3APEIABM%3E2.0.CO%3B2-4

NOTE: *The reference numbering from the original has been maintained in this citation list.*