

Game Theory
Evolutionary game theory

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Abstract

This paper describes the main concepts from biological game theory and some modifications that have been suggested to make them more applicable in economic contexts. Several examples are given to illustrate that these concepts allow the formalization of intuitive insights that could not be formalised by traditional, rationalistic, game theory. In particular, the evolution of language, the evolution of cooperation and convergence towards risk-dominant equilibria are discussed.

Key words: Evolution; Noncooperative game theory; Evolutionary stable strategies; Equilibria

JEL classification: C7; D8

1. Introduction

Kreps (1990) argues that non-cooperative game theory has been of value to economists because it has given them a language for modelling strategic interaction and because it has enabled the transfer of intuitive insights from simple contexts to more complicated ones. Most insights have been obtained by using the concept of Nash equilibrium and the increased popularity of game theoretic ideas has led to an increased awareness of the fact that the answer to the question 'When and why is equilibrium analysis appropriate?' is not a straightforward one. Unfortunately, game theory has not yet offered useful alternatives to Nash equilibrium and until recently there has been little progress on the front of equilibrium selection, i.e. what to do when there are multiple equilibria (Kreps, 1990).

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Nash (1950) already provided two interpretations of his concept. The first, the 'mass-action' interpretation, assumes that there is a population of agents for each player role in the game. Participants accumulate empirical information on the attractiveness of the various pure strategies and, if play settles down, it must be at a Nash equilibrium. [See Weibull (1993) for a nice quotation from Nash's thesis.] Shapley (1964) however, showed that play need not settle down, there may be a limit cycle. The second interpretation views equilibrium as a 'self-enforcing agreement' (or rational prediction). If a prediction on the basis of rationality is unique, then in order for it to actually materialize, if it is known, it has to be a Nash equilibrium. [See Aumann and Brandenburger (1991) for how much knowledge is needed.] Nash (1950, p. 23) already remarks that this is 'quite strongly a rationalistic and idealising interpretation', and that for it to be valid, it is also essential to resolve the equilibrium selection problem. Hence, its relevance for economics may be questioned.

A third justification for Nash equilibrium has its origins in biology and has been obtained first in Maynard Smith and Price (1973). In this interpretation there is no conscious choice at all: Individuals are programmed to play certain strategies, more successful strategies reproduce faster than others so that eventually only the most successful strategies survive: If the population reaches a stable state, all strategies must do equally well, hence the state must be a Nash equilibrium. An advantage of this biological approach is that it not only specifies the stable outcomes, but that it gives an explicit process by means of which these outcomes could possibly be reached. Of course, economists are well aware of the pitfalls involved in importing ideas from biology into their own field (Penrose, 1952; Hirschleifer, 1977) and it certainly is not clear to what extent the biological 'replicator equations' are relevant in economics. At the same time these ideas are so attractive - they allow to formalize intuitive insights that traditional analysis could not formalize - that it is difficult to resist them. In this paper we describe these basic ideas, indicate some modifications that have been suggested to make them more applicable in economic contexts, and illustrate some of the insights that they provide.

2. Basic concepts from biological game theory

Consider a large (infinite) population, each member of which is programmed to play a certain strategy s from a set S . Each period individuals from the population are randomly matched in pairs and if an s -individual meets a t -individual, the payoff (number of offspring) to s is $u(s, t)$, where $u: S \times S \rightarrow \mathbb{R}_+$ is a given fitness function. A monomorphic population in which all individuals play s^* is stable, i.e. resistant against mutations, if each

mutant s that enters in the population with small frequency is selected against. Hence, s^* is stable if in the perturbed population in which a small fraction ε plays s , the individuals playing s^* have more offspring, i.e.

$$(1 - \varepsilon)u(s^*, s^*) + \varepsilon u(s^*, s) > (1 - \varepsilon)u(s, s^*) + \varepsilon u(s, s) \quad (\text{all } s \neq s^*, \text{ small } \varepsilon). \quad (2.1)$$

Note that (2.1) assumes uniform random mixing, i.e. the probability of meeting some type does not depend on your own type, there is no viscosity: If mutants would mainly interact with themselves, they might enter more easily (Hamilton, 1964; Myerson et al., 1991). Also stability is tested only against single mutants, mutants are assumed to appear infrequently so that each one is driven out before another appears. Obviously (2.1) is equivalent to the following pair of conditions:

$$u(s, s^*) \leq u(s^*, s^*) \quad (\text{all } s), \quad (2.2)$$

$$\text{if } u(s, s^*) = u(s^*, s^*), \text{ then } u(s^*, s) > u(s, s) \quad (\text{all } s \neq s^*), \quad (2.3)$$

and s^* is said to be an *evolutionarily stable strategy* (ESS) if these conditions hold. (Maynard Smith and Price, 1973; Maynard Smith, 1982). Condition (2.2) shows that (s^*, s^*) is a Nash equilibrium if s^* is an ESS and because of (2.3) not every symmetric Nash equilibrium corresponds to an ESS. In fact, every ESS induces a proper (hence, perfect) equilibrium (Van Damme, 1987, Theorem 9.3.4).

Next assume the population is polymorphic and let $n_i(t)$ be the number of s -individuals at time t . Then, one period later, the number of \bar{s} -individuals is given by $n_i(t+1) = n_i(t)(1 + \sum_s p_s u(\bar{s}, s))$ where $p_s = p_s(t)$ denotes the fraction of s -types in the population, i.e. the probability of meeting such a type. Assuming that the time between periods is small, we can rewrite the law of motion as $\dot{n}_i = n_i u(\bar{s}, p)$ where $u(\bar{s}, p) = \sum_s p_s u(\bar{s}, s)$ denotes the expected fitness of \bar{s} -types in the population characterized by p . Writing $n = \sum_s n_s$ and differentiating the identity $p_s n = n_s$, we obtain the following dynamic for the population proportions:

$$\dot{p}_s = p_s(u(s, p) - u(p, p)) \quad (\text{all } s), \quad (2.4)$$

where $u(p, p) = \sum_s p_s u(s, p)$ denotes the average fitness of the population. Eq. (2.4) is called the *replicator equation*. It expresses the idea that strategies grow in the population if they do better than average, strategies that do best grow fastest. One immediately sees that a Nash equilibrium is a stationary point of the dynamical system. Conversely, each stable stationary point is a Nash equilibrium and an asymptotically stable fixed point is a perfect equilibrium (Bomze, 1986). In fact, if we allow for mixed strategies to be inherited (as we implicitly assumed above) then asymptotically stable fixed points of the replicator dynamics correspond exactly to ESS (Bomze and

	C	D
C	2 2 0 v	
D	v 0 1 1	

Fig. 1. Stag hunt game $g(v)$ ($0 < v < 2$).

Van Damme, 1992; Hines, 1980; Zeeman, 1981). (If only pure strategies can be inherited, being an ESS is sufficient, but not necessary for asymptotic stability (Taylor and Jonker, 1978).

Let us conclude this section with an example. In the stag hunt game $g(v)$ of Fig. 1 both C and D are ESS. The replicator dynamic for this special case is $\dot{x} = x(1-x)((3-v)x-1)$ where $x = p_c$ is the fraction C in the population. Hence, the basis of attraction of C is $x > (3-v)^{-1}$. If the initial fraction of cooperators is less than $c(v) = (3-v)^{-1}$, the population moves to 'all D'.

3. Issues and problems in an economic context

As we saw biological game theory has attractive things to offer: A refinement of Nash equilibrium that is based on minimal rationality requirements and an explicit dynamic justifying this static equilibrium notion. Especially in those areas of traditional game theory where the current state of affairs is unsatisfactory, one would like to borrow and profit from the ideas and concepts of the biological branch of game theory. Specifically, one may think of the following issues:

1. *Evolution of rationality.* Does evolution wipe out irrational behavior, i.e. are individuals that play dominated strategies eliminated from the population? Can the process of iterative elimination of (weakly) dominated strategies be justified by appealing to evolutionary processes?
2. *Evolution towards equilibrium.* In an economic context, can the use of Nash equilibrium be justified by appealing to an evolutionary process? Hence, does evolution force coordination of individuals' (rational) actions? In extensive form games, does evolution lead to more refined notions such as subgame perfect equilibria?
3. *Evolution of norms.* Does evolution lead players to a specific type of equilibrium, i.e. does evolution resolve the problem of equilibrium selection? In particular what can we say about the evolution of language? What about the evolution of cooperation in repeated games? In static games, do evolutionary pressures lead to efficient equilibria?

Of the papers in this session, Weibull's concentrates on the first issue, the Binmore/Samuelson paper focuses on the second, while this paper will

attempt to illustrate some aspects related to the third issue. Of course, it is not clear whether the concepts from Section 2 can be imported into the economic domain without substantial modification. The replicator dynamics (2.4) may be inadequate to model social interaction in which individuals possess some consciousness and try to use available information to their advantage. Obviously, utility is different from fitness and cultural evolution (through learning, imitation and experimentation) may be governed by a different law of motion (Boyd and Richerson, 1985; Selten, 1991). Furthermore, also the assumption of uniform random matching that underlies (2.4) may be appropriate only in a limited number of economic contexts. Hence, a considerable amount of technical development may be needed before one can sensibly claim that these ideas and concepts are relevant for applications in economics. Some work has started in this direction.

An important limitation of the basic model is that it only deals with two-person, symmetric, static interactions. When players take different roles (such as buyers and sellers), we have an asymmetric context with different populations. Selten (1980) showed that, in the asymmetric case, the conditions analogous to (2.2), (2.3) can be satisfied only at a strict Nash equilibrium. (If there would be an alternative best reply to the equilibrium, a mutant playing this best reply could invade since it would never meet itself.) Many games do not admit such equilibria, hence, they fail to have ESS. Nonexistence is even more prevalent in extensive form games: An ESS has to reach all information sets in order to exclude alternative best responses (Selten, 1983). Theorists have been reluctant to give up the idea of equilibrium and they have come up with concepts with somewhat better existence properties. Maynard Smith already introduced the notion of a *neutrally stable strategy*, which is defined by replacing the strict inequality in (2.3) by a weak one. Also set-valued concepts have been introduced. S^* is an ESS set if it is a minimal set such that (2.1) is satisfied for all $s^* \in S^*$ and all $s \notin S^*$, hence, it is a minimal noninvasion set. [Cf. Thomas (1985) for a related concept.]

It has also been argued that (2.3) might be too stringent a requirement in economic contexts. This condition requires stability against all mutants, including 'stupid' ones, and one might argue that, when mutants arise because of conscious experimentation, such stupid ones will not be introduced in the population and, hence, pose no threat. We might be satisfied, therefore, if stability against 'sensible' mutants is guaranteed. The following example (Swinkels, 1992) may make this more clear. In fig. 1, let $v=3$ so that the game becomes a prisoners' dilemma. Suppose individuals have the choice between taking up an outside option O and playing the game and that, if at least one individual takes up O, each individual has payoff 3/2. All Nash equilibria result in (O, O) but there is no ESS: A population of 'all O' can be invaded by a C-mutant. The mutant does equally well against O as O does

and it does strictly better when it meets itself. However, a D -mutant can prey on the C -mutant and it might be that, if agents have consciousness, D 's are introduced as soon as C 's are around. In this case, C 's are eliminated immediately and they pose no threat. Swinkels (1992) defines 'sensible' mutants as mutants that are best responses to the 'perturbed' population in which they are present in small numbers and an *equilibrium evolutionary stable strategy*, EES, is a strategy that is stable against such mutants. He also defines the set valued analogue of this concept.

4. Two applications using the static approach

It is an intuitive idea that communication and/or repetition helps players to coordinate on efficient equilibria. Although (D, D) might make perfectly good sense in the game $g(v)$ of fig. 1 if players cannot communicate, it would seem that players would talk themselves into the efficient equilibrium (C, C) if they could talk. [However, see Aumann (1992).] Similarly, there is the intuition that repeated play not only allows players to cooperate but that it even forces players to do so (Axelrod, 1984; Axelrod and Hamilton, 1981). It has, however, proved hard to formalize these intuitions and the formal literature dealing with these topics has been somewhat disappointing. For 'repeated games', the 'folk theorem' tells us that basically anything can happen: Repetition allows more efficient outcomes to be reached, but also less efficient ones. Communication games are plagued by 'babbling equilibria': If everybody else talks nonsense and refuses to listen, each player can do no better than talk nonsense and refuse to listen, hence, play will be as if there was no communication. In this section we indicate how the evolutionary approach can yield results that are more in line with our intuition.

4.1. The evolution of language

Consider the coordination game from fig. 1 and extend this game by allowing players to exchange messages (from some set M) before the play of the game. A strategy $s = (m, f)$ now consists of a message m together with a rule $f(\cdot)$ telling which action $f(m')$ to choose in response to the opponent's message m' . Using a message is assumed to be costless. [Hurkens (1993) considers the case of costly messages. He derives an efficiency result for curb equilibria (Basu and Weibull, 1991) and for persistent equilibria (Kalai and Samet, 1984). The evolutionary foundations of these concepts still have to be investigated.] Obviously, if M contains at least two messages, no pure strategy s^* can be an ESS or an EES: There can be drift at each unused message. Kim and Sobel (1991) work with EES sets and establish existence and efficiency for EES sets. For more general games there appears to be a

trade-off between efficiency and existence – one might lose one or the other – so more work is needed. I now sketch the essential part of the Kim/Sobel construction. I restrict myself to pure strategies – to obtain the result for mixed strategies, two populations are needed, see Kim/Sobel. [For related results, see Bhaskar (1992), Fudenberg and Maskin (1991), Matsui (1991) and Wärneryd (1991)].

Consider a population that plays the inefficient strategy D in the communication game. We may assume that there has been drift such that there is at least one unused message m' . A mutant may now use this message to signal its intention to switch to the more efficient strategy C . Hence, the mutant plays $s' = (m', f)$ where $f(m') = C$ and $f(m) = D$ for each message m that is used by the original population. Whether the mutant is successful or not depends on how the original population reacts to the 'unconventional' signal m' : If m' is treated as if it were an equilibrium message, then s' does better than members of the original population; if the use of m' is punished, then the mutant does worse and dies out. Since, in the original population, the response at m' can drift freely as long as there are no mutants around, it is possible that the mutant enters exactly when the original population had drifted to not punishing m' , hence, it is indeed possible for the mutant to take over. [Actually, if we would enlarge the model by taking the complexity of strategies into account – more complex strategies being costlier than simpler ones – then the original population would necessarily move to not punishing m' and the mutant s' would take over more easily (Wärneryd, 1993).] Hence, the inefficient equilibrium is not stable and only the efficient equilibrium can be stable. Kim and Sobel (1991) show that there indeed is an EES set with outcome (C, C) .

4.2. The evolution of cooperation

In fig. 1, let $2 < v < 4$, so that the game is a prisoners' dilemma. Let this game be repeated infinitely often with players evaluating streams of payoffs according to the limit of the means. Aumann (1981) considered the case where players are restricted to simple reactive strategies. Such a strategy specifies an action with which to start as well as a rule $f(\cdot)$ of how to continue after each action of the opponent in the past period. As there are eight such strategies, we obtain an 8×8 matrix which can, by repeated elimination of weakly dominated strategies be reduced to the strategy $(C; C, D)$, i.e. start by cooperating and in each later period do what the other did in the previous period. Hence, only tit-for-tat survives. See Weibull (1993) for the extent to which such iterative elimination corresponds to an evolutionary process. Note that the 8×8 game does not have an ESS but that tit-for-tat is neutrally stable.

Binmore and Samuelson (1993) investigate the consequences of allowing more general strategies that can be represented as finite automata. Following Abreu and Rubinstein (1988), they assume that complexity costs enter payoffs in lexicographic fashion. Now a population consisting of tit-for-tat players is no longer stable: The strategy 'always cooperate' does strictly better since it has lower complexity. But, of course, this strategy can be invaded as well. Nevertheless, Binmore and Samuelson show that a population can be stable only if all its members cooperate.

Fudenberg and Maskin (1990) also work with strategies of finite complexity but they do not take complexity costs into account. They, however, assume that mistakes might be made in executing the strategy, i.e. if the intended action is a , something might go wrong and the actual action might be $a' \neq a$. Such mistakes allow cooperative strategies to invade a population playing the game inefficiently. The essential idea is similar to the one discussed in the previous subsection. Consider a monomorphic population programmed as a finite automaton. The population goes through finitely many different states and all of these states are reached with positive probability because of the mistakes. In the state with the lowest average payoff, deviating mutants cannot be punished. Hence, if the payoff in this state would not be efficient, the mutant could gain by identifying itself in this state and by switching to cooperation if it meets itself. Hence, only cooperation is neutrally stable. [Maskin (1993) shows how to extend the argument to discounted games. In this case some viscosity (i.e. clustering of mutants) is needed in order to force cooperation.]

5. Examples with noisy evolutionary dynamics

The reader is referred to Weibull (1993) for results concerning the deterministic replicator dynamics (2.4). Any strict Nash equilibrium is an asymptotically stable stationary point of this dynamic, hence, the evolutionary process does not help in selecting among such equilibria. Foster and Young (1990) and Fudenberg and Harris (1992) show that a particular selection may be obtained by adding stochastic noise to the system. We illustrated the basic ideas by considering the mathematically more tractable case of a finite population in discrete time. The model is inspired by Kandori et al. (1993). [Also see Young (1993a, b) who applies the same techniques to analyze a noisy version of the Brown/Robinson process.]

Assume the stag hunt game for fig. 1 is played by a large but finite population of size N , an even number. In each period $t = 1, 2, \dots$, the members of the population are randomly matched in pairs to play. The state of the system at t is characterized by how many members, c_t , cooperate at that time. Assume that at each point in time with positive probability at least

one member observes the current state of the system, that each individual who observes the state at t chooses, at time $t+1$, a best response to c_t and that players who do not receive new information do not switch actions. The evolution of the system is now described by a Markov chain. We have that, if $c_t < (N-2)/(3-v)$ [resp. $c_t > (N+1)/(3-v)$] a person who is informed at t chooses to play D (resp. C) at time $t+1$. Hence, the system has two stable stationary states corresponding to the two strict equilibria of the game. For large populations, we have that, if the initial fraction of cooperators is more than $c(v) = (3-v)^{-1}$, we converge to 'all C ', otherwise we converge to 'all D '. The result is in line with the one obtained by (2.4).

Now add noise to the system. Suppose players make mistakes in executing their strategies. When a player intends to play a , he actually plays a' with small, but positive probability ε , with mistake probabilities of different players being independent. The Markov chain is now ergodic, all states have a strictly positive probability of being observed in the limit as time tends to infinity. When ε is very small, however, it is much more likely to observe the system in the states 'all C ' or 'all D ' than in any other state. (I am assuming that ε is of smaller order than the probability that at least one player observes the state of the system.) Because of the mistakes ('mutations') the system can move from 'all C ' to 'all D ' and vice versa. To move away from 'all C ' we need the simultaneous mutation (to D) of a fraction of at least $1-c(v)$ of the population, while to move away from 'all D ', the simultaneous mutation of a fraction $c(v)$ is needed. If $1-c(v) < c(v)$, then the second possibility is less likely by an order of magnitude than the first, hence, in this case an outside observer will mostly see the system in 'all D '. If $c(v) < 1/2$, then, on the other hand, we will mostly see 'all C '. Hence, if we take the limit for ε to zero, we see that the system will be in 'all C ' if $v < 1$ and in 'all D ' if $v > 1$. In the long run, the system is observed to be in the risk-dominant equilibrium (Harsanyi and Selten, 1988).

Of course, it might take very long to reach the long-run equilibrium. Ellison (1992) notes that speed of convergence depends on the structure of social interaction. Assume that the N individuals live along the shores of a lake and that each individual is matched either with his left or with his right neighbor. If $v > 1$, then a person who has at least one neighbor playing D finds it optimal to play D as well, hence, to move from 'all C ' to 'all D ' only one mutation is needed. Once there is one D , the regular dynamics takes the system to 'all D '. Hence, the system will move quickly from 'all C ' to 'all D '. It can leave from there only if two next neighbors mutate at the same time and this is unlikely. Hence, 'all D ' is stable and this state is reached quickly.

The above stories justify equilibrium selection according to the risk-dominance criterion. It is easy to modify the dynamics so as to obtain selection of the Pareto-efficient equilibrium (Vega-Redondo, 1993). Return to the case with uniform random matching but assume the following infor-

mation structure and adjustment process. With positive probability a person sees the outcome of a match in which he did not take part. If players in this match coordinate on the same action and if this action yields a higher payoff than what the person himself gets, he switches to it, otherwise he stays put. With execution errors ('mutants') as above, we only need two mutants in order for the dynamics to take us away from 'all D '. As at least three mutants are needed to upset 'all C ', the long-run equilibrium is 'all C '.

Another modification of the process yields the outcome 'all D ' as long as $v > 0$. Assume a player observes only the payoffs in his own match and that he mimics the action of his partner if this partner is more successful. Then any C -type that is matched with a D -type switches to D , so that only one mutant is needed to upset 'all C ' and, hence, the long-run equilibrium is 'all D '.

6. Conclusion

The previous section has shown that the results – the equilibria – depend on the evolutionary dynamics. Our motive for including these examples was to illustrate just this phenomenon, which forces us to discuss (and study) which processes make sense in which settings. In my opinion, evolutionary game theory should not be judged by the results it has yielded thus far but by the promise it holds for the future. It enables a shift in emphasis, a move from static equilibrium to dynamic processes, a step towards a richer theory. What is striking about the work done to date in this area is especially the crude modelling of boundedly rational behavior and of innovations ('mutants') and the constancy of the underlying game through time. For a richer modelling of behavior rules and innovations, the classifier systems of Holland (1992) offer an attractive alternative and I refer to Marimon et al. (1990) for an application of these to the evolution of money. It is less clear how to deal with the second issue. Recalling that the success of man lies mainly in its ability to alter his environment, i.e. to change the rules of the game, makes it clear that a study of the evolution of the rules of the game is no less of a pressing, and challenging issue.

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