

# Idiosyncratic Investments, Outside Opportunities and the Boundaries of the Firm

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# Abstract

This paper adopts the incomplete contracting perspective to study a firm's continuous choice between producing an essential input in-house (full integration), buying it from an outside supplier (non-integration) and doing a combination of both (tapered integration), when (i) an idiosyncratic capacity investment is required to produce the essential input and (ii) under non-integration outside opportunities are better. It is shown that the firm's boundary choice depends crucially on its commitment power. If the firm can precommit to a particular provision mode, tapered integration will be chosen more frequently. Also, with commitment power the firm will never subcontract only a small portion of its input needs. In-house capacity is smaller and outside capacity larger if the firm can precommit. Total capacity is never larger in the commitment than in the non-commitment case.

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# 1 Introduction

Until the mid-nineties, most of the buns used by *McDonald's* in Austria were imported from Germany. The long way of transportation and the resulting logistical problems were a thorn in the flesh of Andreas Hacker, then active head of *McDonald's-Austria*. He urged the small Austrian baker Kurt Mann into a daring investment. Mann was supposed to build a complete new burger-buns-factory near Vienna (Austria) at the cost of 140 million Austrian Schilling (at that time about 14 million dollars). The catch was: the 140 million ATS capacity investment was highly idiosyncratic (the alternative use value of standardized *McDonald's* buns is fairly low)<sup>1</sup> and *McDonald's* refuses to make binding commitments with its suppliers on principle: no contract, no guarantees for purchasing, not even a written commitment guaranteeing not to quasi expropriate the future subcontractor *ex post* with future investments in capacity by *McDonald's* itself. To quote Martin Knoll, the current chief of *McDonald's-Austria*: “Contracts are hollow words, we do not like to deal with voluminous contracts”.<sup>2</sup> Mann had to take an enormous risk. However, he decided in favor of the investment. Today his firm has about 320 employees, it delivers about ninety million buns per year to *McDonald's* and its yearly turnover amounts to 300 million ATS (about 22 million dollars).

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<sup>1</sup>*McDonald's* buns are a high-tech product and very special machines are needed for their production: The maximal deviation from the exact directories for the buns height is one millimeter. Each bun must weigh exactly 60 grams, etc.

<sup>2</sup>The original German cite (“Verträge sind Schall und Rauch, wir beschäftigen uns nicht gern mit voluminösen Verträgen”) and many other details of our story in the main text originate in an article by Karl Riffert with the title “Mann o Mann” in issue 2/2000 of the Austrian business magazine *Trend*.

The *McDonald's* example raises several interesting questions. Among others: Why was no contract agreed upon, specifying in advance the quantity and quality of buns to be delivered for all realizations of demand? Why did the future subcontractor trust in *McDonald's* to keep its non-enforceable promise not to provide additional capacities itself to produce buns in the future? And finally: why did *McDonald's* not make this idiosyncratic investment itself?

The present paper focuses on the last of these questions: it studies a firm's choice between producing an essential input in-house (full integration), buying it from an outside supplier (non-integration) and doing a combination of both (tapered integration) in presence of idiosyncratic investments in capacities. To capture the situation described above we adopt the incomplete contracting perspective of Grossman and Hart (1986) and Hart and Moore (1988).<sup>3</sup> That is, we assume (1) that high transaction costs prevent the parties from writing detailed *ex ante* contracts; (2) that *ex ante* profit sharing agreements are infeasible since the relevant choice variables are unverifiable; and (3) that *ex post* trading decisions are contractable.

In such a situation it is well known that *ex post* bargaining involves the threat of hold-up leading to insufficient *ex ante* investment in the relationship-specific capital.<sup>4</sup> Thus, idiosyncratic investment together with contractual incompleteness provides an internalisation advantage. So, for subcontracting to be optimal in some circumstances there must be an offsetting cost of in-

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<sup>3</sup>See Hart and Moore (1999) and Maskin and Tirole (1999) for a beginning of a rigorous foundation of the theory of incomplete contracts based on the assumption that complicated states of nature cannot be verified.

<sup>4</sup>See e.g. Klein, Crawford and Alchian (1978), or Williamson (1979, 1985).

house production.

An often discussed offsetting cost of integration is that under non-integration outside opportunities are better.<sup>5</sup> In the present paper we do not explicitly formalize the loss-of-the-outside-opportunities cost of integration, although this could easily be done.<sup>6</sup> Instead, we use the short-cut assumption that the independent subcontractor (but not the integrated firm) has access to an intermediate good market where he can sell the input at a constant price. To capture the idea that the capacity is specifically designed for producing the input needed by the firm (idiosyncratic investment) we assume that the subcontractor is unable to recover his cost by serving only the intermediate good market.<sup>7</sup> We use the resulting trade-off between transaction cost benefits and

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<sup>5</sup>The idea that independent subcontractors have better outside opportunities than vertically integrated firms has been around in literature at least since Arrow (1975). In a large part of the literature this basic idea is hidden behind other assumptions: Porter (1980), Williamson (1985) and Lewis and Sappington (1991), for example, assume that outside subcontractors have a production cost advantage compared to fully integrated firms. This production cost advantage is motivated by the argument that subcontractors are often able to aggregate diverse demands, thereby realizing economies of scale and scope. This argument assumes, of course, that vertically integrated firms are unable to do the same, i.e., that their outside opportunities are worse.

<sup>6</sup>To see this suppose that idle capacities can only be used to produce an alternative input which is useless outside the respective industry. Then a vertically integrated firm would have to sell the input to rival firms. Further suppose that the quality of the input can be reduced by the producer without cost. Then integration may cause a firm to supply its downstream competitors with lower quality than a technologically inferior independent producer would supply. In such a situation integration leads non-integrated downstream firms to shift their purchases away from the integrated firm.

<sup>7</sup>The idea behind this assumption is that idle capacities can only be used to produce

outside opportunity losses of vertical integration to study how changes in the subcontractor's bargaining power and his outside opportunities affect a firm's boundary choice.

We show that for each outside opportunity (that is, for each price at the intermediate good market) there exists a unique critical level of bargaining power for the subcontractor such that full integration prevails if the subcontractor's bargaining power is smaller than this critical level and tapered integration whenever it is larger. Non-integration, although always the socially optimal provision mode, never emerges as an equilibrium outcome.

Also, neither the firm's nor the subcontractor's profit is monotone in the subcontractor's bargaining power: If the subcontractor's bargaining power is very low (very high, respectively) a Pareto improvement can be obtained by raising (reducing) it. In an intermediate range, the subcontractor may prefer to have less power while the firm would prefer that the subcontractor is tougher!

It turns out that another important factor, namely whether the firm can precommit to a particular capacity level or not, affects the firm's boundary choice.<sup>8</sup> If the firm can precommit, then tapered integration will be chosen an alternative, low margin input which can more efficiently be produced with a different technology. (For instance, production of standard burger-buns does not require the special machines used by Mann to satisfy *McDonald's* input requirement.) Our assumption has the implication that the subcontractor would not invest in capacity, but for the prospect of selling a significant amount of the input to the downstream firm. Williamson (1985, p.95) calls this kind of asset specificity "dedicated asset specificity".

<sup>8</sup>In the *McDonald's* example reputational effects seem to allow the firm to effectively precommit itself to a certain capacity level: *McDonald's* faces the same situation re-

more frequently. Also, with commitment power the firm will never subcontract only a small portion of its input needs. In-house capacity is smaller and outside capacity larger if the firm can precommit. Total capacity is never larger in the commitment than in the non-commitment case.

A model of the boundaries of the firm displaying a similar basic trade-off as our model (namely that between governance cost advantages and production cost disadvantages of vertical integration) has earlier been studied by Williamson (1985). A main difference to the present work is that Williamson does not explicitly deal with the parties' investment decisions. These decisions enter rather indirectly through the specification of governance costs. By contrast, we explicitly model the strategic interactions of the parties at the investment stage of the game thereby deriving the "governance costs" endogenously.

The present paper is also related to Grossman and Hart (1986) and Hart and Moore (1990)'s incomplete contracting theory of the firm. Consistent with this work we identify a firm with the assets that its owner controls and take the position that ownership confers residual rights of control, i.e., the right to decide how the firm's assets are to be used if no *ex ante* contract has been signed. We depart from the Grossman, Hart and Moore (GHM) framework by assuming that outside opportunities change by bringing assets inside the firm. Another major difference concerns the investment/integration decisions. While in the GHM framework the degree of vertical integration is co-peatedly in many different countries. In such a context it is well known that the firm may be willing to incur the short-run cost of not adjusting its capacity *ex post* to get the Stackelberg payoff in future relationships.

operatively determined *ex ante* followed by the non-cooperative investments of the two parties, the non-cooperative investments themselves determine the degree of vertical integration in our model. Also, investments are strategic complements in the GHM framework while they are strategic substitutes in the model studied in the present paper.

The rest of the paper is organized as follows: Section 2 introduces the basic model. Section 3 derives the profit functions and discusses the resulting properties of the reaction functions. Section 4 presents the main results. The paper ends with some concluding remarks in Section 5. Some proofs are given in the appendix.

## 2 The Model

A downstream firm has exclusive access to a final good market. On this market it can sell a maximum quantity of  $X$  at a constant price  $p$ . The quantity  $X$  is a random variable uniformly distributed on the interval  $[0, \bar{X}]$  with  $0 < \bar{X} < \infty$ .<sup>9</sup> Production of the final good requires an essential input that the downstream firm,  $M$  (for manufacturer), can either provide internally or buy from an upstream firm,  $S$  (for subcontractor). Both  $M$  and  $S$  have access to the same input technology. This technology requires an idiosyncratic capacity investment. Capacity can be purchased from a competitive outside

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<sup>9</sup>To motivate this assumption, one can imagine that the firm is a monopolist facing a demand curve made up of unit demands. Potential customers have one of two possible valuations for one unit of the good: either 0 or  $p$ . The number of customers with valuations  $p$  is the random variable  $X$  mentioned in the text.

market at a constant price  $q > 0$ . If firm  $i \in \{M, S\}$  has a certain capacity level, it can produce any quantity of the input not exceeding capacity at a constant cost which is normalized to zero. For ease of exposition, we assume that  $M$  needs exactly one unit of the input (in addition to other inputs) to produce one unit of the final good and that  $M$ 's cost of transforming the input into the final good is zero. Furthermore,  $p > q$ , so that installation of sufficient capacity and carrying out production would always be worthwhile if  $M$  knew the quantity demanded at the outset.  $S$  (but not  $M$ ) has access to an intermediate good market. At this market he can sell the input at a constant price  $\phi > 0$ . To capture the idea that capacity is specific to the production of the input needed by  $M$  we assume that  $\phi$  is strictly smaller than  $q$ .

The assumed time and information structure is as follows: At stage 1  $M$  and  $S$  choose their capacity levels  $k^M$  and  $k^S$ . We consider two different cases. In the first case each firm  $i \in \{M, S\}$  chooses its capacity level taking the capacity level of the second firm as given. We refer to this as the Cournot-Nash case, emphasizing the simultaneous nature of the underlying game and its formal similarity to the model of Cournot competition. The second model considered is Stackelberg leadership by  $M$ . Here, the assumption is that  $M$  can commit to a given capacity level while anticipating the optimal response of  $S$ . In both cases both firms know the distribution of  $X$  but not its actual realization when making their capacity decision. Later, at stage 2,  $X$  is realized and becomes publicly observable. Then bargaining takes place between the two firms on whether inputs will be delivered from  $S$  to  $M$  and, if so, how many units and at which price. Bargaining takes the following form: With

probability  $\alpha$ ,  $S$  makes a take-it-or-leave-it offer to  $M$ ; with probability  $1 - \alpha$ ,  $M$  makes a take-it-or-leave-it offer to  $S$ . This bargaining procedure implies that  $S$  obtains, on average, a fraction  $\alpha$  of the bargaining surplus. The latter is defined as the value in excess of the threat point payoffs. We take the non-cooperative solution in which  $M$  relies exclusively on internal provision and where  $S$  serves only the intermediate good market as the threat point in the bargaining process.<sup>10</sup> Production takes place in stage 3. If an agreement between  $M$  and  $S$  was reached in stage 2, this agreement is carried out. Then markets are served.

### 3 Preliminary Results

To solve the firms' stage 1 capacity choice problem we first look at the stage 2 bargaining problem taking the capacity levels of the two parties as given.

#### 3.1 The Stage Two Bargaining Problem

Let the subcontractor's stage 2 stand-alone payoff for a given price at the intermediate good market be denoted by  $A^S(k^S|\phi)$ . This payoff is given by  $A^S(k^S|\phi) = \phi k^S$ . Similarly, the manufacturer's stage 2 stand-alone payoff is given by  $A^M(k^M|p, X) = p \min\{X, k^M\}$ . To determine the bargaining surplus, denoted by  $B(k^M, k^S|\phi, p, X)$ , we have to distinguish three different regions in the demand space. These regions (denoted by  $R_1$  to  $R_3$ ) are defined in Figure 1.

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<sup>10</sup>See De Meza and Lockwood (1998) and Chiu (1999) for a different interpretation of threat points and outside options in bargaining.

*Insert Figure 1*

Consider first Region 1. If demand falls in this region, then  $M$  can produce the whole input requirement internally. Hence, the bargaining surplus in this region is zero. In Region 3, on the other hand,  $M$  is able to buy  $k^S$  units of the essential input from  $S$  that couldn't have been produced in-house. Thus, in Region 3 the bargaining surplus is the market price of the final good minus the price at the intermediate good market times  $S$ 's capacity level. A similar reasoning for the intermediate range  $R_2$  reveals that the bargaining surplus at stage 2 is

$$B(k^M, k^S | \phi, p, X) = \begin{cases} 0 & \text{for } X \text{ in } R_1 \\ (p - \phi)(X - k^M) & \text{for } X \text{ in } R_2 \\ (p - \phi)k^S & \text{for } X \text{ in } R_3. \end{cases}$$

Given our assumption on the outcome of the bargaining process the payoffs of the two firms at stage 2 are

$$A^S(k^S | \phi) + \alpha B(k^M, k^S | \phi, p, X) \quad (1)$$

for the subcontractor and

$$A^M(k^M | p, X) + (1 - \alpha) B(k^M, k^S | \phi, p, X) \quad (2)$$

for the manufacturer.

### 3.2 The Subcontractor's Capacity Choice Problem

If we substitute the values for  $A^S(k^S | \phi)$  and  $B(k^M, k^S | \phi, p, X)$  in the subcontractor's stage 2 payoff function (1), take the expectation with respect to

$X$  (taking into account that  $\bar{X}$  is in  $R_3$ <sup>11</sup>), and subtract  $S$ 's capacity costs  $qk^S$  we obtain  $S$ 's stage 1 profit function denoted by  $\Pi^S(k^M, k^S|\phi, \alpha, p, q)$ . Whenever there is no risk of confusion we will use the shorthand notation  $\Pi^S(k^M, k^S|\phi, \alpha)$ . As is easily verified, for  $\alpha \in [0, 1]$  and  $\phi \in (0, q)$ ,  $\Pi^S(k^M, k^S|\phi, \alpha)$  is given by

$$\Pi^S(k^M, k^S|\phi, \alpha) = \phi k^S + \alpha(p - \phi) \left( \bar{X} - k^M - \frac{k^S}{2} \right) \frac{k^S}{\bar{X}} - qk^S.$$

This term is easily understood. Since the subcontractor has the option to sell the input at the intermediate good market, he earns  $\phi$  for each unit of capacity for sure. For units sold to the manufacturer he gets an additional amount of  $\alpha(p - \phi)$ . In expectation, the subcontractor will sell  $(\bar{X} - k^M - k^S/2)k^S/\bar{X}$  units to the manufacturer. From the resulting expected gross profit the subcontractor subtracts the capacity cost.

The derivative of the subcontractor's (expected net) profit function with respect to  $k^S$  is the (expected net) shadow value of an additional unit of capacity. Setting this shadow value equal to 0, solving for  $k^S$  and taking into account that  $S$ 's capacity cannot be negative yields  $S$ 's reaction function

$$k^S(k^M|\phi, \alpha) = \left[ \frac{\alpha(p - \phi) - (q - \phi)}{\alpha(p - \phi)} \bar{X} - k^M \right]^+, \quad (3)$$

where  $[y]^+$  stands for  $\max\{0, y\}$ . According to this reaction function, the subcontractor will either invest zero, or he will choose the difference between the amount of capacity he would buy if he was allowed to serve the demand

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<sup>11</sup>That  $\bar{X}$  is in  $R_3$  follows from the fact that (a)  $k^M$  is necessarily smaller than  $\bar{X}$  since capacity units in excess of  $\bar{X}$  are costly for the manufacturer without providing her with any benefit, and (b) choosing  $k^S > \bar{X} - k^M$  can never be optimal for the subcontractor since capacity-costs exceed the profit opportunities at the outside market.

at the final good market alone ( $k^S(0|\phi, \alpha)$ ), and the amount of capacity provided by the manufacturer by herself ( $k^M$ ).

### 3.3 The Manufacturer's Capacity Choice Problem

If we substitute the values for  $A^M(k^M, p, X)$  and  $B(k^M, k^S, \phi, p, X)$  in the manufacturer's stage 2 payoff function (2), take the expectation with respect to  $X$  and subtract  $M$ 's stage 1 capacity costs  $qk^M$  we obtain  $M$ 's stage 1 profit function. We denote this function by  $\Pi^M(k^M, k^S|\phi, \alpha, p, q)$  and use the shorthand notation  $\Pi^M(k^M, k^S|\phi, \alpha)$ . It is given by

$$\begin{aligned} \Pi^M(k^M, k^S|\phi, \alpha) &= p\left(\bar{X} - \frac{k^M}{2}\right)\frac{k^M}{\bar{X}} - qk^M \\ &\quad + (1 - \alpha)(p - \phi)\left(\bar{X} - k^M - \frac{k^S}{2}\right)\frac{k^S}{\bar{X}}. \end{aligned} \quad (4)$$

#### 3.3.1 The Cournot-Nash Case

In the Cournot-Nash case, the manufacturer chooses her capacity level taking the subcontractor's capacity investment as given. Setting the partial derivative of  $\Pi^M(k^M, k^S|\phi, \alpha)$  with respect to  $k^M$  equal to 0, solving for  $k^M$  and taking into account that  $M$ 's capacity cannot be negative yields  $M$ 's reaction function

$$k^M(k^S|\phi, \alpha) = \left[ \frac{p - q}{p}\bar{X} - (1 - \alpha)\frac{p - \phi}{p}k^S \right]^+. \quad (5)$$

That is, for any given vector  $(\phi, \alpha)$ ,  $k^M(k^S|\phi, \alpha)$  gives the manufacturer's optimal response to the subcontractor's capacity choice. A pair  $(k^M, k^S)$  satisfying (3) and (5) is a Cournot-Nash equilibrium of the simultaneous-

move capacity game, which we denote by  $(\tilde{k}^M(\phi, \alpha), \tilde{k}^S(\phi, \alpha))$ . It has:

$$\begin{aligned} \tilde{k}^M(\phi, \alpha) &= \begin{cases} \bar{X} - \frac{\alpha q - (1-\alpha)(q-\phi)}{\alpha(\alpha(p-\phi)+\phi)} \bar{X} & \text{if } \alpha \geq \tilde{\alpha}(\phi) \\ \bar{X} - \frac{q}{p} \bar{X} & \text{otherwise} \end{cases} \\ \tilde{k}^S(\phi, \alpha) &= \begin{cases} \frac{\alpha(p-\phi)q - p(q-\phi)}{\alpha(p-\phi)(\alpha(p-\phi)+\phi)} \bar{X} & \text{if } \alpha \geq \tilde{\alpha}(\phi) \\ 0 & \text{otherwise,} \end{cases} \end{aligned} \quad (6)$$

where

$$\tilde{\alpha}(\phi) := \frac{p(q-\phi)}{q(p-\phi)}. \quad (7)$$

Note that  $\tilde{k}^M(\phi, \alpha) \in (0, \bar{X})$ .<sup>12</sup> Also, note that  $\lim_{\phi \rightarrow q} \tilde{\alpha}(\phi) = 0$  and  $\lim_{\phi \rightarrow 0} \tilde{\alpha}(\phi) = 1$ .

### 3.3.2 Stackelberg Leadership

In the Stackelberg case the manufacturer commits to a capacity level anticipating the subcontractor's optimal response. Hence,  $M$ 's problem is to maximize  $\Pi^M(k^M, k^S(k^M|\phi, \alpha)|\phi, \alpha)$  with respect to  $k^M$ , where  $k^S(k^M|\phi, \alpha)$  is as in (3).

If  $k^S(k^M|\phi, \alpha)$  is replaced by zero we get  $M$ 's stand-alone profit function  $\Pi^M(k^M, 0|\phi, \alpha)$ , which, of course, is independent of  $\phi$  and  $\alpha$ . Setting the derivative with respect to  $k^M$  equal to 0 and solving for  $k^M$  yields the  $M$ 's stand-alone capacity level denoted by  $k_1^M$ :

$$k_1^M = \frac{p-q}{p} \bar{X}. \quad (8)$$

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<sup>12</sup>For  $\alpha \leq \tilde{\alpha}(\phi)$  this is obvious. For  $\alpha > \tilde{\alpha}(\phi)$ , the first part ( $\tilde{k}^M(\phi, \alpha) > 0$  for  $\alpha > \tilde{\alpha}(\phi)$ ) follows since  $q-\phi < p-\phi$  and  $0 \leq (1-\alpha)^2$  implies that  $q\alpha - (1-\alpha)(q-\phi) < \alpha(\alpha(p-\phi)+\phi)$ ; the second part ( $\tilde{k}^M(\phi, \alpha) < \bar{X}$ ) is true since  $\tilde{k}^M(\phi, \alpha) < \frac{p-q}{p} \bar{X} < \bar{X}$  by (5).

$M$ 's optimal stand-alone profit thus equals

$$\Pi^M(k_1^M, 0) = \frac{(p-q)^2}{2p} \bar{X}. \quad (9)$$

If  $k^S(k^M|\phi, \alpha)$  is replaced by the term inside the brackets in (3) we get  $M$ 's profit function in an interior solution. Setting the derivative with respect to  $k^M$  equal to 0 and solving for  $k^M$  yields  $M$ 's optimal capacity choice in an interior Stackelberg equilibrium. We denote this capacity level by  $k_2^M(\phi, \alpha)$ . It is given by

$$k_2^M(\phi, \alpha) = \bar{X} - \frac{q}{\alpha(p-\phi) + \phi} \bar{X}. \quad (10)$$

By (3),  $S$ 's best response to  $k_2^M(\phi, \alpha)$  is

$$k_2^S(\phi, \alpha) = \frac{q}{\alpha(p-\phi) + \phi} \bar{X} - \frac{q-\phi}{\alpha(p-\phi)} \bar{X} = \frac{\phi(\alpha(p-\phi) - (q-\phi))}{\alpha(p-\phi)(\alpha(p-\phi) + \phi)} \bar{X}, \quad (11)$$

where  $k_2^S(\phi, \alpha) \geq 0$  iff  $\alpha \geq \hat{\alpha}(\phi)$  and where

$$\hat{\alpha}(\phi) := \frac{q-\phi}{p-\phi}. \quad (12)$$

Note that  $\lim_{\phi \rightarrow 0} \hat{\alpha}(\phi) = \frac{q}{p} < 1$  and  $\lim_{\phi \rightarrow q} \hat{\alpha}(\phi) = 0$ .

The next step is to compare  $\Pi^M(k_1^M, 0|\phi, \alpha)$  with  $\Pi^M(k_2^M(\phi, \alpha), k_2^S(\phi, \alpha)|\phi, \alpha)$ . If  $\Pi^M(k_1^M, 0|\phi, \alpha) > \Pi^M(k_2^M(\phi, \alpha), k_2^S(\phi, \alpha)|\phi, \alpha)$  then the Stackelberg equilibrium, denoted by  $(\hat{k}^M(\phi, \alpha), \hat{k}^S(\phi, \alpha))$ , is given by  $(k_1^M, 0)$ . Otherwise an interior Stackelberg equilibrium prevails and  $(\hat{k}^M(\phi, \alpha), \hat{k}^S(\phi, \alpha))$  is given by  $(k_2^M(\phi, \alpha), k_2^S(\phi, \alpha))$ .<sup>13</sup>

<sup>13</sup>Note that  $k_2^S(\phi, \alpha) > 0$  whenever  $\Pi^M(k_2^M, k_2^S|\phi, \alpha) > \Pi^M(k_1^M, 0|\phi, \alpha)$  since otherwise  $\Pi^M(k_2^M, k_2^S|\phi, \alpha) \leq \max_{k^M} \{p(\bar{X} - \frac{k^M}{2}) \frac{k^M}{\bar{X}}\} = \Pi_1^M$ . The reverse statement “ $(\hat{k}^M(\phi, \alpha), \hat{k}^S(\phi, \alpha)) = (k_2^M(\phi, \alpha), k_2^S(\phi, \alpha))$  whenever  $\alpha \geq \hat{\alpha}(\phi)$ ” is not true, however, as will become clear later.

## 4 Equilibrium Capacities

In the sequel we distinguish three different provision modes: Under full integration (mode F) the manufacturer relies entirely on internal provision of the essential input; that is, only the manufacturer makes a strictly positive capacity investment at stage 1 and the subcontractor remains inactive. Tapered integration (mode T) refers to a situation in which both firms are able to produce at stage 2; that is, both  $k^M$  and  $k^S$  are strictly positive. Under non-integration (mode N) only the subcontractor makes a positive capacity investment at stage 1 and the manufacturer has to satisfy her input needs from outside.

Let us begin the analysis by considering the problem from a social planner's point of view. Since the subcontractor can do anything the manufacturer can and since the subcontractor has in addition the option to sell the input at the intermediate good market, the unique socially optimal provision mode in our model is non-integration.<sup>14</sup> However, as the following result shows, egoistic motives do never guide the parties toward the socially optimal provision mode (not to mention the socially optimal capacity levels!).

**Proposition 4.1** *Under the assumptions of our model neither the Stackelberg nor the Cournot-Nash equilibrium involves non-integration (mode N).*

PROOF: In mode N,  $k^M = 0$  and  $k^S = k^S(0) = \left[ \frac{\alpha(p-\phi) - (q-\phi)}{\alpha(p-\phi)} \bar{X} \right]^+$  which is strictly positive iff  $\alpha > \hat{\alpha}(\phi)$ . Consider the Stackelberg case first. Here,

<sup>14</sup>The formal proof for this claim consists of maximizing  $\Pi^M(k^M, k^S | \phi, \alpha) + \Pi^S(k^M, k^S | \phi, \alpha)$  with respect to  $k^M$  and  $k^S$ , and showing that this yields  $k^M = 0$  and  $k^S = (p - q)\bar{X}/(p - \phi)$ .

$k_2^M(\phi, \alpha)$  equals zero iff  $\alpha \leq \hat{\alpha}(\phi)$ . But then  $k^S = 0$  too, implying that  $\Pi^M(0, 0) = 0$ , which cannot be optimal for  $M$  since she has always the option to choose her stand-alone capacity level guaranteeing a payoff of at least  $\Pi_1^M(k_1^M, 0) > 0$ . Next, consider the Cournot case. Here, the manufacturer's optimal capacity level is always positive, as we have shown in Footnote 12 above.  $\square$

An explanation for this result is readily provided: If the manufacturer knew her input requirement with certainty in advance, she would always prefer to produce this quantity in-house rather than purchasing it from the subcontractor as in-house production prevents her from being exploited by the subcontractor in the bargaining process. The same logic applies in our probabilistic framework. Here, the manufacturer has an incentive to invest in in-house capacity to satisfy the demand which will arise with high probability. The subcontractor will also invest in capacity and satisfy any greater demand which will arise with lower probability iff (i) he is sufficiently more efficient than the manufacturer in bearing the risk of excess capacity, and (ii) his bargaining power guarantees a sufficiently large share of the efficiency gain for him. For a given level of bargaining power this is the case if the price at the intermediate good market is high enough. This is confirmed by the following result:

**Proposition 4.2** *In both the Cournot-Nash and the Stackelberg case for each  $\phi \in (0, q)$  there exists a unique  $\alpha^I(\phi) \in [\hat{\alpha}(\phi), \tilde{\alpha}(\phi)]$  such that full integration (mode F) prevails for all  $\alpha \leq \alpha^I(\phi)$  and tapered integration (mode T) prevails for all  $\alpha > \alpha^I(\phi)$ . In the Cournot-Nash case  $\alpha^I(\phi) = \tilde{\alpha}(\phi)$  as defined in (7).*

In the Stackelberg case  $\alpha^I(\phi) =: \alpha^{ST}(\phi) \in (\hat{\alpha}(\phi), \tilde{\alpha}(\phi))$ . In both cases, the range of values of  $\alpha$  for which mode F prevails is decreasing in  $\phi$ ; that is,  $\alpha^I(\cdot)$  is a strictly decreasing function. Furthermore,  $\lim_{\phi \rightarrow 0} \alpha^I(\phi) = 1$  and  $\lim_{\phi \rightarrow q} \alpha^I(\phi) = 0$ .

PROOF: In the Appendix.

*Insert Figure 2*

If  $\alpha$  equals 1 the entire bargaining surplus goes to the subcontractor. Anticipating this, the manufacturer chooses her stand-alone capacity level  $k_1^M$ . In determining this capacity level,  $M$  sets her stage 1 shadow value of an additional unit of capacity equal to the capacity cost. For  $\alpha = 1$  the stage 1 shadow value of an additional unit of capacity in firm  $S$  necessarily exceeds that of capacity in  $M$  for all  $\phi > 0$ : If the unit is used to satisfy  $M$ 's input needs,  $S$  nets the same surplus as  $M$  would earn if she employs her own in-house capacity. If a capacity unit is not used to satisfy  $M$ 's input requirement, it idles in  $M$ 's firm; yet, in  $S$ 's firm it is used to serve the intermediate good market. Hence, for  $\alpha = 1$  we have  $k^M = k_1^M > 0$  and  $k^S > 0$  for all  $\phi > 0$ , implying that mode T prevails. Now consider the other extreme. If  $\alpha = 0$  the subcontractor cannot expect to extract any surplus in dealing with the manufacturer. Since the price at the intermediate good market doesn't cover capacity cost, he will not invest. Hence, for  $\alpha = 0$  we have  $k^M = k_1^M > 0$  and  $k^S = 0$ , implying that mode F prevails. For intermediate values of  $\alpha$  the Cournot-Nash and the Stackelberg case behave qualitatively differently: While  $\alpha^I(\phi) = \tilde{\alpha}(\phi)$  in the Cournot-Nash case,  $\hat{\alpha}(\phi) < \alpha^I(\phi)$  in the Stackelberg case. This implies that capacities are changing contin-

uously in  $\alpha$  and  $\phi$  at the boundary between mode F and mode T in the Cournot-Nash, but that there is a discontinuous jump in capacity levels in the Stackelberg case. We will discuss this jump in more detail below.

Proposition 4.2 has interesting implications. For each  $\phi$ , define  $\alpha^M(\phi) = \arg \max_{\alpha \in [0,1]} \{\Pi^M(k^M(\phi, \alpha), k^S(\phi, \alpha))\}$ , where  $(k^M(\phi, \alpha), k^S(\phi, \alpha))$  are the respective equilibrium capacities. In the same way, for each  $\phi$ , define  $\alpha^S(\phi) = \arg \max_{\alpha \in [0,1]} \{\Pi^S(k^M(\phi, \alpha), k^S(\phi, \alpha))\}$ , where  $(k^M(\phi, \alpha), k^S(\phi, \alpha))$  are the respective equilibrium capacities. Since the subcontractor's capacity is strictly positive if and only if  $\alpha > \alpha^I(\phi)$  we know that  $\alpha^S(\phi) \in (\alpha^I(\phi), 1]$ . Also, since  $\Pi^M(\cdot)$  exceeds M's stand alone profit for  $\alpha \in (\alpha^I(\phi), 1)$ ,  $\alpha^M(\phi)$  must lie in  $(\alpha^I(\phi), 1)$ . So, for all  $\alpha \leq \alpha^I(\phi)$ , or more precisely for all  $\alpha < \min\{\alpha^M(\phi), \alpha^S(\phi)\}$ , a strict Pareto improvement can be obtained by raising  $\alpha$  to  $\min\{\alpha^M(\phi), \alpha^S(\phi)\}$ . The possibility of a Pareto improvement arises since the increase in  $\alpha$  has a positive impact on the sum of profits (that is, on  $\Pi^M + \Pi^S$ ). The manufacturer gains since the increase in total profit more than compensates her for getting a smaller part of it. And the subcontractor is better off since both the total profit and his share in it are augmented. Similarly, for all  $\alpha > \max\{\alpha^M(\phi), \alpha^S(\phi)\}$  a strict Pareto improvement can be obtained by reducing  $\alpha$  to  $\max\{\alpha^M(\phi), \alpha^S(\phi)\}$ . Again, the possibility of a Pareto improvement arises since the adjustment in  $\alpha$  has a positive impact on the size of the cake. For levels of  $\alpha$  between  $\min\{\alpha^M(\phi), \alpha^S(\phi)\}$  and  $\max\{\alpha^M(\phi), \alpha^S(\phi)\}$  one player prefers a lower, the other a higher level of  $\alpha$ . Intuitively one would expect that the subcontractor's optimal  $\alpha$  lies above the manufacturer's, since  $\alpha$  determines the share of bargaining surplus that goes to  $S$ . This needs not be the case, however, as the example below shows.

In this example for each  $\phi < 1.1$  there exists a range of values for  $\alpha$  where the subcontractor would prefer to have less power while the manufacturer would prefer that the subcontractor is tougher!

**Example:** Consider the Stackelberg case and let  $p = 10$  and  $q = 2$ . Then  $\alpha^I(\phi)$ ,  $\alpha^M(\phi)$  and  $\alpha^S(\phi)$  are as follows:

*Insert Table 1*

The intuition behind this result is as follows. In the range(s) under consideration a higher  $\alpha$  induces the manufacturer to increase her capacity level. Increasing  $k^M$  is profitable for  $M$  since buying from  $S$  becomes more expensive when his bargaining power improves, and since  $S$  is prepared to hold a higher  $k^S$  for any  $k^M$  if  $\alpha$  is higher. The increase in  $k^M$  increases the manufacturer's stage 2 stand-alone payoff but decreases the bargaining surplus. The manufacturer profits since the increase in her stand-alone payoff more than compensates her for getting less from dealing with the subcontractor. The subcontractor, on the other hand, loses since the impact from the decrease in the bargaining cake exceeds that from the increase in his share in it.

Our next result deals with capacities under tapered integration.

**Lemma 4.1** *In both the Cournot-Nash and the Stackelberg case total capacity in mode  $T$  is independent of the manufacturer's capacity investment. It is strictly increasing in  $\alpha$  and in  $\phi$  and equals the manufacturer's stand-alone capacity  $k_1^M$  iff  $\alpha = \tilde{\alpha}(\phi)$  as defined in (7).*

PROOF: The first claim is an immediate consequence of (3). To prove the

rest note that by (3), total capacity, denoted by  $k(\phi, \alpha)$ , equals

$$k(\phi, \alpha) = \frac{\alpha(p - \phi) - (q - \phi)}{\alpha(p - \phi)} \bar{X}.$$

Taking the derivative with respect to  $\alpha$  and with respect to  $\phi$  confirms the first part of the second claim and a comparison with (8) the second.  $\square$

Given the reaction function of the subcontractor as recorded in (3) the first claim in Lemma 4.1 is not surprising. In mode T the subcontractor buys that amount of capacity which he would buy if he were allowed to deliver the whole quantity demanded at the final good market *minus* the amount provided by the manufacturer herself. The reason is simple: The subcontractor knows that for any given demand at the final good market the manufacturer will first employ her in-house capacity. Thus, in determining his own capacity investment  $k^S$ , the subcontractor will compare his (gross) shadow value of the  $(k^M + k^S)$ st unit of capacity with capacity cost.

How will this shadow value change in  $\phi$  and in  $\alpha$ ? For a given level of bargaining power an increase in the price at the intermediate good market has two effects on the subcontractor's shadow value of capacity and therewith on the amount of capacity the subcontractor would buy if he were allowed to serve all the demand at the final good market. First, capacity that is not used to serve the manufacturer's final good market has a higher value; that is, a high price at the intermediate good market insures the subcontractor against low demand on the manufacturer's end product market. We call this the *insurance effect*. Second, an increase in the price at the intermediate good market improves the subcontractor's threat point in the ex post bargaining

with the manufacturer and thereby his bargaining position. We call this second effect the *bargaining position effect*. For the subcontractor both the insurance and the bargaining position effect are strictly positive since both the profit for selling to the manufacturer and the profit for selling to the outside market are increased. Thus, total capacity in mode T will increase in  $\phi$ . An increase in  $\alpha$ , on the other hand, increases the subcontractor's share in the joint surplus and therewith again his incentive to increase the cake.

**Lemma 4.2** *In the Cournot-Nash case the manufacturer's capacity choice in mode T is equal to her stand-alone capacity iff  $\alpha = \tilde{\alpha}(\phi)$  or  $\alpha = 1$  and strictly smaller otherwise. It is first decreasing and then increasing in  $\alpha$  and always decreasing in  $\phi$ . In the Stackelberg case the manufacturer's capacity choice in mode T is equal to her stand-alone capacity level iff  $\alpha = 1$  and strictly smaller otherwise. It is strictly increasing in  $\alpha$  and in  $\phi$ .*

PROOF: In the Appendix.

*Insert Figure 3*

Figure 3 traces the equilibrium capacities as a function of the subcontractor's bargaining power ( $\alpha$ ) holding the price at the intermediate good market ( $\phi$ ) constant. The behavior of the Cournot-Nash equilibrium correspondence is depicted in thickly dotted, that of the Stackelberg equilibrium in bold lines. In the Cournot-Nash equilibrium M's capacity level in mode T is first decreasing and then increasing in S's bargaining power. This is quite surprising since it is often argued that an increase in the subcontractor's bargaining strength will decrease the manufacturer's share in the joint surplus and thereby increase her incentive to invest in in-house capacity. Yet there is

a second effect. *Ceteris paribus*, an increase in his bargaining power increases the subcontractor's investment incentive for each  $\phi$  and this decreases M's incentive to invest since capacity investments are strategic substitutes. In the Cournot-Nash case the two effects point in opposite directions and the overall impact is ambiguous. In the Stackelberg case, the manufacturer takes into account that the subcontractor is prepared to hold a higher capacity level if  $\alpha$  is high even if  $k^M$  is relatively high too. He therefore adjusts his capacity accordingly. Increasing her capacity is profitable for M since buying from S becomes more expensive when his bargaining power improves.

*Insert Figure 4*

The effect of an increase in the price at the intermediate good market on equilibrium capacities is shown in Figure 4. In the Cournot-Nash equilibrium the manufacturer's capacity level in mode T is strictly decreasing in the price at the intermediate good market, while it is strictly increasing in the Stackelberg equilibrium. The reason for the striking difference in the reaction to changes in  $\phi$  is similar to that discussed for changes in  $\alpha$ . The insurance effect mentioned earlier decreases M's incentive to invest in in-house capacity since S is prepared to hold a higher capacity level even if his share in the joint surplus is relatively low. The bargaining position effect, however, increases M's incentive to invest to avoid being exploited in the bargaining process. In the Cournot-Nash case the insurance effect dominates the bargaining position effect and  $\tilde{k}^M$  is decreasing in  $\phi$ . In the Stackelberg case, on the other hand, the manufacturer takes into account that with a high  $\phi$  the subcontractor is prepared to hold a higher capacity even if  $k^M$  is high too and she adjusts her capacity level accordingly. Increasing her capacity is profitable for M

since buying from S becomes more expensive when his bargaining position improves.

Our next result reveals more about the discontinuity of capacity choices and profits at the boundary between mode F and mode T in the Stackelberg case:

**Proposition 4.3** *In the Stackelberg case a move from full integration (mode F) to tapered integration (mode T) is accompanied by (i) a discontinuous downward jump of  $\hat{k}^M$ ; (ii) a discontinuous upward jump of  $\hat{k}^S$ ; (iii) a discontinuous downward jump of  $\hat{k}^M + \hat{k}^S$ ; (iv) a continuous increase of  $\Pi^M$ ; (v) a discontinuous upward jump of  $\Pi^S$ .*

PROOF: Using that the change from mode F to mode T takes place at  $\alpha = \alpha^{ST}(\phi) \in (\hat{\alpha}(\phi), \tilde{\alpha}(\phi))$ , (i) follows from the fact that  $\hat{k}^M(\phi, \alpha^{ST}(\phi)) < \hat{k}^M(\phi, \tilde{\alpha}(\phi)) < k_1^M$  by Proposition 4.2 and Lemma 4.2. Property (ii) follows from the fact that  $\hat{k}^S(\phi, \alpha^{ST}(\phi)) > \hat{k}^S(\phi, \hat{\alpha}(\phi)) = 0$  since  $\alpha^{ST}(\phi) > \hat{\alpha}(\phi)$ . To show (iii) we use Lemma 4.1. The claim then follows from the fact that  $k(\phi, \alpha^{ST}(\phi)) < k(\phi, \tilde{\alpha}(\phi)) = k_1^M$ . Property (iv) is obvious, and (v) follows from the fact that  $\alpha^{ST}(\phi) > \hat{\alpha}(\phi)$  and therefore  $\hat{k}^S(\phi, \alpha^{ST}(\phi)) > 0$  since  $\hat{k}(\phi, \alpha) > 0$  whenever  $\alpha > \hat{\alpha}(\phi)$  by definition of  $\hat{\alpha}(\phi)$  and by (11).  $\square$

Proposition 4.3 tells us that, in contrast to the Cournot-Nash case, in the Stackelberg case neither the manufacturer's nor the subcontractor's capacity level is continuous in the price at the intermediate good market. The driving force behind this result is a discontinuity in the marginal benefit of an additional unit of capacity from the manufacturer's point of view: In a full integration environment ( $k^M > 0$  and  $k^S = 0$ ) an extra unit of capacity

yields the manufacturer an extra payoff (in ex ante terms) of  $p\frac{\bar{X}-k^M}{X}$ . In a tapered integration environment her marginal value of an additional unit of capacity is necessarily strictly smaller for any  $\alpha < 1$  since any capacity-increase by the manufacturer is accompanied by a capacity-decrease by the subcontractor. Hence, from M's point of view, there is a downward jump in the marginal value of capacity when moving from mode F to mode T. This implies that for any  $\alpha < 1$  the maximal amount of capacity provided by the manufacturer under mode T is distinctly smaller than her stand-alone capacity level chosen under mode F. But then the subcontractor's capacity level under mode T cannot be arbitrarily small, since otherwise M's profit under T would be smaller than that under F, contradicting the presumption that T is the chosen mode in equilibrium.

**Proposition 4.4** *For any pair  $(\phi, \alpha)$ , the amount of capacity provided by the manufacturer in the Stackelberg equilibrium is never larger than the corresponding level provided by her in the Cournot-Nash equilibrium. For values of  $\alpha$  with  $\alpha > \alpha^{ST}(\phi)$  the difference in the manufacturer's capacity choices, denoted by  $\Delta k^M(\phi, \alpha) := \tilde{k}^M(\phi, \alpha) - \hat{k}^M(\phi, \alpha)$ , is decreasing in  $\alpha$  and in  $\phi$  and approaches zero if one of these variables approaches its upper bound.*

PROOF: For  $\alpha > \tilde{\alpha}(\phi)$ , the difference between the manufacturer's capacity choice in the Stackelberg case and the corresponding level in the Cournot-Nash case is  $\Delta k^M(\phi, \alpha) = \frac{1-\alpha}{\alpha} \frac{q-\phi}{\alpha(p-\phi)+\phi}$ . This expression is strictly positive for  $\alpha < 1$  and  $\phi < q$  and approaches zero if one of these variables approaches its upper bound. Taking the derivative with respect to  $\alpha$  leads to

$$\frac{\partial \Delta k^M(\phi, \alpha)}{\partial \alpha} = -\frac{q-\phi}{\alpha^2(\alpha(p-\phi)+\phi)^2}((2-\alpha)\alpha(p-\phi)+\phi) < 0.$$

Taking the derivative with respect to  $\phi$  yields

$$\frac{\partial \Delta k^M(\phi, \alpha)}{\partial \phi} = -\frac{1-\alpha}{\alpha} \frac{(1-\alpha)q + \alpha p}{(\alpha(p-\phi) + \phi)^2} < 0.$$

For  $\alpha \in [\alpha^{ST}(\phi), \tilde{\alpha}(\phi)]$  the claim follows from Proposition 4.2 and from Lemma 4.2. And for  $\alpha < \alpha^{ST}(\phi)$  the result is a triviality.  $\square$

Proposition 4.4 is quite intuitive. If the manufacturer has commitment power she takes into account that any capacity increase by her is accompanied by a capacity decrease by the subcontractor. She therefore commits to a lower capacity level anticipating a higher response by the subcontractor. That having commitment power becomes less important as  $\alpha$  and  $\phi$  grow larger is not surprising given the result recorded in Lemma 4.1: For any given  $k^M$  the subcontractor's capacity investment is strictly increasing in these variables. If the subcontractor's capacity level is high anyway inducing an even higher response is less profitable. And that the difference in the manufacturer's capacity choices approaches zero if either  $\alpha$  or  $\phi$  approaches its upper bound is also obvious. If  $\alpha$  approaches 1 the manufacturer does not benefit from the subcontractor's presence. She will therefore choose her stand-alone capacity level in both the Cournot-Nash and the Stackelberg case. And if  $\phi$  approaches  $q$  the subcontractor will adjust his capacity such that total capacity equals  $\bar{X}$ . Knowing this, the manufacturer's trade-off in her boundary choice is between (i) investing in in-house capacity at a cost of  $q$  per unit risking that capacity will remain idle if demand is too low, and (ii) spending  $q + \alpha(p - q)$  per unit only if demand is sufficiently high. Facing this trade-off she will choose  $k^M = \bar{X} - \frac{q}{\alpha(p-q)+q}\bar{X}$  independently of whether she can precommit to a particular capacity level or not.

**Proposition 4.5** *Total capacity in the Stackelberg equilibrium is less than or equal to total capacity in the Cournot-Nash equilibrium.*

PROOF: By Proposition 4.2 and Lemma 4.1 total capacity in the Stackelberg equilibrium equals total capacity in the Cournot-Nash equilibrium for  $\alpha \geq \tilde{\alpha}(\phi)$  and for  $\alpha < \hat{\alpha}(\phi)$ . Consider therefore the range  $\alpha \in (\hat{\alpha}(\phi), \tilde{\alpha}(\phi))$ . If  $\alpha \in (\hat{\alpha}(\phi), \alpha^{ST}(\phi)]$  then  $\tilde{k}^M(\phi, \alpha) = \hat{k}^M(\phi, \alpha) = k_1^M$  and  $\tilde{k}^S(\phi, \alpha) = \hat{k}^S(\phi, \alpha) = 0$ . If, however,  $\alpha \in (\alpha^{ST}(\phi), \tilde{\alpha}(\phi))$  then  $(\tilde{k}^M(\phi, \alpha), \tilde{k}^S(\phi, \alpha)) = (k_1^M, 0)$  and  $(\hat{k}^M(\phi, \alpha), \hat{k}^S(\phi, \alpha)) = (k_2^M(\phi, \alpha), k_2^S(\phi, \alpha))$  with  $k_2^M(\phi, \alpha) + k_2^S(\phi, \alpha) = k(\phi, \alpha) < k_1^M$ .  $\square$

Proposition 4.5 can best be discussed by means of Figure 3. If  $\alpha$  falls short of  $\alpha^{ST}(\phi)$  then full integration (with  $k^M = k_1^M$  and  $k^S = 0$ ) prevails both in the Cournot-Nash and the Stackelberg case. Tapered integration is the chosen provision mode in both environments if  $\alpha$  exceeds  $\tilde{\alpha}(\phi)$ . In this case the manufacturer's capacity level is strictly larger in the Cournot-Nash than in the Stackelberg case. However, the subcontractor adjusts his capacity choice such that total capacity is the same in both environments. We are left with the interval  $(\alpha^{ST}(\phi), \tilde{\alpha}(\phi))$ . If  $\alpha$  falls in this interval then full integration prevails in the Cournot-Nash and tapered integration in the Stackelberg case. In the latter the manufacturer uses her commitment power to choose a relatively low capacity level in order to induce a relatively high response by the subcontractor. Together the two parties provide less capital than would be optimal under full integration.

## 5 Concluding Remarks

This paper has studied a firm's continuous choice between full integration, different degrees of tapered integration and non-integration in a model that combines elements of Williamson (1979, 1985)'s transaction cost- and Grossman and Hart (1986)'s incomplete contracting theory of vertical integration. We have shown that the firm's boundary choice depends crucially on its commitment power. If the firm can precommit to a particular provision mode, tapered integration will be chosen more frequently. Also, with commitment power the firm will never subcontract only a small portion of its input needs. In-house capacity is smaller and outside capacity larger if the firm can precommit. Total capacity is never larger in the commitment than in the non-commitment case.

In both the commitment and the non-commitment case there exists a unique critical level of bargaining power for the subcontractor such that full integration prevails if the subcontractor's bargaining power is smaller than this critical level and tapered integration whenever it is larger. This critical level of bargaining power for the subcontractor is strictly decreasing in his outside opportunities. Non-integration – although the socially optimal provision mode for all combinations of bargaining power and outside opportunities – can never emerge as an equilibrium outcome.

Other interesting results concern the profits of the involved parties. We have shown that neither the firm's nor the subcontractor's profit is monotone in the subcontractor's bargaining power: If the subcontractor's bargaining power is very low, a Pareto improvement can be obtained by raising it. Similarly,

if the subcontractor's bargaining power is very high, a Pareto improvement can be obtained by reducing it. In an intermediate range the subcontractor may prefer to have less power while the manufacturer would prefer the subcontractor to be tougher!

## 6 Appendix

### Proof of Proposition 4.2 for the Cournot-Nash Case

In the Cournot-Nash case  $\alpha^I(\phi) = \tilde{\alpha}(\phi)$ . This follows from (6) and (7). To verify the second claim, note that

$$\frac{\partial \tilde{\alpha}(\phi)}{\partial \phi} = -\frac{p(p-q)}{q(p-\phi)^2} < 0.$$

The rest follows from the definition of  $\tilde{\alpha}(\phi)$ .  $\square$

### Proof of Proposition 4.2 for the Stackelberg Case

In the Stackelberg case the manufacturer chooses mode T iff  $\Pi^M(k_2^M, k_2^S | \phi, \alpha) > \Pi^M(k_1^M, 0) =: \Pi_1^M$ .<sup>15</sup> First note that by (4),  $\Pi^M(k_2^M(\phi, 1), k_2^S(\phi, 1) | \phi, 1) = \Pi_1^M$ . Next note that at  $\alpha = 1$  we have  $\frac{d}{d\alpha} \Pi^M(k_2^M(\phi, \alpha), k_2^S(\phi, \alpha) | \phi, \alpha) < 0$  for any  $\phi$ . To see this first observe that  $\frac{d\Pi^M}{dk^M} = 0$  for  $k^M = k_2^M(\phi, \alpha)$  and  $k^S = k^S(k_2^M(\phi, \alpha) | \phi, \alpha) = k_2^S(\phi, \alpha)$  by the first order condition of the optimization problem. Therefore, for  $\alpha = 1$

$$\begin{aligned} \frac{d\Pi^M}{d\alpha} (k_2^M(\phi, \alpha), k_2^S(\phi, \alpha) | \phi, \alpha) &= \\ &= \frac{\partial \Pi^M}{\partial \alpha} + \left( \frac{\partial \Pi^M}{\partial k^M} + \frac{\partial \Pi^M}{\partial k^S} \frac{\partial k_2^S}{\partial k^M} \right) \frac{\partial k_2^M}{\partial \alpha} + \frac{\partial \Pi^M}{\partial k^S} \frac{\partial k_2^S}{\partial \alpha} \\ &= \frac{\partial \Pi^M}{\partial \alpha} + \frac{\partial \Pi^M}{\partial k^S} \frac{\partial k_2^S}{\partial \alpha} \\ &= -(p-\phi) \left( \bar{X} - \hat{k}_2^M - \frac{\hat{k}_2^S}{2} \right) \frac{\hat{k}_2^S}{\bar{X}} < 0 \end{aligned}$$

since  $\frac{\partial \Pi^M}{\partial k^S} = 0$  for  $\alpha = 1$ . Hence, for any  $\phi \in (0, q)$ , there must exist values for  $\alpha$  such that the manufacturer's profit is higher in the mode T than in mode

<sup>15</sup>Whenever there is no risk of confusion we use the shorthand notation  $k_2^M$  and  $k_2^S$  instead of  $k_2^M(\phi, \alpha)$  and  $k_2^S(\phi, \alpha)$ .

F. Furthermore, using that  $k_2^S(\phi, \hat{\alpha}(\phi)) = 0$  and  $k_2^M(\phi, \hat{\alpha}(\phi)) = 0$  we have  $\Pi^M(k_2^M(\phi, \hat{\alpha}(\phi)), k_2^S(\phi, \hat{\alpha}(\phi)) | \phi, \hat{\alpha}(\phi)) = 0 < \Pi_1^M$ . Thus, there must exist at least one  $\alpha \in (\hat{\alpha}(\phi), 1)$  such that  $\Pi_1^M = \Pi^M(k_2^M(\phi, \alpha), k_2^S(\phi, \alpha) | \phi, \alpha)$ . Next we show that this  $\alpha$  is unique. By (4),  $\Pi^M(k_2^M, k_2^S | \phi, \alpha) = \Pi_1^M$  is equivalent to

$$\left( \frac{p(2\bar{X} - k_1^M - k_2^M)}{2\bar{X}} - q \right) (k_1^M - k_2^M) = (1 - \alpha)(p - \phi) \left( \bar{X} - k_2^M - \frac{k_2^S}{2} \right) \frac{k_2^S}{\bar{X}}.$$

To determine the left hand side of this equation first notice that

$$k_1^M - k_2^M = \frac{q(1 - \alpha)(p - \phi)}{p(\alpha(p - \phi) + \phi)} \bar{X}.$$

Also,

$$\frac{2\bar{X} - k_1^M - k_2^M}{2\bar{X}} = \frac{q}{2} \left( \frac{\alpha(p - \phi) + p + \phi}{p(\alpha(p - \phi) + \phi)} \right)$$

by (8) and (10). Thus, the left hand side of the above equation equals

$$\frac{1}{2p} \left( \frac{q(1 - \alpha)(p - \phi)}{\alpha(p - \phi) + \phi} \right)^2 \bar{X}.$$

To determine the right hand side note that

$$\bar{X} - k_2^M - \frac{k_2^S}{2} = \frac{q}{2(\alpha(p - \phi) + \phi)} \bar{X} + \frac{q - \phi}{2\alpha(p - \phi)} \bar{X}.$$

Therefore, the right hand side of the above equation is given by

$$\begin{aligned} (1 - \alpha)(p - \phi) \frac{1}{2} \left[ \left( \frac{q}{\alpha(p - \phi) + \phi} \right)^2 - \left( \frac{q - \phi}{\alpha(p - \phi)} \right)^2 \right] \bar{X} &= \\ = (1 - \alpha)(p - \phi) \frac{1}{2} \frac{(q\alpha(p - \phi))^2 - ((q - \phi)(\alpha(p - \phi) + \phi))^2}{(\alpha(p - \phi)(\alpha(p - \phi) + \phi))^2} \bar{X}. \end{aligned}$$

Putting these pieces together and dividing by  $\frac{(1 - \alpha)(p - \phi)}{2} \bar{X}$  we observe that

“ $\Pi^M(k_2^M, k_2^S | \phi, \alpha) = \Pi_1^M$  for  $\alpha \neq 1$ ” is equivalent to:

$$\begin{aligned} \frac{q^2(1 - \alpha)(p - \phi)}{p} &= \frac{(q\alpha(p - \phi))^2 - ((q - \phi)(\alpha(p - \phi) + \phi))^2}{(\alpha(p - \phi))^2} \\ &= q^2 - \frac{(q - \phi)^2(\alpha(p - \phi) + \phi)^2}{(\alpha(p - \phi))^2} \end{aligned}$$

which again is equivalent to

$$\frac{(\alpha(p - \phi) + \phi)q^2}{p} = \frac{(q - \phi)^2(\alpha(p - \phi) + \phi)^2}{(\alpha(p - \phi))^2}.$$

Dividing by  $\alpha(p - \phi) + \phi > 0$  we deduce that  $\alpha^{ST}(\phi)$  is uniquely determined by the two relations

$$\begin{aligned} \alpha^{ST}(\phi)^2(p - \phi)^2q^2 &= p(q - \phi)^2(\alpha^{ST}(\phi)(p - \phi) + \phi) \\ \alpha^{ST}(\phi) &> \hat{\alpha}(\phi) > 0. \end{aligned} \tag{13}$$

Since M has always the option to choose the Cournot equilibrium capacities we also have  $\alpha^{ST}(\phi) \leq \tilde{\alpha}(\phi)$ . To show that  $\alpha^{ST}(\phi) < \tilde{\alpha}(\phi)$  for  $\phi \in (0, q)$  we replace  $\alpha^{ST}(\phi)$  in (13) by  $\tilde{\alpha}(\phi)$  to see that this does not solve the equation. This completes the proof of the first claim. To prove the second claim we take the derivative of both sides of (13). We get

$$\begin{aligned} &\left(2\alpha^{ST}(\phi)(p - \phi)^2q^2 - p(q - \phi)^2(p - \phi)\right) \frac{\partial \alpha^{ST}(\phi)}{\partial \phi} = \\ &\left[2(p - \phi)q^2\alpha^{ST}(\phi)^2 - 2(q - \phi)p(\alpha^{ST}(\phi)(p - \phi) + \phi)\right] + (1 - \alpha^{ST}(\phi))p(q - \phi)^2. \end{aligned}$$

Replacing  $(p - \phi)q^2\alpha^{ST}(\phi)^2$  by the respective expression derived from (13) the right hand side of this equation can be rewritten as

$$\begin{aligned} &p(q - \phi)^2 \left[2(\alpha^{ST}(\phi)(p - \phi) + \phi) \left(\frac{1}{p - \phi} - \frac{1}{q - \phi}\right) + (1 - \alpha^{ST}(\phi))\right] = \\ &p(q - \phi)^2 \left[-\frac{(p - q)}{(p - \phi)(q - \phi)}2\alpha^{ST}(\phi)p + (1 - \alpha^{ST}(\phi)) \left(1 - 2\frac{\phi(p - q)}{(p - \phi)(q - \phi)}\right)\right]. \end{aligned}$$

Taking into account that  $1 - 2\frac{\phi(p - q)}{(p - \phi)(q - \phi)} < 1$  and that  $\alpha^{ST}(\phi) > \hat{\alpha}(\phi)$  this must be smaller than  $p(q - \phi)^2 \left[1 - \left(\frac{(p - q)2p}{(p - \phi)(q - \phi)} + 1\right) \hat{\alpha}(\phi)\right]$  which is smaller than zero since  $\hat{\alpha}(\phi) = \frac{q - \phi}{p - \phi}$  by definition. Hence,  $\frac{\partial \alpha^{ST}(\phi)}{\partial \phi} < 0$  whenever  $2\alpha^{ST}(\phi)(p - \phi)^2q^2 > p(q - \phi)^2(p - \phi)$ . Again replacing  $(p - \phi)^2q^2\alpha^{ST}(\phi)$  by the respective expression derived from (13) this is equivalent to  $2p(q -$

$\phi)^2((p - \phi) + \frac{\phi}{\alpha^{ST}(\phi)}) > p(q - \phi)^2(p - \phi)$ . Dividing by  $p(q - \phi)^2$  we get  $(p - \phi) + 2\frac{\phi}{\alpha^{ST}(\phi)} > 0$  which is always true. This proves the second claim. The last claim is obvious since for  $\phi = 0$  we have  $\tilde{\alpha}(0) = 1$  and  $k_2^S(0, \alpha) = 0$  and  $k_2^S(0, \alpha) = k_1^M$ . For  $\phi = q$  we have  $\tilde{\alpha}(q) = \hat{\alpha}(q) = 0$ .  $\square$

## Proof of Lemma 4.2

First notice that by (5) the difference between the manufacturer's stand-alone capacity  $k_1^M$  and her capacity choice in an interior equilibrium for  $\alpha \geq \tilde{\alpha}(\phi)$  is  $k_1^M - \tilde{k}^M(\phi, \alpha) = (1 - \alpha)\frac{(p-\phi)}{p}\tilde{k}^S(\alpha, \phi)$ , which is zero for  $\alpha = \tilde{\alpha}(\phi)$  and for  $\alpha = 1$  and strictly positive otherwise. The derivative of  $\tilde{k}^M(\phi, \alpha)$  with respect to  $\alpha$  is

$$\frac{\partial \tilde{k}^M(\phi, \alpha)}{\partial \alpha} = \frac{(p - \phi)(2q - \phi)}{\alpha^2(\alpha(p - \phi) + \phi)^2} h^M(\alpha) \bar{X},$$

where  $h^M(\alpha) = \alpha^2 - 2\alpha\frac{q-\phi}{2q-\phi} - \frac{\phi(q-\phi)}{(p-\phi)(2q-\phi)}$ . Hence,  $\tilde{k}^M(\phi, \alpha)$  is increasing in  $\alpha$  whenever  $h^M(\alpha) > 0$  and decreasing in  $\alpha$  whenever  $h^M(\alpha) < 0$ . The quadratic function  $h^M(\alpha)$  is negative for  $\alpha = 0$  and has a minimum at  $\alpha = \frac{q-\phi}{2q-\phi} > 0$ . To prove that  $\tilde{k}^M(\phi, \alpha)$  is first decreasing and then increasing in  $\alpha$  we therefore have to show that  $h^M(\tilde{\alpha}(\phi)) < 0$  and  $h^M(1) > 0$ . The first requirement is equivalent to  $p(q - \phi)(qp - \phi(p - q)) < q(p - \phi)(qp - \phi(p - q))$  which is true since  $\tilde{\alpha}(\phi) < 1$  and  $qp > \phi(p - q)$ . The second requirement is equivalent to  $(p - \phi)q > (q - \phi)p$  which is true since  $p > q$ .

The derivative of  $\tilde{k}^M(\phi, \alpha)$  with respect to  $\phi$  is

$$\frac{\partial \tilde{k}^M(\phi, \alpha)}{\partial \phi} = -\frac{1 - \alpha}{\alpha} \frac{(1 - \alpha)q + \alpha(p - q)}{(\alpha(p - \phi) + \phi)^2} \bar{X} < 0.$$

To verify the last claim first notice that  $k_2^M(\phi, \alpha) = k_1^M - \frac{q(1-\alpha)(p-\phi)}{p(\alpha(p-\phi)+\phi)} \bar{X}$ , by

(8) and (10). The derivative of  $k_2^M(\phi, \alpha)$  with respect to  $\alpha$  is

$$\frac{\partial k_2^M(\phi, \alpha)}{\partial \alpha} = \frac{(p - \phi)q}{(\alpha(p - \phi) + \phi)^2} \bar{X} > 0.$$

The derivative of  $k_2^M(\phi, \alpha)$  with respect to  $\phi$  is

$$\frac{\partial k_2^M(\phi, \alpha)}{\partial \phi} = \frac{(1 - \alpha)q}{(\alpha(p - \phi) + \phi)^2} \bar{X} > 0.$$

□

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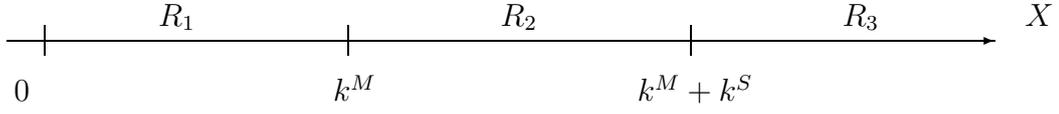


Figure 1: Capacities and Demand

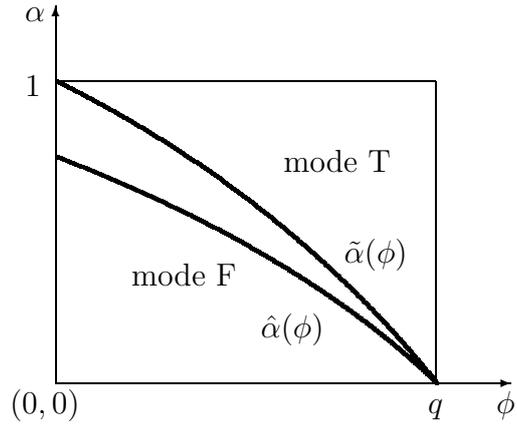


Figure 2: Equilibrium Provision Modes

$\phi$	$\hat{\alpha}(\phi)$	$\tilde{\alpha}(\phi)$	$\alpha^{ST}(\phi)$	$\alpha^M(\phi)$	$\alpha^S(\phi)$
0.05	0.20	0.98	0.96	0.98	0.96
0.25	0.18	0.90	0.81	0.90	0.81
0.45	0.16	0.81	0.67	0.80	0.67
0.65	0.14	0.72	0.55	0.71	0.55
0.85	0.13	0.63	0.44	0.61	0.49
1.05	0.11	0.53	0.34	0.51	0.46
1.25	0.09	0.43	0.25	0.41	0.43
1.45	0.06	0.32	0.17	0.31	0.39
1.65	0.04	0.21	0.11	0.21	0.35
1.85	0.02	0.09	0.04	0.11	0.30

**Table I:** Numerical Example

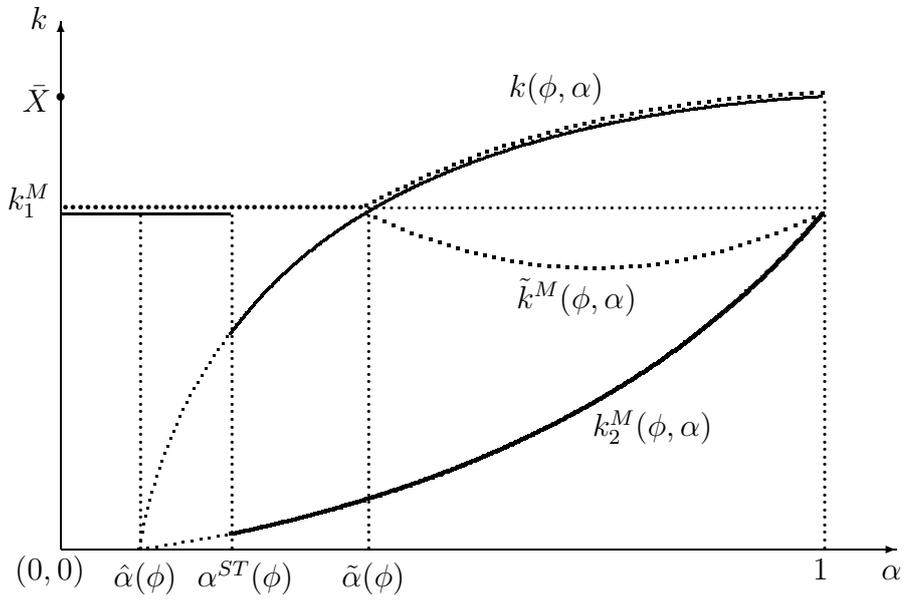


Figure 3: Equilibrium Capacities for a Given Outside Price  $\phi$

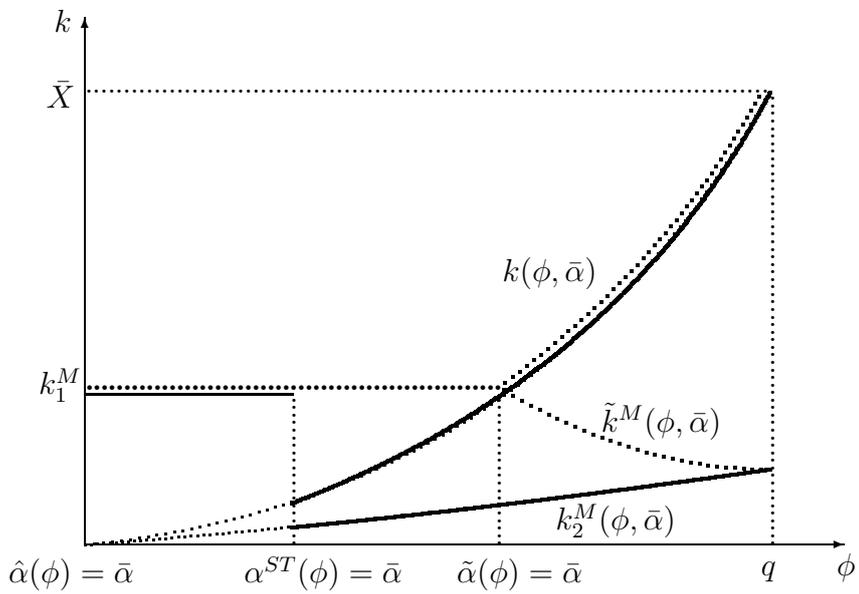


Figure 4: Equilibrium Capacities for a Given Bargaining Power  $\bar{\alpha} \leq \frac{q}{p}$