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Voluntary Contributions when the Public Good Is not Necessarily Normal

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We argue that there are interesting examples of privately provided public goods that do not satisfy the assumption of strict normality, and reconsider voluntary-contribution games in a more general framework. We show that, in general, (1) equalizing transfers between individuals with identical tastes can increase total supply of the public good, and (2) more of the public good can be supplied if agents move sequentially rather than simultaneously. These results are in sharp contrast to earlier conclusions derived in the literature under the assumption of strict normality.

Keywords: public goods, private provision, nonnormality.

JEL classification: H41.

1 Introduction

A standard assumption in the theory of private provision of public goods is strict normality of the public good(s) for all consumers at all levels of wealth. This assumption seems to be motivated mainly by technical reasons since together with strict normality of private consumption it guarantees uniqueness of Nash equilibrium in the simultaneous-move game.

In this paper, we argue that there are interesting examples of privately provided public goods for which the strict-normality assumption is not justified. Indeed, this assumption does not naturally apply to a variety of (small) private organizations that promote the interests of certain groups in the population and that depend on voluntary contributions. As an example, consider private organizations that promote the erection of public parks, or public playgrounds for children.[1] Certainly, such organizations do provide a public good. However, it is

1 In Germany, for instance, there is a considerable number of private organizations *(Bürgerinitiativen)* pursuing various goals of this kind.

not clear that demand for this good is everywhere strictly increasing with income. To the contrary, the possibility of substitution by private goods suggests that the demand for public parks is decreasing at high income levels. Indeed, high-income groups probably substitute "consumption" of public parks by, e.g., consumption of their own private gardens. Consequently, one should expect the following qualitative behavior of the demand for such public goods. Up to a certain critical income level demand increases with income. However, beyond that critical level demand decreases, i.e., the public good becomes inferior at high income levels. In the following, we refer to such a qualitative behavior of individual demand as unimodality of the individual Engel curve. As a second example, consider private organizations that promote the interests of groups in the population that suffer from certain illnesses. Such organizations campaign, among other things, for special concessions granted to these groups by public health care. Again, the public good provided does not seem to be strictly normal at every level of wealth. The reason is that most people in high-income classes do not depend on the public health-insurance system but own private insurance contracts. Hence, one may expect a similar qualitative behavior of individual demand for the public good as in the first example.

Clearly, one may think of many other examples of privately provided public goods that do not satisfy the common strict-normality assumption. Indeed, not even the standard example in the literature, donations to charity, is entirely unambiguous in this respect. While, in principle, the normality assumption has intuitive appeal in this context, there are also motives for giving to charity that do not support this assumption. For instance, one aspect of the good consumed through donations to charity is "absence of the threat of crime," or, put differently, "a feeling of safety in the social environment." For income that is not well above average there seems to be no possibility to substitute donations to charity in order to consume that kind of safety. High income, on the other hand, allows for such substitution. For instance, one can experience "a feeling of safety" by employing private security services, hiring bodyguards, and so on. Hence, if it is conceivable that some people donate to charity solely because of their desire to consume "absence of crime" the strict-normality assumption loses its plausibility even in this context.[2]

The purpose of the present paper is to reconsider the theory of

2 There is also some empirical evidence suggesting that higher-income individuals do not necessarily have a greater inclination to support the poor than lower-income individuals. For example, Gramlich and Rubinfeld (1982) use data from Michigan households to estimate income and price elasticities

private provision of public goods without imposing the normality assumption. Specifically, it is assumed that individual Engel curves are either strictly increasing everywhere (corresponding to strict normality), or unimodal, i.e., strictly increasing up to a certain critical income level and decreasing above that level. Our analysis reveals that many results obtained under the assumption of strict normality of public good(s) fail to hold in this more general setting. First, we prove that, in contrast to the strictly normal case analyzed by Varian (1994), sequential contributions to a public good can yield a larger equilibrium amount than simultaneous contributions. This suggests that the sequential-contribution setup might be relevant even beyond its descriptive adequacy in certain contexts: namely, as the result of a design strategy for a central agency that, in collecting individual donations, can decide to reveal information to prospective contributors in order to increase total supply of the public good. Secondly, we show that, in contrast to the results obtained by Bergstrom et al. (1986) for the strictly normal case, equalizing transfers between individuals with identical tastes can increase total supply of the public good, both in simultaneous- and sequential-move equilibrium.

The paper is organized as follows. Section 2 introduces our basic assumptions on individual preferences and demand, and discusses the resulting properties of individual reaction functions. Section 3 addresses the issue of existence and multiplicity of equilibria, both in the simultaneous- and the sequential-move game. Section 4 compares the equilibria in the corresponding games while Sect. 5 provides the analysis of wealth redistributions. A discussion of welfare effects implied by the various comparative-statics results is found in the concluding Sect. 6.

2 Voluntary-contribution Games

Consider a simple economy with two individuals, indexed by $i = 1, 2$, and two goods. Each individual's utility is given by a strictly quasi-concave utility function $u^i(c_i, G)$, where c_i denotes i's consumption of a private good and G the consumption of a purely public good. Each individual has an initial endowment of m_i^e units of the private good. For simplicity, let the price of the private good be equal to 1. Hence, one may think of m_i^e as consumer i's income. The public good is produced from the private good at a cost of one unit private good per unit of public good. Throughout, it is assumed that each consumer's preferences are strictly monotone and continuous. However, in contrast

for public spending. They find that on a microlevel income elasticity is positive for school spending but negative for welfare spending.

to most of the literature we do not assume that the public good is "normal" at every level of wealth. Instead, we will assume that for each individual there exists some income $\hat{m}_i$ such that the "demand" for the public good decreases in m_i for incomes larger than $\hat{m}_i$. Specifically, let $\tilde{G}_i(\cdot)$ denote consumer i's Engel curve for the public good, i.e., $\tilde{G}_i(m_i)$ is the amount of the public good supplied by agent i with income m_i given that she is the only supplier.[3] Given a current income m_i^{e}, $\tilde{G}_i(m_i^{\mathrm{e}})$ will also be referred to as agent i's standalone contribution. Our basic assumption is as follows.

Assumption 1 (unimodality): For individual i, either $\tilde{G}_i(\cdot)$ is strictly increasing everywhere, or there exists $\hat{m}_i > 0$ such that $\tilde{G}_i(\cdot)$ is strictly increasing in the interval $[0, \hat{m}_i)$ and decreasing in $[\hat{m}_i, \infty)$. Furthermore, in the latter case, $\tilde{G}_i(\cdot)$ is strictly decreasing in $[\hat{m}_i, \hat{\hat{m}}_i)$ for some $\hat{\hat{m}}_i > \hat{m}_i$.

Note that Assumption 1 includes the standard assumption of strict normality of the public good everywhere as a special case. However, for the most part of this paper we will be interested in the case of "proper" unimodality, i.e., in settings where the public good is strictly inferior at some (high) incomes. For private consumption, on the other hand, we will assume noninferiority everywhere. Note that this implies that $\tilde{G}_i(\cdot)$ can never increase at a rate greater than 1.

The public good is supplied by voluntary contributions of the consumers. For each i, let g_i denote consumer i's contribution to the public good. In the following, we will consider two different models of voluntary contributions. The first is the simultaneous-move game analyzed by Warr (1983) and Bergstrom et al. (1986), among others. In this game, the basic assumption is that the individuals choose their contributions simultaneously, taking the activities of the other agent(s) as given. Consequently, consumer i's decision problem is

$$\max_{c_i, g_i} u^i(c_i, g_i + g_j) \tag{1}$$

$$\text{s.t. } c_i + g_i = m_i^{\mathrm{e}} \quad \text{and} \tag{2}$$

$$g_i \geq 0 \,, \tag{3}$$

where $j \neq i$. A pair (c_1^*, c_2^*) and (g_1^*, g_2^*) satisfying (1)–(3) for $i = 1, 2$

3 Note that since prices for both goods are held fixed throughout the paper, demand and Engel curves coincide.

is a Nash equilibrium of the corresponding contribution game. In order to distinguish the equilibrium of this game from that of the game described below, we will refer to it as Cournot–Nash equilibrium, emphasizing the simultaneous nature of the underlying game and its formal similarity to the model of Cournot competition in industrial organization.

The second model considered is Stackelberg leadership of one individual. This model was first examined in Varian (1994) and further analyzed in Buchholz et al. (1997), and Kerschbamer and Puppe (1997a) (for the case of incomplete information, see also Haslbeck, 1995). Here, the assumption is that one individual, the Stackelberg leader, can commit to a certain quantity of contribution while anticipating the optimal response of the follower. Such a possibility of unilateral commitment may be due to the timing of decisions, i.e., when contributions are made sequentially with the Stackelberg leader ("her") moving first. Suppose that individual 1 is the leader. Then, her problem is: $\max_{c_1, g_1} u^1(c_1, g_1 + g_2^{\mathrm{R}}(g_1))$, s.t. $c_1 + g_1 = m_1^{\mathrm{e}}$ and $g_1 \geq 0$, where $g_2^{\mathrm{R}}(\cdot)$ denotes the follower's ("his") reaction function, i.e., for any given g_1, $g_2^{\mathrm{R}}(g_1)$ is the solution of the follower's problem (1)–(3).

In order to solve problem (1)–(3) it is convenient to rewrite (2) by adding g_j on both sides of the equation. This yields $c_i + G = m_i^{\mathrm{e}} + g_j$. Using this, it is easily verified that i's reaction function can be written as follows. For all $g_j \geq 0$, $j \neq i$,

$$g_i^{\mathrm{R}}(g_j) := g_i^{\mathrm{R}}(m_i^{\mathrm{e}}, g_j) := \max\{\tilde{G}_i(m_i^{\mathrm{e}} + g_j) - g_j, 0\} \, . \tag{4}$$

In (4), $\tilde{G}_i(\cdot)$ denotes, as before, individual i's Engel curve for the public good, i.e., the unique solution to $\max_G u^i(m_i - G, G)$. Consequently, for all $g_i^{\mathrm{R}}(g_j) > 0$, individual i's best response to the other agent's contribution g_j can be thought of as i's demand for the public good if her income was $m_i^{\mathrm{e}} + g_j$, minus the other agent's contribution. Clearly, given continuity of u^i, $g_i^{\mathrm{R}}(\cdot)$ will be a continuous function. In order to describe the qualitative behavior of $g_i^{\mathrm{R}}(\cdot)$ implied by our assumptions on individual preferences, let $\hat{g}_j := \hat{g}_j(m_i^{\mathrm{e}}) := \max\{\hat{m}_i - m_i^{\mathrm{e}}, 0\}$, where $i \neq j$ and m_i^{e} is i's actual income. If $g_i^{\mathrm{R}}(\hat{g}_j)$ is positive, our assumptions on individual demand for the public good imply that $g_i^{\mathrm{R}}(\cdot)$ is decreasing at a rate less than 1 in the interval $[0, \hat{g}_j)$, and at a rate greater than 1 for values of g_j greater than $\hat{g}_j$ [until $g_i^{\mathrm{R}}(\cdot)$ becomes zero].[4] Next, define

4 Hence, just as $\hat{m}$ demarcates the "normal" region of the Engel curve (positive slope) from the "inferior" region (negative slope), $\hat{g}$ demarcates the "flat" region of the reaction function (slope greater than -1) from the "steep"

$\bar{g}_j := \bar{g}_j(m_i^e) := \min\{ g_j : \tilde{G}_i(m_i^e + g_j) = g_j \}$. Note that by continuity of $g_i^R(\cdot)$ the minimum exists, so that $\bar{g}_j$ is well defined. Given that $\tilde{G}_i(\cdot)$ can never increase at a rate greater than 1, i.e., given that private consumption is noninferior everywhere, one obtains that $g_i^R(g_j) = 0$ if and only if $g_j \geq \bar{g}_j$. That is, at point $\bar{g}_j$ agent j contributes so much that agent i chooses to contribute zero. Therefore, $\bar{g}_j$ is called agent j's complete-crowding-out contribution. Finally, define m_i^0 by $m_i^0 := \hat{m}_i - \tilde{G}_i(\hat{m}_i)$. Obviously, $0 \leq m_i^0 < \hat{m}_i$. The following observation summarizes the qualitative behavior of the reaction function.

Fact 1: a. Suppose that $0 < m_i^e \leq m_i^0$. Then, the reaction function $g_i^R(\cdot)$ is decreasing at a rate less than 1 in the interval $[0, \bar{g}_j)$ and $g_i^R(g_j) = 0$ for all $g_j \geq \bar{g}_j$ (see Fig. 1a).

b. Suppose that $m_i^0 < m_i^e < \hat{m}_i$. Then, the reaction function $g_i^R(\cdot)$ is decreasing at a rate less than 1 in the interval $[0, \hat{g}_j)$. Furthermore, the reaction function is decreasing at a rate greater than 1 in the interval $[\hat{g}_j, \bar{g}_j)$, and it is zero for all $g_j \geq \bar{g}_j$ (see Fig. 1b).

c. Suppose that $m_i^e \geq \hat{m}_i$. Then, the reaction function $g_i^R(\cdot)$ is decreasing at a rate greater than 1 in the interval $[0, \bar{g}_j)$, and it is zero for all $g_j \geq \bar{g}_j$ (see Fig. 1c and d).

In order to verify this, consider first case a. First, we show that $\hat{g}_j < \bar{g}_j$ if and only if $m_i^e > m_i^0$. Indeed, $\hat{g}_j < \bar{g}_j$ if and only if $g_i^R(\hat{g}_j) > 0$. However, by (4) this is equivalent to $\tilde{G}_i(\hat{m}_i) - (\hat{m}_i - m_i^e) > 0$, i.e., to $m_i^e > m_i^0$. Hence, in case a, $\bar{g}_j \leq \hat{g}_j$. This implies that the reaction function becomes zero before it reaches the region where its slope can become smaller than -1. Since private consumption is noninferior everywhere, $g_i^R(\cdot)$ can nowhere be increasing. Consequently, $g_i^R(\cdot)$ looks qualitatively as shown in Fig. 1a.

Given our definitions and notation, case b is straightforward: By assumption, $\tilde{G}_i(m_i)$ is strictly increasing at a rate less or equal to 1 in the interval $[0, \hat{m}_i)$. Hence, if $m_i^0 < m_i^e < \hat{m}_i$, then by (4), $g_i^R(\cdot)$ is decreasing at a rate less than 1 in the interval $[0, \hat{g}_j)$. Similarly, since $\tilde{G}_i(m_i)$ is decreasing in the interval $[\hat{m}_i, \infty)$, $g_i^R(\cdot)$ is decreasing at a rate greater than or equal to 1 in the interval $[\hat{g}_j, \bar{g}_j)$, and it is zero for all $g_j \geq \bar{g}_j$ (see Fig. 1b).

region (slope smaller than -1). The hat has been chosen as a graphical representation of the qualitative change of the corresponding function.

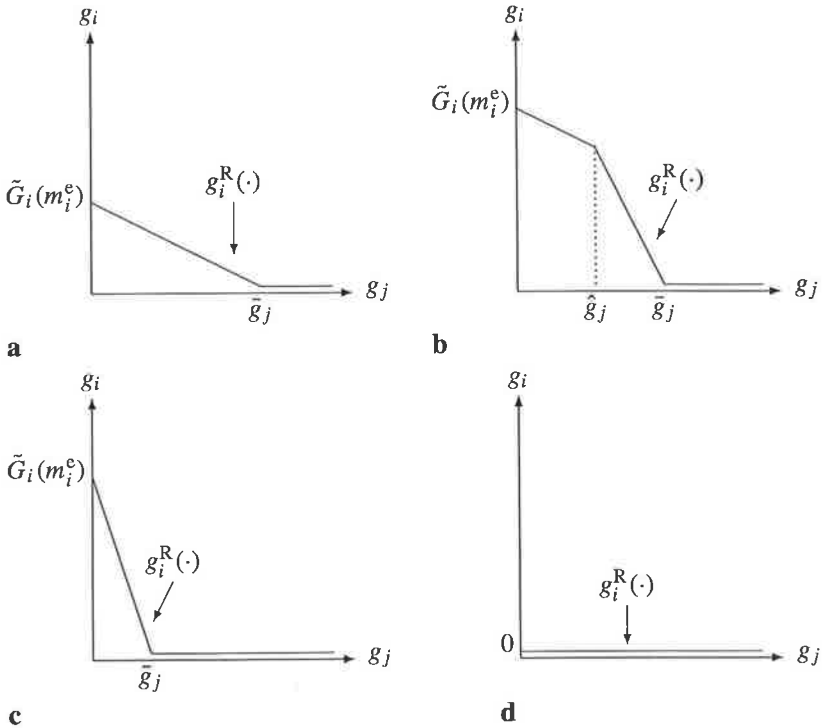

Fig. 1: Qualitative behavior of g_i^R

Finally, in case c, when $m_i^e \geq \hat{m}_i$, $\tilde{G}_i(m_i^e + \cdot)$ is decreasing everywhere. Hence, if $\tilde{G}_i(m_i^e) > 0$, $g_i^R(\cdot)$ is decreasing at a rate greater or equal to 1 until it hits the axis (Fig. 1c). On the other hand, there is also the possibility that $\tilde{G}_i(m_i^e) = 0$ in which case $g_i^R(\cdot) = 0$ everywhere (Fig. 1d).

3 Equilibria

Existence of Cournot–Nash and Stackelberg equilibria follows from standard arguments. However, in contrast to the case where both private consumption and public good are strictly normal, there will, in general, be a multiplicity of Cournot–Nash equilibria.

3.1 Cournot–Nash Equilibria

In order to illustrate the possibility of multiple Cournot–Nash equilibria, consider the following example.

Example 1: Suppose that the two individuals have identical preferences given by the following utility function.[5] For all $c_i, G \geq 0$, $(c_i, G) \neq (0, 0)$, $u(c_i, G) = 2\ln(c_i + 2G) + c_i$. It is easily verified that u is strictly quasi-concave and that the associated individual Engel curves are given by

$$\tilde{G}(m_i) = \begin{cases} m_i & \text{if } m_i \leq 1, \\ 2 - m_i & \text{if } 1 \leq m_i \leq 2, \\ 0 & \text{if } 2 \leq m_i. \end{cases}$$

Calculating i's reaction function gives

$$g_i^{\mathrm{R}}(g_j) = \begin{cases} m_i^{\mathrm{e}} & \text{if } g_j \leq 1 - m_i^{\mathrm{e}}, \\ 2 - m_i^{\mathrm{e}} - 2g_j & \text{if } 1 - m_i^{\mathrm{e}} < g_j < 1 - m_i^{\mathrm{e}}/2, \\ 0 & \text{if } 1 - m_i^{\mathrm{e}}/2 \leq g_j, \end{cases}$$

where $i \neq j$. Suppose that $m_1^{\mathrm{e}} = 0.9$ and $m_2^{\mathrm{e}} = 1$. Then there exist exactly three Cournot–Nash equilibria with the following distribution of individual contributions: (0.9, 0), (0, 1), and (0.3, 0.4) (see Fig. 2).[6]

3.2 Stackelberg Equilibria

In contrast to Cournot–Nash equilibrium, multiplicity of Stackelberg equilibria is also an issue when the public good is strictly normal. First, we consider the case in which the public good is inferior for the follower at his initial level of wealth. The following result shows that in this case there can be at most two Stackelberg equilibria.

Fact 2: Suppose that in a Stackelberg game with individual 1 as leader,

5 We are indebted to Klaus Nehring for suggesting the form of the utility function used in this example.

6 Note that an issue of stability arises here. Indeed, in the above example the interior equilibrium is not stable with respect to any dynamics where agents adjust their contribution in direction to their best responses. However, the focus of our analysis is not on stability and the issue is not relevant for our main results.

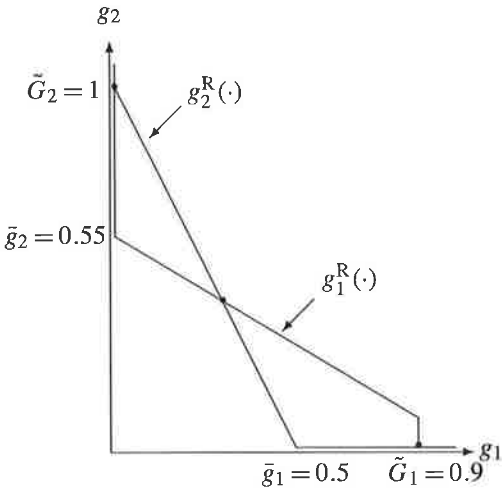

Fig. 2: Multiple Cournot–Nash equilibria

the follower's Engel curve $\tilde{G}_2(\cdot)$ is decreasing in the interval $[m_2^e, \infty)$. Then, there exist at least one and at most two Stackelberg equilibria. The two potential equilibria are $(g_1^S, g_2^S) = (0, \tilde{G}_2(m_2^e))$ and $(g_1^S, g_2^S) = (\tilde{G}_1(m_1^e), 0)$.

Proof: Obviously, among all $(g_1, 0)$, the leader's standalone contribution $g_1 = \tilde{G}_1(m_1^e)$ maximizes her utility. Consequently, $(\tilde{G}_1(m_1^e), 0)$ is at least a candidate for equilibrium (though not always actually an equilibrium). Next, suppose that the follower chooses his contribution from the strictly positive region of his reaction function $g_2^R(\cdot)$. We show that in this case $g_1^S = 0$ is optimal for the leader. The leader maximizes

$$u^1\left(m_1^e - g_1, g_1 + g_2^R(g_1)\right) = u^1\left(m_1^e - g_1, \tilde{G}_2(m_2^e + g_1)\right) .$$

By assumption, $\tilde{G}_2(m_2^e + \cdot)$ is decreasing. Hence, by strict monotonicity of preferences, $u^1(m_1^e, \tilde{G}_2(m_2^e)) > u^1(m_1^e - g_1, \tilde{G}_2(m_2^e + g_1))$ for all $g_1 > 0$. Consequently, $g_1^S = 0$ is optimal for the leader. □

Fact 2 shows that – if the public good is inferior for the follower at his initial level of wealth – there cannot exist a Stackelberg equilibrium in which both agents contribute a positive amount. For the intuition behind this fact, let $T(g_1) := g_1 + g_2^R(g_1)$ denote total provision of the public good when the leader chooses g_1. The follower being in his inferior region at $m_2 = m_2^e$ implies that his reaction function is decreasing at a rate greater than, or equal to, 1 in the whole positive range, so that $T(g_1)$ is decreasing in g_1 for all $g_1 < \bar{g}_1$. Consequently, choosing a contribution in the interval $(0, \bar{g}_1)$ can never be optimal for the leader, since any reduction of her contribution would increase not only her private consumption but also the total quantity of the public good.

Remark: Suppose, as in Fact 2, that $\tilde{G}_2(\cdot)$ is nonincreasing in $[m_2^e, \infty)$. A sufficient condition for $(0, \tilde{G}_2(m_2^e))$ to be the unique Stackelberg equilibrium is that $\tilde{G}_2(m_2^e) \geq \tilde{G}_1(m_1^e)$. This follows at once from monotonicity of preferences.

By Fact 2, if the follower is in his "inferior region" there is only one equilibrium in which he contributes a positive amount. On the other hand, it is quite clear that if $\tilde{G}_2(\cdot)$ is strictly increasing at m_2^e there may be several Stackelberg equilibria in which the follower chooses a positive contribution. A sufficient condition that rules this out is concavity of $\tilde{G}_2(\cdot)$ in the region where it is increasing.

Assumption 2 (concavity): Individual i's Engel curve $\tilde{G}_i(\cdot)$ is a concave function in its increasing part, i.e., on the interval $[0, \hat{m}_i)$.

Fact 3: Suppose that the follower's Engel curve satisfies Assumptions 1 and 2. Then there exist at least one and at most two Stackelberg equilibria. Furthermore, there exists at most one Stackelberg equilibrium (g_1^S, g_2^S) in which the follower (individual 2) contributes a positive amount. In any such equilibrium, $g_1^S \leq \hat{g}_1$.

Proof: As in Fact 2, a possible candidate for equilibrium is $(\tilde{G}_1(m_1^e), 0)$. Next, suppose that the follower chooses his contribution from the strictly positive region of his reaction function $g_2^R(\cdot)$. By Fact 2, the leader's utility is strictly decreasing in the interval $(\hat{g}_1, \bar{g}_1]$ in that case. Hence, in any Stackelberg equilibrium with $g_2^S > 0$ one must have $0 \leq g_1^S \leq \hat{g}_1$. Suppose there were two equilibrium values $g_1^S \neq g_1'^S$ with $0 \leq g_1^S, g_1'^S \leq \hat{g}_1$. Let

$$u_{\max}^1 := u^1\big(m_1 - g_1^S, \tilde{G}_2(m_2^e + g_1^S)\big) = u^1\big(m_1 - g_1'^S, \tilde{G}_2(m_2^e + g_1'^S)\big) \ .$$

Consider the value $g_1^0 := (g_1^S + g_1'^S)/2$. By strict quasi-concavity of u^1, $u^1(m_1 - g_1^0, \frac{1}{2}[\tilde{G}_2(m_2^e + g_1^S) + \tilde{G}_2(m_2^e + g_1'^S)]) > u^1_{max}$. Since $g_1^S, g_1'^S \leq \hat{g}_1$ we know that $0 \leq m_2^e + g_1^S, m_2^e + g_1'^S \leq \hat{m}_2$. Hence by concavity of $\tilde{G}_2(\cdot)$ in that region, $\frac{1}{2}[\tilde{G}_2(m_2^e + g_1^S) + \tilde{G}_2(m_2^e + g_1'^S)] \leq \tilde{G}_2(m_2^e + g_1^0)$.

Consequently, by monotonicity of u^1, $u^1(m_1^e - g_1^0, \tilde{G}_2(m_2^e + g_1^0)) > u^1_{max}$, which is obviously not possible. Hence, a utility-maximizing value $g_1^S \in [0, \hat{g}_1]$ must be unique. □

To illustrate the role of concavity of the follower's Engel curve in Fact 3, consider potential equilibria with $g_2^S > 0$ and $g_1^S \leq \hat{g}_1$, and let $T(g_1) = g_1 + g_2^R(g_1)$ denote total provision of the public good. Concavity of the follower's Engel curve in $[0, \hat{m}_2]$ implies concavity of his reaction function in $[0, \hat{g}_1]$, and hence concavity of $T(\cdot)$ in this range. In words, if the leader increases her own contribution in the considered interval, each additional dollar of contribution has a smaller effect on the total quantity of the public good than the previous one. From the leader's point of view this is equivalent to an increasing marginal cost of providing the public good. Since her marginal evaluation for the public good is decreasing there must be a unique optimal value g_1 in the considered range. This potential candidate for Stackelberg equilibrium has to be compared with the second candidate $g_1 = \tilde{G}_1(m_1^e)$. The one with higher utility for the leader becomes the actual Stackelberg equilibrium.

4 Comparison of Cournot–Nash and Stackelberg Equilibria

This section is devoted to the comparison of Cournot–Nash and Stackelberg equilibria. In particular, we show that, contrary to the case where the public good is strictly normal, total provision in the sequential-move game may be higher than in the corresponding simultaneous-move game. The following preliminary result does not depend on specific assumptions on individual demand for the public good and can be proved along the lines of theorem 2 in Varian (1994).

Fact 4: The amount of the public good provided by the leader in any Stackelberg equilibrium is never larger than the smallest amount she will provide in any Cournot–Nash equilibrium.

Note that by Fact 4, if $(0, \tilde{G}_2(m_2))$ is a Cournot–Nash equilibrium in the simultaneous-move game, then $(0, \tilde{G}_2(m_2))$ is the unique Stackelberg equilibrium of the sequential-move game. Furthermore, one has the following corollary.

Corollary 1: Suppose that there is a Cournot–Nash equilibrium (g_1^*, g_2^*) with $0 \leq g_1^* \leq \hat{g}_1$. Then total provision of the public good in any Stackelberg equilibrium is not greater than $g_1^* + g_2^*$.

Indeed, by Fact 4, in any Stackelberg equilibrium (g_1^S, g_2^S), one must have $g_1^S \leq g_1^*$. The conclusion thus follows from the fact that $g_1 + g_2^R(g_1)$ is increasing in the interval $[0, \hat{g}_1]$.

If the public good is a normal good, Corollary 1 implies that the total amount of the public good provided in a Stackelberg equilibrium is never larger than that provided in any Cournot–Nash equilibrium. This conclusion, however, is not valid under our present assumption that the public good may become inferior at high income levels. To illustrate the possibility that in a Stackelberg equilibrium total provision of the public good is greater than in any Cournot–Nash equilibrium consider the following example.

Example 2: Suppose that with the utility function specified in Example 1, incomes are given by $m_1^e = 0.5$ and $m_2^e = 1.4$, respectively. In this case, the unique Cournot–Nash equilibrium is $(0.5, 0)$ (see Fig. 3). On the other hand, it is easily verified that $\tilde{G}_2(m_2^e) = 0.6 > \tilde{G}_1(m_1^e) = 0.5$, hence by the remark on Fact 2, $(0, 0.6)$ is the unique Stackelberg equilibrium. Thus total provision of the public good is greater in Stackelberg than in Cournot–Nash equilibrium.

The following result gives sufficient conditions for total provision of the public good never being less in Stackelberg equilibrium than in any Cournot–Nash equilibrium.

Theorem 1: Suppose that in a Stackelberg game with individual 1 as leader, the follower's Engel curve $\tilde{G}_2(\cdot)$ is decreasing in $[m_2^e, \infty)$, i.e., the follower is in his "inferior" region. Furthermore, suppose that $\tilde{G}_2(m_2^e) \geq \tilde{G}_1(m_1^e)$. Then, total provision of the public good in the unique Stackelberg equilibrium is at least as great as in any Cournot–Nash equilibrium.

Proof: Under the assumptions of Theorem 1, $(g_1^S, g_2^S) = (0, \tilde{G}_2(m_2^e))$ is the unique Stackelberg equilibrium (cf. Fact 2 and the following

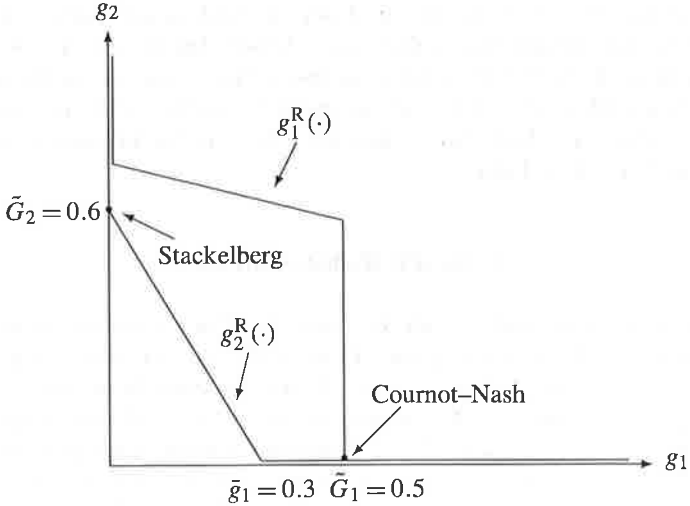

Fig. 3: Cournot–Nash and Stackelberg equilibria

remark). By $\tilde{G}_2(m_2^e) \geq \tilde{G}_1(m_1^e)$, it suffices to show that $\tilde{G}_2(m_2^e)$ is always at least as great as the total amount of public good provided in any Cournot–Nash equilibrium (g_1^*, g_2^*) with $0 \leq g_1^* < \bar{g}_1$. In any such equilibrium, total amount of public good is

$$g_2^R(g_1^*) + g_1^* = \tilde{G}_2(m_2^e + g_1^*) \ .$$

Since $\tilde{G}_2(\cdot)$ is decreasing in $[m_2^e, \infty)$ the conclusion follows. □

Remark: Suppose that $\tilde{G}_2(\cdot)$ is decreasing in $[m_2^e, \infty)$ and strictly so at m_2^e. Furthermore, assume that $\tilde{G}_2(m_2^e) > \tilde{G}_1(m_1^e)$. Then total provision of the public good in Stackelberg equilibrium is strictly greater than in any Cournot–Nash equilibrium if and only if $\tilde{G}_2(m_2^e) < \bar{g}_2$. This is easily verified along the lines of Theorem 1 noting that $\tilde{G}_2(m_2^e) < \bar{g}_2$ holds if and only if $(0, \tilde{G}_2(m_2^e))$ (the unique Stackelberg equilibrium) is not a Cournot–Nash equilibrium.

It is emphasized that the assumption of the follower being in his "inferior region" alone is not sufficient for the conclusion of Theorem 1. That is, the additional condition $\tilde{G}_2(m_2^e) \geq \tilde{G}_1(m_1^e)$ cannot be dropped. For instance, with $m_1^e = 0.8$ and $m_2^e = 1.5$ in Example 1, it can be verified that (0.8, 0) is the unique Cournot–Nash equilibrium, whereas (0, 0.5) is the unique Stackelberg equilibrium. On the other hand, the assumption of the follower being in his "inferior region" at his initial level of wealth is also not necessary for the conclusion of Theorem 1. What is necessary, however, is that $m_2^e + g_1^*$ is in the inferior region of the follower's Engel curve.

5 Wealth Redistributions

We are now ready to deal with the issue of income redistributions. The section is divided into two parts. In the first part, we state some general facts about redistribution that are independent from our specific assumptions on individual demand. These facts are closely related to the respective neutrality results for interior Cournot–Nash and Stackelberg equilibria. The second part investigates redistributions under the assumption of identical preferences. In particular, we demonstrate that under unimodality of individual Engel curves equalizing transfers can increase the total supply of the public good.

5.1 General Facts about Redistributions

In this section, unless otherwise indicated, we will make no specific assumptions about individual demand other than those implied by quasi-concavity of individual utility functions. For the record, we first state Warr's (1983) well-known neutrality result for interior Cournot–Nash equilibria.

Fact 5: Let (g_1^*, g_2^*) be an interior Cournot–Nash equilibrium (i.e., g_1^*, $g_2^* > 0$) corresponding to the income distribution (m_1, m_2). Furthermore, let Δm be such that $g_1^* - \Delta m, g_2^* + \Delta m \geq 0$. Then, $(g_1^* - \Delta m, g_2^* + \Delta m)$ is a Cournot–Nash equilibrium corresponding to the income distribution $(m_1 - \Delta m, m_2 + \Delta m)$.

Contrary to a claim in Varian (1994), the corresponding neutrality result for interior Stackelberg equilibria is more subtle. Indeed, in Stackelberg equilibrium transfers from the follower to the leader can discontinuously increase total supply of the public good even when the amount transferred falls short of the follower's original contribution.[7] As a con-

7 For a simple example showing this, see Kerschbamer and Puppe (1997a).

sequence, the following neutrality result for interior Stackelberg equilibria only applies in one direction, namely to transfers from the leader to the follower (for a proof, see Kerschbamer and Puppe, 1997b).

Fact 6: Let (g_1^S, g_2^S) be an interior Stackelberg equilibrium (with individual 1 as leader) corresponding to the income distribution (m_1, m_2). Let $\Delta m > 0$ be a positive transfer from the leader to the follower such that $g_1^S - \Delta m \geq 0$. Then $(g_1^S - \Delta m, g_2^S + \Delta m)$ is a Stackelberg equilibrium corresponding to the income distribution $(m_1 - \Delta m, m_2 + \Delta m)$.

Facts 5 and 6 allow in a straightforward manner to deduce upper bounds for total provision of the public good in equilibrium. Denote by M total income, i.e., $M = m_1 + m_2$. Furthermore, for $i = 1, 2$, let $\tilde{G}_i^{\max}$ denote the maximal standalone contribution of individual i that can be obtained by redistributing wealth, i.e., $\tilde{G}_i^{\max} := \max_{0 \leq m_i \leq M} \tilde{G}_i(m_i)$.

Corollary 2: In any interior Cournot–Nash equilibrium with aggregate income M, total supply of the public good is never larger than $\min\{\tilde{G}_1^{\max}, \tilde{G}_2^{\max}\}$. Moreover, total supply of the public good in an arbitrary Cournot–Nash equilibrium is never larger than $\max\{\tilde{G}_1^{\max}, \tilde{G}_2^{\max}\}$.

Proof: Let (g_1^*, g_2^*) be an interior equilibrium corresponding to the income distribution (m_1, m_2). By Fact 5, $(0, g_1^* + g_2^*)$ and $(g_1^* + g_2^*, 0)$ are Cournot–Nash equilibria corresponding to the income distributions $(m_1 - g_1^*, m_2 + g_1^*)$ and $(m_1 + g_2^*, m_2 - g_2^*)$, respectively. Hence, $g_1^* + g_2^* = \tilde{G}_2(m_2 + g_1^*) \leq \tilde{G}_2^{\max}$, and $g_1^* + g_2^* = \tilde{G}_1(m_1 + g_2^*) \leq \tilde{G}_1^{\max}$. This proves the first assertion. From this, the second assertion follows at once. □

The proof of the corresponding result for Stackelberg equilibrium is similar, noting that the argument is valid only for transfers from the leader to the follower.

Corollary 3: Let aggregate income be given by M, and let individual 1 be the leader in a Stackelberg game. In any interior Stackelberg equilibrium total supply of the public good is never larger than $\tilde{G}_2^{\max}$. Moreover, total supply of the public good in an arbitrary Stackelberg equilibrium is never larger than $\max\{\tilde{G}_1^{\max}, \tilde{G}_2^{\max}\}$.

5.2 Redistributions with Identical Tastes

Consider now redistributions among individuals with identical tastes. Suppose that individual Engel curves satisfy Assumption 1, and let M denote aggregate income. Since we assume identical tastes now, we may omit subscripts referring to individuals in all symbols related to individual Engel curves and reaction functions. For instance, $\hat{m}$ denotes (as before) the unique maximizer of the individuals' Engel curve $\tilde{G}(\cdot)$ (if a maximizer exists at all, i.e., if there is an "inferior" region; otherwise, of course, $\hat{m} = \infty$).

First, observe that for any distribution of income, total supply of the public good in equilibrium (Cournot–Nash or Stackelberg with individual 1 as leader) is never larger than $\tilde{G}(m^*)$, where $m^* := \min\{M, \hat{m}\}$. This follows at once from Corollaries 2 and 3 since $\tilde{G}(m^*)$ is the maximal standalone contribution. Secondly, it is easily verified that for the distribution $(M - m^*, m^*)$, the unique Stackelberg equilibrium is $(0, \tilde{G}(m^*))$ in which total supply of the public good attains its maximal value.

In the case of a strictly normal public good, these conclusions are well in line with the finding of Bergstrom et al. (1986) that with identical tastes equalizing transfers can never increase total supply of the public good in Cournot–Nash equilibrium. Indeed, in that case the most unequal distribution which gives everything to one person yields the highest possible amount of public good (both, in the unique Cournot–Nash and the unique Stackelberg equilibrium). On the other hand, if the public good becomes inferior at some income level, and if there is enough income to redistribute, then an equalizing transfer may well induce an increase of total provision of the public good in Stackelberg equilibrium. For instance, if $M = 2\hat{m}$ the distribution that maximizes total supply of the public good is the equal distribution. To further illustrate the possible effects of redistribution consider again Examples 1 and 2. Obviously, the transition from the income distribution (0.5, 1.4) (cf. Example 2) to the distribution (0.9, 1) (cf. Example 1) involves an equalizing transfer. As a consequence of this transfer, total quantity of the public good supplied in the unique Stackelberg equilibrium rises from 0.6 to 1. Moreover, in this example total quantity of the public good supplied in any Cournot–Nash equilibrium after the transfer is strictly greater than total supply before the transfer.

6 Conclusion

We have shown that some of the standard results in the theory of private provision of public goods hinge on the questionable assumption of strict normality of the public good at every level of wealth. Once this assumption is relaxed, equalizing transfers among individuals with identical tastes may increase total supply of the public good, both in Stackelberg and in Cournot–Nash equilibrium. Furthermore, total provision of the public good may be higher in Stackelberg equilibrium than in any Cournot–Nash equilibrium.

Our results raise the question as to the welfare effects of income redistributions on the one hand, and of moving from a simultaneous- to a sequential-move game on the other. As to the welfare analysis of income redistributions, it has already been observed in Buchholz et al. (1997) that in the sequential-move setup income transfers from the follower to the leader can be strictly Pareto improving. Clearly, such a possibility of Pareto improvement cannot exist in the simultaneous-move game. Concerning the welfare comparison between Cournot–Nash and Stackelberg equilibria, it is easily verified that – unless the equilibria coincide – the leader's utility must be strictly higher in Stackelberg than in any Cournot–Nash equilibrium, while the opposite is true for the follower. Hence, no actual Pareto improvement can be obtained by moving from a simultaneous- to a sequential-move game, even if the Stackelberg equilibrium entails a larger amount of the public good. However, in the latter case there is room for potential Pareto improvements in the following sense: Given the Stackelberg quantity of the public good, there may exist a distribution of private consumption such that the resulting allocation Pareto-dominates the Cournot–Nash equilibrium. Note, however, that such a possibility of potential Pareto improvement may not always exist. For instance, it can be checked that the Cournot–Nash equilibrium in Example 2 is efficient although it involves a smaller amount of the public good than the corresponding Stackelberg equilibrium. The reason is that any Pareto improvement would necessarily entail a reduction of individual 1's private consumption which is not possible since 1's private consumption is already zero in equilibrium. Clearly, if the private good is strictly normal this can never happen, and any Cournot–Nash equilibrium necessarily entails underprovision of the public good. However, even in that case a Stackelberg equilibrium with a greater amount of public good may not be potentially Pareto superior to Cournot–Nash equilibrium. For instance, it may happen that one individual's private consumption in equilibrium is just too small to make the necessary adjustments.

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