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Are Two a Good Representative for Many?*

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This paper studies a discrete formulation of the screening model with type-dependent reservation utilities. It closes the gap between various pooling results derived in continuous-type models and the separating results obtained for the binary case. We show that binary models do not capture the most interesting features of general models with countervailing incentives. However, as soon as at least three types are introduced all interesting results of the continuum case can be replicated in a discrete framework. Two kinds of pooling can appear in the three-type model: "overtake"-pooling, which can be ruled out by standard monotonicity conditions; and "crash"-pooling, which is really a consequence of countervailing incentives. *Journal of Economic Literature* Classification Numbers: C72, D82.

1. INTRODUCTION

Throughout the hidden-information strand of the principal-agent literature either two types or a continuum of types is taken as representative for a world that typically would rather involve a large discrete type-set. While (under certain regularity conditions) both approaches yield qualitatively similar results when types can be ordered in a "proper way," this ceases to be the case as soon as countervailing incentives arise (for example, because of type-dependent reservation utilities).

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With countervailing incentives the continuous-type papers, as, e.g., Champsaur and Rochet [4], Feenstra and Lewis [5], and Lewis and Sappington [10, 11], get a variety of different results, depending on the technical assumptions in the underlying modeling approaches. So, in [10] the optimal contract is characterized by upward distortion in production for high-cost types, downward distortion for low-cost types, and efficient production at a single interior point and at the two extremes of the type distribution. By contrast, in [4] the classical result of no distortion at the top and downward distortion for all lower types is simply turned upside down. Also, all types but a single one might earn strictly positive rents as in [4, 10], or a nondegenerate interval of types might be held to its reservation utility as in [5, 11]. Apart from those differences the solutions in most continuous-type papers share a common property: The optimal contract involves a range of pooling even when standard monotonicity conditions are imposed.

In contrast to the continuous-type papers the discrete papers by Caillaud *et al.* [2], Laffont and Tirole [7, 8], Rochet [14], and Stiglitz [16] —all restricting their attention to a binary type-set—reach quite uniform conclusions. They show that, depending on the strength of countervailing incentives, either classical, reversed classical, or first best results are obtained. Irrespective of which of these cases prevails the second best contract is necessarily fully separating.

The first contribution reaching some level of generality in analyzing countervailing incentives is Maggi and Rodriguez-Clare [12]. These authors show that the properties of second best contracts in a continuous-type framework crucially depend on whether the agent's reservation utility is concave, linear, or convex in his type. If it is strictly concave, or linear respectively, the qualitative results of [10], or [5, 11] respectively, obtain and the optimal contract entails pooling. However, if the agent's reservation utility is convex in type, the equilibrium is separating.

Maggi and Rodriguez-Clare's paper adds considerably to our understanding of the continuous-type environment. However, since reality is discrete and continuity a mathematical fiction the question remains whether the properties of the continuum case carry over to a discrete framework. If the type-space is restricted to two, as in previous work, the answer is no.[1] We show that, as soon as at least three types are introduced, all the qualitative results of the continuous-type papers can be derived in a simple and easy understandable discrete framework without imposing artificial regularity conditions. Thus, the answer to the question stated in the title is, that in order to represent a world with a large discrete type-space, one

[1] The observation that the two-type model is very particular has earlier been made in the literature, for example, in [9].

should rather choose between a three-type discrete and a continuous-type model than restrict oneself to two types. In short, two are not a good representative for many.

The plan of the paper is as follows. Section 2 describes the model. Section 3 treats the binary setting. Section 4 studies the three-type case and derives the main results. We conclude in Section 5.[2]

2. THE MODEL

We focus on a situation where one player, the principal (P), owns a productive technology and employs a second player, the agent (A), to run it on her behalf. The output of the technology is called gross benefit and is denoted by $x \in \mathbb{R}_+$. Producing x causes a cost, or disutility, to the agent which is denoted by $g_i(x)$. The index $i \in I = \{1, ..., n\}$ represents all the private information which A has about his environment, his ability, and his preferences, and is referred to as A's type. A knows his type before a contract is agreed upon, but P does not. P's beliefs about i are represented by a known probability vector $\pi = (\pi_1, ..., \pi_n)$ on $I = \{1, ..., n\}$.

For all $i \in I$, $g_i(\cdot)$ is strictly positive, strictly increasing, and strictly convex on $\mathbb{R}_+$. Producing a gross benefit of 0 causes no cost to the agent so that $g_i(0) = 0$ for all $i \in I$. A higher realization of i is assumed to imply lower total and marginal cost for all $x > 0$, and the difference in costs between two different types is assumed to be increasing at a nondecreasing rate with output. For $i < j$ we define

$$\Delta g_{ij}(\cdot) \equiv g_i(\cdot) - g_j(\cdot). \tag{1}$$

The above assumptions imply that $\Delta g_{ij}(\cdot)$ is a positive, strictly increasing and convex function.[3]

The principal designs a contract to maximize her expected utility. She is assumed to be riskneutral with a utility function represented by $V = x - t$, where t is the transfer (compensation) paid from P to A. The agent's utility

[2] Before proceeding, we would like to acknowledge an earlier, fairly general analysis of the discrete screening model, presented by Spence [15]. Although Spence doesn't consider type-dependent reservation utilities the present work nevertheless makes considerable use of his insights.

[3] Caillaud and Hermalin [1] impose a similar set of assumptions and call it the "screening condition". In the present context this (quite strong) version of the screening condition is, for example, satisfied if (i) gross benefit is the sum of some (exogenous) productivity parameter and the agent's (endogenous) effort, (ii) higher types have greater productivity parameters, and (iii) the agent's cost, or disutility, of effort increases at an increasing rate with effort. Weaker versions of the screening condition (without the convexity assumption) have been used in the literature, e.g., in [13].

is his compensation minus the cost of producing output: $U_i = t - g_i(x)$. The agent's outside opportunities are represented by his reservation utility, R. Since the realization of this variable might depend on A's type R is indexed by i.

By the revelation principle, we can restrict attention to direct truthful contracts of the form $(\mathbf{x}, \mathbf{t}) = ((x_1, t_1), ..., (x_n, t_n))$, where x_i is the output level designated for type i, and t_i is the compensation paid from P to A if A chooses x_i.[4] To avoid lots of conditional statements it is useful to assume that the gross benefit generated by any type is sufficiently large so that hiring the agent is always profitable. With this assumption and the definition $u_i = t_i - g_i(x_i)$ we can write P's problem as

$$\max_{(x_i, u_i)} \sum_{i=1}^{n} \pi_i(x_i - g_i(x_i) - u_i) \qquad \text{s.t.} \qquad (M)$$

$$u_i \geqslant u_j + g_j(x_j) - g_i(x_j) \qquad \forall i, j \in I \qquad (IC_{ij})$$

$$u_i \geqslant R_i \qquad \forall i \in I \qquad (IR_i)$$

We denote the solution to the first best problem, where P maximizes (M) subject to (IR_i) only, by $(\mathbf{x}^*, \mathbf{u}^*)$. If we assume that $g_1'(0) < 1$ the first best solution satisfies $g_i'(x_i^*) = 1$ and $u_i^* = R_i$ for all $i \in I$. Furthermore, $x_1^* < x_2^* < \cdots < x_n^*$.

Turning to the second best some preliminary results are worth mentioning. First, notice that the incentive compatibility constraints (IC_{ij}) and (IC_{ji}) are, for any pair $i < j$, equivalent to

$$\Delta g_{ij}(x_i) \leqslant u_j - u_i \leqslant \Delta g_{ij}(x_j). \qquad (2)$$

As $\Delta g_{ij}(\cdot)$ is a positive, strictly increasing function this relation can be met only if $x_i \leqslant x_j$ (and thus $u_i \leqslant u_j$). An immediate consequence is:

LEMMA 2.1. *A necessary and sufficient condition for an output vector* $\mathbf{x} = (x_1, ..., x_n)$ *to be implementable is* $x_1 \leqslant x_2 \leqslant \cdots \leqslant x_n$.

Proof. Necessity follows from (2). For sufficiency let $R_{\max}$ be the maximal element in $\{R_1, R_2, ..., R_n\}$. Put $u_1 = R_{\max}$, and for any $i > 1$, $u_i = u_{i-1} + \Delta g_{g_{i-1}, i}(x_{i-1})$. Since $x_{i-1} \leqslant x_i \leqslant x_{i+1}$, $(IC_{i, i+1})$ and $(IC_{i, i-1})$ are satisfied. For any $j > i+1$ add constraints $(IC_{i, i+1})$, $(IC_{i+1, i+2})$, ..., $(IC_{j-1, j})$. Using $x_{i+1} \leqslant x_{i+2} \leqslant \cdots \leqslant x_j$ shows that (IC_{ij}) is satisfied. The argument for $j < i-1$ is similar. Hence, all the incentive compatibility constraints are satisfied. ▮

[4] This is not totally correct, since P might wish to offer a contract in which output-levels and transfers are stochastic functions of A's verifiable report. To keep the analysis simple we ignore the possibility of randomization throughout the paper.

Before proceeding, it is necessary to introduce an assumption guaranteeing convexity of P's second best maximization approach. The problem can best be seen from (2). With constant reservation utilities only the second inequality in this relation matters, and since Δg_{ij} is a convex function, P's problem is well behaved. However, with countervailing incentives either of the two inequalities might be binding, possibly leading to violations of the second order conditions. To guarantee that these conditions are always satisfied we assume in what follows that $\Pi_i g_i''(x) > \Pi_{i-1} g_{i-1}''(x)$, where $\Pi_i = \sum_{j=1}^{i} \pi_j$.[5] With this assumption we get:

LEMMA 2.2. *All adjacent incentive compatibility constraints cannot be simultaneously binding.*

Proof. The result follows from the first order conditions, (2), and $x_1^* < x_n^*$.

Implication. Pooling can only be a local, never a global phenomenon. That is, if $x_i = x_j$ *for* $i \neq j$ *then there exists some type* k *such that* $x_k \neq x_j$.

3. THE BINARY CASE ($n = 2$)

In this section we summarize the results for the case in which the agent's type-set is binary ($I = \{1, 2\}$). Proofs for these results in various degrees of generality can be found in Caillaud *et al.* [2], Laffont and Tirole [7, 8], Rochet [14], and Stiglitz [16].[6] Proposition 3.1 tells us that in a binary environment the optimal contract never entails pooling. This follows immediately from the implication to Lemma 2.2.

PROPOSITION 3.1. *In the binary case the second best contract is never pooling (independently of* ΔR_{21}*).*

The next result shows that the properties of the second best contract strongly depend on $\Delta R_{21} \equiv R_2 - R_1$. In this result reference is made to Fig. 1. In this figure an arrow from the top to the bottom is drawn whenever

[5] The proof for the claim that this assumption implies that the second order conditions are satisfied is available from the authors upon request. An alternative assumption that does not involve probabilities would be that the differences in productivities between different types are increasing at a constant rate in output. This alternative assumption is very restrictive, however, and not needed in the present context.

[6] In [2, 7, 8] countervailing incentives arise because the agent's reservation utility is type-dependent. In [14], a bi-dimensional type-set leads to countervailing incentives. In [16], various causes of countervailing incentives are studied in the context of optimal taxation.

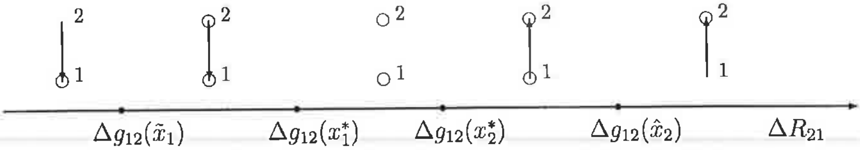

FIG. 1. The binary case.

(IC_{21}) is binding and an arrow in the opposite direction whenever (IC_{12}) is binding. A circle around type i indicates that (IR_i) is binding.[7]

PROPOSITION 3.2. *Consider the binary case. For a given difference in reservation utilities, ΔR_{21}, the particular subset of constraints that are binding is as depicted in Fig.* 1.

Figure 1 shows that as ΔR_{21} increases it passes over 4 different threshold levels, defining 5 regions, each characterized by the constraints that are binding. The benchmark levels $\tilde{x}_1$ $(<x_1^*)$ and $\hat{x}_2$ $(>x_2^*)$ are independent of ΔR_{21} and as defined in Table 1.[8]

If ΔR_{21} is negative (type decreasing reservation utilitiy), zero (constant reservation utility), or positive but fairly small the higher-type agent has an incentive to pose as the lower-type one in order to signal that his disutility of effort is high so that a high transfer is required to compensate him for production. To counteract this tendency a rent is conceded to type 2. This rent makes it optimal to distort type one's output level downward to $\tilde{x}_1$. Downward distortion relaxes (IC_{21}) because the higher-type agent has a higher marginal productivity. Type 2, on the other hand, is not jeopardized by any other type so that there is no distortion at the top.

If ΔR_{21} increases we get to a threshold $(\Delta g_{12}(\tilde{x}_1))$ where the individual rationality constraint for type 2 becomes binding. In the following region the transfer to type 2 must increase in R_2 in order to satisfy (IR_2). This relaxes (IC_{21}) making distortions in x_1 less profitable. Consequently x_1 is gradually increased towards x_1^*. If the increase in ΔR_{21} continues (IC_{21}) becomes slack and the first best solution becomes feasible.

The first best is no longer implementable if ΔR_{21} passes $\Delta g_{12}(x_2^*)$. Then the binding incentive problem becomes to prevent the lower-type agent from exaggerating his productivity in order to signal that his reservation

[7] The notation of Fig. 1 is borrowed from Rochet [14].

[8] In the notation of Table 1, the symbol ˆ stands for *up*- the symbol ˜ for *downward distorted*. The variable $\tilde{x}_i$, for example, denotes the (downward distorted) output level intended for type i if he is jeopardized by his upper neighbor only. Similarly, the variable $\tilde{\tilde{x}}_i$ denotes the (downward distorted) output level intended for type i if he is jeopardized by all higher types. The simultaneous presence of a ˆ and a ˜ symbol means that the respective type is jeopardized by both, lower *and* higher types, so that up- *and* downward distorting forces determine his second best output level.

TABLE 1

Definitions

$\tilde{x}_i$	$g_i'(\tilde{x}_i) = 1 - \frac{\pi_{i+1}}{\pi_i} \Delta g'_{i,i+1}(\tilde{x}_i)$
$\hat{x}_i$	$g_i'(\hat{x}_i) = 1 + \frac{\pi_{i-1}}{\pi_i} \Delta g'_{i-1,i}(\hat{x}_i)$
$\tilde{\tilde{x}}_i$	$g_i'(\tilde{\tilde{x}}_i) = 1 - \frac{1 - \Pi_i}{\pi_i} \Delta g'_{i,i+1}(\tilde{\tilde{x}}_i)$
$\hat{\hat{x}}_i$	$g_i'(\hat{\hat{x}}_i) = 1 + \frac{\Pi_i - \pi_i}{\pi_i} \Delta g'_{i-1,i}(\hat{\hat{x}})$
$\hat{\tilde{x}}$	$g_i'(\hat{\tilde{x}}_i) = 1 - \frac{\pi_{i+1}}{\pi_i} \Delta g'_{i,i+1}(\hat{\tilde{x}}_i) + \frac{\pi_{i-1}}{\pi_i} \Delta g'_{i-1,i}(\hat{\tilde{x}}_i)$

utility is high, i.e., to signal that a high transfer is needed to compensate him for staying with the principal. The optimal contract mitigates this incentive problem by distorting upwards the output level intended for type 2. Doing so relaxes (IC_{12}) because the lower-type agent is less productive. Type 1, on the other hand, is not jeopardized by any other type so that there is no distortion at the bottom. In the first region after $\Delta g_{12}(x_2^*)$ both individual rationality constraints are binding and x_2 increases gradually towards $\hat{x}_2$. Then x_2 remains constant at $\hat{x}_2$ and the transfer to the lower-type agent is increased leaving this type a strictly positive rent.

4. A GOOD REPRESENTATIVE FOR MANY

In contrast to the binary setting summarized in the previous section most of the continuous-type models with countervailing incentives share the feature that the solution involves ranges of pooling even if standard regularity conditions are imposed. Also, for a given shape of the reservation-utility function the agent may have an incentive to either understate or overstate his private information, depending on its realization. Moreover, the characteristics of the optimal contract strongly depend on specific assumptions on the probability distribution over types and on the derivative of the reservation utility with respect to type (cf., for example, Jullien [6], and Maggi and Rodriguez-Clare [12]). Since the rationale for using continuous-type models is that they are a useful idealization of a situation with a large finite number of types, an important question is whether the continuum results carry over to a discrete framework. In this section we show that, by introducing a third type, the most important qualitative results of continuous-type papers can be replicated in our discrete framework without imposing artificial regularity conditions. Thus, the answer to the question stated in the title is that in order to represent a world with a large finite type-space, one should rather choose between a three-type discrete and a continuous-type model than restrict oneself to two types.

We begin our analysis of the three-type case with a distinctive feature of our discrete framework, namely that there exist conditions under which the optimal contract is necessarily fully separating independently of the differences in reservation utilities between adjacent types, that is, independently of $\Delta R_{i+1,i} \equiv R_{i+1} - R_i$. As we will argue later (see footnote 11), finding such conditions becomes increasingly difficult as the number of types increases.[9]

PROPOSITION 4.1. *In the three-type case the second best contract is never pooling (independently of ΔR_{21} and ΔR_{32}) iff*

$$\pi_2 \, \Delta g'_{12}(x_1^*) > \pi_3 \, \Delta g'_{23}(x_1^*) \tag{3}$$

$$\pi_1 \, \Delta g'_{12}(x_3^*) < \pi_2 \, \Delta g'_{23}(x_3^*). \tag{4}$$

Sketch of Proof of Propositions 4.1 *and* 4.2. The technique of proving these propositions is (a) to compute the solution to (M) for all possible combinations of binding constraints; (b) to check in which subarea of the two-dimensional space stretched by the possible differences in reservation utilities each of the solutions found in step (a) is feasible; (c) to show that the subareas calculated in step (b) do not intersect and cover the entire $\mathbb{R}^2$; and (d) to search for conditions under which all subareas involving a pooling solution disappear. ∎

Conditions (3) and (4) avoid bunching and allow us to discuss the most important forces at work with countervailing incentives with the help of a simple figure:

PROPOSITION 4.2 (Separating). *Consider the three-type case. Suppose that* (3) *and* (4) *hold. Then, for a given pair of differences in reservation utilities,* ($\Delta R_{21}, \Delta R_{32}$), *the particular subset of incentive compatibility constraints that are binding is as depicted in Fig.* 2.

Figure 2 divides the two-dimensional space stretched by the possible differences in reservation utilities ($\Delta R_{21}, \Delta R_{32}$) in 9 different areas, each characterized by a different combination of binding incentive compatibility

[9] The proofs of the following results are straightforward but extremely lengthy and are available from the authors upon request.

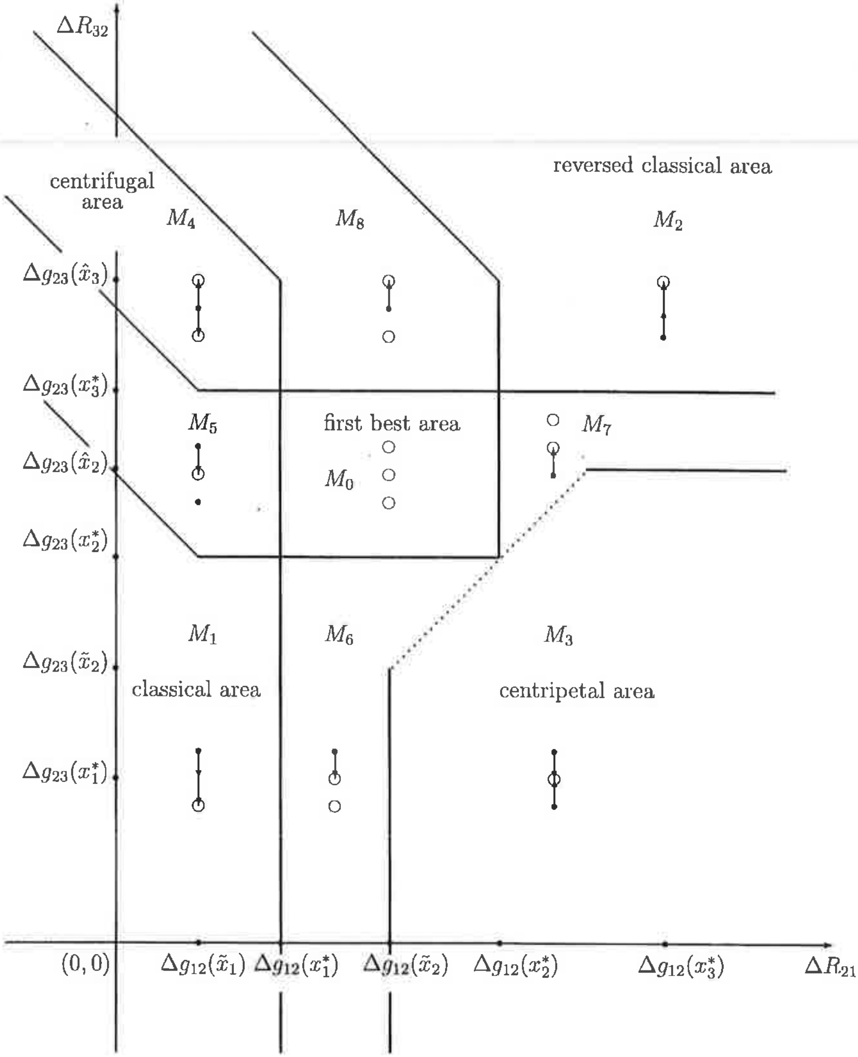

FIG. 2. Three types in a separating environment.

constraints.[10] The size and location of these areas depend on A's disutility of effort function $g(\cdot)$ and on P's probability belief π. The dotted line in Fig. 2 is given by $\Delta R_{32} = \Delta g_{23}(\Delta g_{12}^{-1}(\Delta R_{21}))$ and is the only line in the figure that is not necessarily a straight line.

[10] In order to keep Fig. 2 simple we only marked the individual rationality constraints that are necessarily binding in the respective area. A more detailed figure indicating in addition all possible combinations of binding individual rationality constraints is available from the authors upon request.

To get some intuition for the forces at work in a separating setup let us start with a familiar environment and then study the adjustments in the optimal contract if we gradually change one of the differences in reservation utilities. If ΔR_{21} and ΔR_{32} are both small ($\Delta R_{21} \leqslant \Delta g_{12}(\tilde{\tilde{x}}_1)$, $\Delta R_{21} + \Delta R_{32} \leqslant \Delta g_{12}(\tilde{\tilde{x}}_1) + \Delta g_{23}(\tilde{x}_2)$) we are in that subregion of the *classical area* M_1, where the (conventional) "pure classical case" is implemented. In the pure classical case $x_1 = \tilde{\tilde{x}}_1$, $x_2 = \tilde{x}_2$ and $x_3 = x_3^*$. The reason for this is straightforward: If the differences in reservation utilities are small, or even negative, then each higher type agent has an incentive to pose as a lower type one in order to signal that a high transfer is required to compensate him for production. To counteract this tendency a rent must be conceded to the higher types. The magnitude of these rents positively depends on the production levels intended for the lower types. Since reducing x_1 below x_1^* decreases the rents for both, type 2 and type 3, x_1 is set equal to $\tilde{\tilde{x}}_1$. The output level x_2, on the other hand, influences only the rent of type 3. Hence $x_2 = \tilde{x}_2$. For type 3 rent extraction is not a concern and so $x_3 = x_3^*$.

Let us hold ΔR_{32} fixed and increase ΔR_{21}. Then we first get to a region (still in M_1) where (IR_2) is binding. In this region u_2 must increase in ΔR_{21} in order to satisfy (IR_2). This relaxes (IC_{21}) making distortions in x_1 less profitable. Thus, x_1 is gradually increased from $\tilde{\tilde{x}}_1$ towards x_1^*. If the increase in ΔR_{21} continues we get to area M_6 where (IC_{21}) is slack and the principal implements the first best for type 1. For types 2 and 3, on the other hand, she implements the classical case for two types, i.e., $x_2 = \tilde{x}_2$ and $x_3 = x_3^*$. This ceases to be optimal if ΔR_{21} passes the threshold $\Delta g_{12}(\tilde{x}_2)$. Then we get to an area where only the incentive compatibility constraints pointing to the centre are binding. We refer to this as the *centripetal area.* In this area P's problem is to prevent the lowest type from exaggerating his productivity in order to signal that his reservation utility is high, while the highest-type agent has still an incentive to understate his productivity in order to signal that his disutility of effort is high. To counteract these tendencies rents could be conceded to both, type 1 and type 3. The magnitudes of these rents would depend upon the output level x_2. The rent of type 1 would decrease in x_2 while that for type 3 would increase in x_2. If ΔR_{21} is high enough rents to both types are indeed paid and a tradeoff between rent extraction for these types and production efficiency for type 2 is made in designing x_2. The resulting production level $\hat{\tilde{x}}_2$ lies in the interval $(\tilde{x}_2, \hat{x}_2)$ and is smaller, equal or larger than x_2^* depending on the differences in productivities and on the probabilities. If ΔR_{21} lies between $\Delta g_{12}(\tilde{x}_2)$ and $\Delta g_{12}(\hat{\tilde{x}}_2)$ then type 1 is held to his reservation utility and x_2 is gradually adjusted from $\tilde{x}_2$ to $\hat{\tilde{x}}_2$.

Next let us hold ΔR_{21} fixed at the high level while ΔR_{32} is still low. If ΔR_{32} increases we get to a threshold (not shown in the figure) where (IR_3)

becomes binding. In the following region u_3 must increase in R_3 in order to satisfy (IR_3). This relaxes (IC_{32}). As (IC_{12}) is still binding x_2 is gradually increased from $\hat{\hat{x}}_2$ towards $\hat{x}_2$. If the increase in ΔR_{32} continues we get to area M_7. In this area (IC_{32}) is slack and P implements the first best for type 3. For types 1 and 2, on the other hand, she implements the two-type reversed classical case, i.e., $x_1 = x_1^*$ and $x_2 = \hat{x}_2$. If the increase in ΔR_{32} still continues we pass the threshold to the *reversed classical area* and get (again after an adjustment region where IR_2 is binding) to the "pure reversed classical case" for three types, where the only binding individual rationality constraint is (IR_3), and where the binding incentive problem is to prevent the lower types from exaggerating their productivities in order to signal that their reservation utilities are high. Here, $x_1 = x_1^*$, $x_2 = \hat{x}_2$, and $x_3 = \hat{\hat{x}}_3$ for a symmetric reason to that given in the "pure classical case."

If we hold ΔR_{32} fixed at the high level and decrease ΔR_{21} we get, again after several adjustment regions, to an area where only the incentive compatibility constraints moving away from the centre are binding. We refer to this as the *centrifugal area.* Here, production is downward distorted for type 1 ($\tilde{x}_1 \leqslant x_1 < x_1^*$), upward distorted for type 3 ($x_3^* < x_3 \leqslant \hat{x}_3$), and efficient for type 2 ($x_2 = x_2^*$). The two extreme types are held to their reservation utilities while type 2 might earn strictly positive rents.

A last area worth mentioning is that in the centre of Fig. 2: If the differences in reservation utilities are in an intermediate range, the agent's incentive to overstate his reservation utility (which requires overstating his type) and his incentive to overstate his disutility of effort (which requires understating his type) are in balance and the *first best solution* is feasible.

We are now in the position to explain the following result:

PROPOSITION 4.3 (Pooling). *Consider the three-type case. Suppose that condition* 3 (*respectively condition* 4) *is violated. Then there exists a nonempty subset of* $M_1 \cup M_3 \subseteq \mathbb{R}^2$ (*respectively* $M_2 \cup M_3 \subseteq \mathbb{R}^2$) *such that, for any* $(\Delta R_{21}, \Delta R_{32})$ *in this set, the second best contract involves pooling between types* 1 *and* 2 (*respectively types* 2 *and* 3).

Sketch of Proof of Proposition 4.3. Figure 2 is drawn under the assumption that $x_1^* < \tilde{x}_2$ and $\hat{x}_2 < x_3^*$. It is obvious that, as soon as, say, the first inequality is violated the classical area, M_1, and the centripetal area, M_3, intersect, which leads to pooling between type 1 and type 2. Thus, the second best contract is never pooling *iff* $x_1^* < \tilde{x}_2$ and $\hat{x}_2 < x_3^*$. To show that these conditions are equivalent to (3) and (4) define a function $f(x) = 1 - g_2'(x) - (\pi_2/\pi_3)\, \Delta g_{23}'(x)$. From the second order condition being satisfied it follows that $f'(\cdot) < 0$. Furthermore, $f(\tilde{x}_2) = 0$ by the definition of $\tilde{x}_2$. Thus, $x_1^* < \tilde{x}_2$ *iff* $f(x_1^*) > 0$, that is, *iff* $1 - g_2'(x_1^*) = \Delta g_{12}'(x_1^*) > (\pi_3/\pi_2)\, \Delta g_{23}'(x_1^*)$. That $\hat{x}_2 < x_3^*$ is equivalent to $\pi_1\, \Delta g_{12}'(x_3^*) < \pi_2\, \Delta g_{23}'(x_3^*)$ can be shown in a similar way.[11] ∎

Pooling occurs if carrying out the tradeoffs between rent extraction and production efficiency discussed above would violate the monotonicity constraint $x_i \leqslant x_j$ for $i < j$. Two basic forces potentially leading to an incompatibility with monotonicity can be distinguished. First, an incompatibility might arise with more than 2 types even in absence of countervailing incentives because with incentive constraints binding in a single direction only, second best production levels moving in the same direction might try to "overhaul" each other. Suppose, for example, that in our three-type environment π_2 is low relative to π_1 (π_2 is low relative to π_3). Then carrying out the tradeoff between rent extraction and production efficiency discussed above might entail $\tilde{x}_2 < \tilde{\tilde{x}}_1$ ($\hat{x}_2 > \hat{\hat{x}}_3$), violating monotonicity. To save monotonicity of the second best output vector for reservation-utility profiles falling in the classical (reversed classical) area, x_1 and x_2 (x_2 and x_3) are bunched on an intermediate output level x_{12} (x_{23}) satisfying $\tilde{x}_2 < x_{12} < \tilde{\tilde{x}}_1$ ($\hat{\hat{x}}_3 < x_{23} < \hat{x}_2$). We refer to this kind of pooling as *classical- or overtake-pooling*. In continuous-type models the occurrence of overtake-pooling is typically prevented by imposing up- and downward hazard-rate conditions. In a discrete framework these conditions are only sufficient to avoid overtake-pooling if the $\Delta g_{h,h+1}'$'s are independent of h.

More interesting and specific to settings with countervailing incentives is a second kind of pooling which we refer to as *centripetal- or crash-pooling*. Crash-pooling arises if incentive constraints binding towards the centre cause potential violations of the monotonicity constraint. For example, if π_3 is large relative to π_2 (π_1 is large relative to π_2) the basic tradeoffs might entail $\tilde{x}_2 < x_1^*$ (or $\hat{x}_2 > x_3^*$). If this happens the centripetal area in Fig. 2 extends into the classical (or reversed classical) area leading to pooling between type 1 (type 3) and type 2 for an open set of values $(\Delta R_{21}, \Delta R_{32})$. (See Fig. 3). Notice that crash-pooling can only occur in the centripetal area which is bounded from above by the dotted line given by $\Delta R_{32} = \Delta g_{23}(\Delta g_{12}^{-1}(\Delta R_{21}))$. So, if the occurrence of overtake-pooling is prevented by imposing adequate regularity conditions (as it is typically done in continuous-type papers) the optimal contract is fully separating if the profile of differences in reservation utilities under consideration lies above the dotted line. This corresponds exactly to the continuous-type results of Jullien [6],

[11] Notice that to avoid pooling in a setting with arbitrary many types we would have to assure that $\hat{x}_i < \tilde{x}_{i+1}$ for all $i \in I \backslash \{n\}$. It is obvious that it becomes increasingly difficult to fulfil this requirement as n increases.

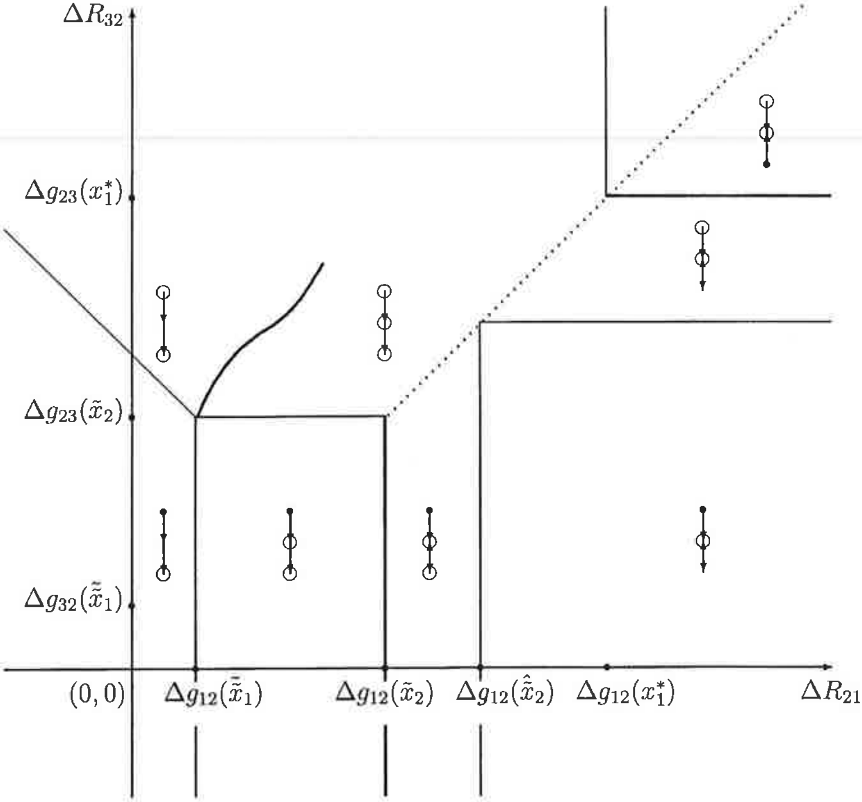

FIG. 3. Pooling between types 1 and 2 if $\tilde{x}_1 < \tilde{x}_2 < \hat{\tilde{x}}_2 < x_1^*$.

and Maggi and Rodriguez-Clare [12].[12] By contrast, if the agent's outside opportunities are such that the centripetal case applies, imposing these regularity conditions does not prevent pooling, a finding consistent with the results of Feenstra and Lewis [5] and Lewis and Sappington [10,11].

[12] Notice, that imposing "homogeneity"—a condition first introduced by Maggi and Rodriguez-Clare [12] and so named by Jullien [6]—corresponds exactly to the requirement that $(\Delta R_{21}, \Delta R_{32})$ lies above the dotted line in Fig. 2: Homogeneity demands that there exists a vector **x** such that the contract **(x, R)** is implementable. This is equivalent to requiring the existence of an **x** such that $[\Delta g_{12}(x_1) \leqslant \Delta R_{21} \leqslant \Delta g_{12}(x_2)$ and $\Delta g_{23}(x_2) \leqslant \Delta R_{32} \leqslant \Delta g_{23}(x_3)]$ which again is equivalent to $[\Delta g_{23}(x_1) \leqslant \Delta g_{23}(\Delta g_{12}^{-1}(\Delta R_{21})) \leqslant \Delta g_{23}(x_2) \leqslant \Delta R_{32} \leqslant \Delta g_{23}(x_3)]$.

5. CONCLUSION

We have shown that the most important qualitative results derived in continuous-type models with countervailing incentives can be replicated in a discrete framework with at least three types. In a three-type model the properties of second best contracts strongly depend on the differences in reservation-utilities between adjacent types. If these differences are small (*classical area*) the second best contract exhibits the conventional features of "no distortion at the top" and downward distortion for lower types. The lowest type gets his reservation utility while higher types might earn rents. If the differences in reservation-utility are large (*reversed classical area*) the conventional results are turned upside down, i.e., the optimal contract is characterized by "no distortion at the bottom" and upward distortion for higher types, as well as by positive rents for lower types and no rents for the highest type.

More interesting phenomena arise if the differences in reservation utilities are strongly de- or increasing. In the first (the *centripetal*) case the second best contract typically has "no distortion at the extremes," upward distortion for lower and downward distortion for higher types. The type in the centre gets his reservation utility while types in the peripheries might earn rents. Exactly the opposite is true in the second (the *centrifugal*) case. Here the second best contract exhibits "no distortion in the centre," upward distortion for higher and downward distortion for lower types. The types at the extremes get their reservation utilities while the moderate type might earn rents.

We have identified two kinds of pooling. *Overtake-pooling* might arise even in absence of countervailing incentives and its occurrence is prevented by imposing standard regularity conditions. *Crash-pooling* is peculiar to models with countervailing incentives and infects only the centripetal area.

Since continuous-type papers typically impose regularity conditions excluding overtake-pooling, their result that the optimal contract is fully separating when the agent's reservation-utility is convex in type (corresponding to reservation-utility profiles in the centrifugal area) while it might entail pooling in the concave (centripetal) case is perfectly in line with our analysis, as are the other characteristics of second best contracts derived in the continuum literature.

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