Incentives vs. Selection in Promotion Tournaments: Is It Possible to Kill Two Birds With One Stone?*

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April 13, 2012

Abstract

This paper investigates whether the performance dimensions 'incentive provision' and 'accuracy in selection' are compatible in tournaments with heterogeneous workers. The analysis compares static one-stage and dynamic two-stage promotion tournaments, as well as two different seeding variants of two-stage promotion tournaments. The results suggest that any tournament with heterogeneous participants provides some incentives for effort and some sorting of types. However, modifications which improve the performance in one will deteriorate the performance in the other dimension, i.e., tournament formats that perform better in terms of incentive provision do worse in terms of selecting the best participant, and vice versa. This suggests that multiple instruments should be used whenever both goals are equally important.

JEL-Classification: M52, J33

Keywords: Promotion Tournaments, Heterogeneity, Incentive Provision, Selection

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1 Introduction

Most employment relationships are characterized by competition among employees for promotion to a better paid, more attractive position. While these promotion tournaments are sometimes just a by-product of a given hierarchical structure, they often are an explicit instrument in the practice of human resource management (HRM) in professional occupations: Think of law firms or consulting firms, for example, where 'up-or-out' promotion policies are the norm and vacant manager or partner position are (almost entirely) filled with insiders. Moreover, some CEOs organize promotion tournaments for their successorship. The most prominent example is certainly Jack Welch (2001), who designed the competition for his succession about six years before he actually left. Several candidates from inside GE knew that they were competing against each other, and that they would either become the next CEO, or would have to leave the firm.[1] 'Up-or-out' promotion policies are also common in the competition between scientists for (rare) positions at universities: In each year, only the (relatively) best performing PhDs become assistant professors, and only the best among the assistant professors receive a tenured position subsequently, while mediocre staff members have to leave. In all these applications, tournaments are used as a means to achieve two goals: First, the prospect of moving up the ladder to higher levels within the same institution is a strong motivator for employees to exert effort in their current job. Therefore, promotion tournaments help to incentivize employees.[2] Second, the selection of the most able candidate(s) is very important due to the 'up-or-out' nature of the competition. It is quite obvious that institutions intend to promote (and keep) productive employees, whereas the inferior candidates should leave.[3] This raises the question whether promotion tournaments between heterogeneous employees can be designed in such a way that they perform optimally along both dimensions. Can tournaments be used as a device to maximize the incentives for effort provision while at the same time minimizing the probability that the "wrong" contestant wins? Or, in other words, can promotion tournaments be designed in such a way that they kill two birds with one stone?

This paper provides a systematic investigation of how the two criteria 'incentive provision' and 'selection performance' are related to each other. In particular, we investigate how modifications of the tournament structure affect the two aforementioned goals. When comparing a *static* one-shot and a

[1]See also Konrad (2010) for an extensive discussion of this example.

[2]The tournament helps to solve a moral hazard problem. The seminal paper for this application is Lazear and Rosen (1981). Alternatively, the tournament may serve as a commitment device for the principal, see, e.g., Malcomson (1984), and Prendergast (1999) for a survey.

[3]This has already been stressed by Sherwin Rosen in his seminal paper on promotion tournaments, where he states that "the inherent logic [of promotion tournaments] is to determine the best contestants and to promote survival of the fittest" (Rosen 1986, p.701). Surprisingly, however, the main focus of his analysis is on incentive maintenance across different hierachie levels.

dynamic two-stage pair-wise elimination structure, our results indicate that the dynamic format performs better in terms of aggregate equilibrium efforts, while the static tournament dominates with respect to selection. Therefore, it seems that an additional hierarchy level is beneficial for incentive provision, but detrimental for the selection performance of a promotion tournament. In addition, we compare two different seeding variants of the dynamic pair-wise elimination structure, one in which similar workers compete against each other on the first stage, and one in which heterogeneous worker types compete on the first stage of the tournament. Again, one structure (the former one) performs better in terms of aggregate equilibrium effort, while the other one dominates with respect to selection. Overall, these results suggest the existence of a fundamental trade-off between incentive provision and selection, both across different tournament formats (static vs. dynamic), and across different seeding variations of the same dynamic tournament structure (homogeneous stage-1 interaction vs. heterogeneous stage-1 interaction). While any tournament with heterogeneous participants provides some incentives for effort and some sorting of types, modifications which improve the performance in one will deteriorate the performance in the other dimension.[4] The reason is that the structural variations considered in this paper have different effects on the strategic disadvantage of weak workers: When the tournament structure amplifies their strategic disadvantage, weak workers are discouraged and tend to reduce their equilibrium effort provision, which then facilitates selection, but reduces overall incentives for effort provision. This reasoning suggests that a trade-off between the two goals is not restricted to the *structure* of a promotion tournament, which is investigated in this paper. In fact, the same problem will be present for any design parameter which affects the strategic disadvantage of weaker types; improvements in the incentive dimension through handicaps (Lazear and Rosen 1981) or type specific prizes (Gürtler and Kräkel 2010), for example, will deteriorate the selection accuracy. From a policy perspective, this implies that multiple instruments should be used whenever both goals are equally important. If the talent of employees is observable, HRM could organize a promotion tournament which maximizes incentives for effort provision between a preselected sample of equally talented employees, for example. Otherwise, some kind of assessment center prior to the promotion competition may serve this function, potentially a tournament that is optimized along its selection dimension. In other words, a promotion tournament alone cannot serve both goals equally well.

The question whether promotion tournaments can provide both incentives and sorting was already

[4]Even though we concentrate on personnel policies, and in particular on the promotion tournament application throughout this paper, this finding is equally important in the context of rent-seeking contests. Note, however, that the interpretation is different in this case: A trade-off between incentive provision and selection in a promotion context (where maximization of aggregate effort is a natural goal) translates into a lack of this trade-off in a rent-seeking contest (where effort inputs are wasteful and the usual objective is their minimization).

addressed by Baker, Jensen, and Murphy (1988). However, they interpreted the sorting function in a different way. Baker et al. (1988) investigate in how far promotion tournaments ensure that employees end up in those jobs for which they are best suited, i.e., they assume that skill and human capital requirement differ qualitatively across hierarchy levels.[5] We consider situations where talents requirements are qualitatively identical across hierarchy levels: Skill requirements in law and consultancy firms, for example, do not change by much with positions. Also, top managers and CEOs perform very similar tasks, and both assistant and tenured professors teach and do research. However, the ability to perform the same task is assumed to differ across workers. Therefore, our paper is more related to work by Tsoulouhas, Knoeber, and Agrawal (2007), which studies a one-stage promotion tournament where insiders and outsiders compete for a CEO position. Assuming that both the quality of the promoted agent and the provision of incentives matter for the designer, they find that the two goals are conflicting if the ability of outsiders is higher than the ability of insiders. While this trade-off is similar to the one established in our paper, the focus of their study is different: They analyze optimal handicapping in a setting where selection involves both insiders and outsiders, but only effort provision by insiders is beneficial for the organization. In contrast, we consider within firm competition in different promotion tournament structures. This paper is also related to the contest design literature. Ryvkin and Ortmann (2008) address the selection performance of different tournament structures, but in contrast to our paper they discard the effect of this variation on incentives. Groh, Moldovanu, Sela, and Sunde (2011) show that the design of dynamic tournaments can involve a trade-off between incentive provision and selection performance, but they focus on different seedings in a dynamic tournament, while we also investigate how dynamic tournaments relate to static ones when participants are heterogeneous.[6] So far, static and dynamic contests have only been compared in the case of homogeneous participants, see, e.g., Gradstein and Konrad (1999).

The remainder of this paper is structured as follows. The next section introduces the tournament environment and discusses the relation of our models to existing work. Section **??** determines the equilibrium behavior of tournament participants for each potential setting. This information is then used in section 2.3 to obtain and compare the equilibrium measures for incentive provision and selection performance. Section 4 concludes.

[5] In their words, '...talents for the next level in the hierarchy are not perfectly correlated with talents to be the best performer in the current job' (p. 602). The best salesman, for example, can be a bad manager, which leads to the so-called Peter Principle. See also Prendergast (1993) and Bernhardt (1995), who also consider the matching performance of promotion tournaments.

[6] Another (technical) difference is that they use a perfectly discriminating all-pay-auction framework, while the analysis in this paper uses a standard Tullock contest success function instead.

2 The Model

2.1 A Promotion Tournament with Heterogeneous Workers

Consider an institution who uses a promotion tournament to fill some vacant higher-level position that is of value P to employees from lower ranks.[7] For simplicity, assume that four risk neutral workers from the same company compete for the open position on the internal labor market, i.e., while working on their actual position, they are evaluated relative to their colleagues, and the employee with the best performance is promoted at the end of the evaluation period. Workers know that they are being evaluated, and in particular, they are perfectly informed about both their own productivity and the productivities of their colleagues.[8] To keep the theoretical analysis tractable, we assume that workers are of two different types: Equal shares are highly productive ("strong") and less productive ("weak"), respectively. Each worker provides effort to increase his/her chances for a promotion. The organizing entity of the promotion tournament, in short the principal, cannot directly observe individual efforts, but receives a noisy ordinal performance signal instead. As a result, the promotion probability p_i for some arbitrary worker i in a tournament interaction is given by the ratio of own effort x_i over effort provided by the immediate competitor(s), X. Formally, the probability is defined as

$$p_i\left(x_i, X\right) = \frac{x_i}{x_i + X}. \tag{1}$$

While the promotion probability of a worker is clearly increasing in his own effort provision, and decreasing in the effort provided by the immediate opponent(s), the chosen formulation implies that the worker with the highest effort does not always win, i.e., effort does not translate directly into performance. The reason is that the performance signal is distorted by random noise.[9] The principal pursues the following two objectives:

1. Maximize aggregate effort by all workers (Incentive Provision).

2. Maximize the probability that a strong, productive worker wins (Selection).

[7] The value of being promoted (P) may include both monetary components (promotions imply higher wages) and non-monetary aspects (e.g., concerns for status or power).

[8] This assumption may be problematic in some settings, for example in assessment centers. Note, however, that this paper focusses on within company promotion tournaments. In most professional occupations, the first promotion possibility for new hires is after one or two years. Therefore, workers who compete on the internal labor market for open positions usually know each other due to ongoing interactions in the workplace. The promotion tournament for the succession of Jack Walch, for example, which is presented in the Introduction, lasted six years.

[9] We use a so-called Tullock (1980) contest success function (CSF) with discriminatory power one, which can be transformed into an all-pay auction ontest success function with multiplicative noise that follows the exponential distribution. See Konrad (2009) for details (p.52f).

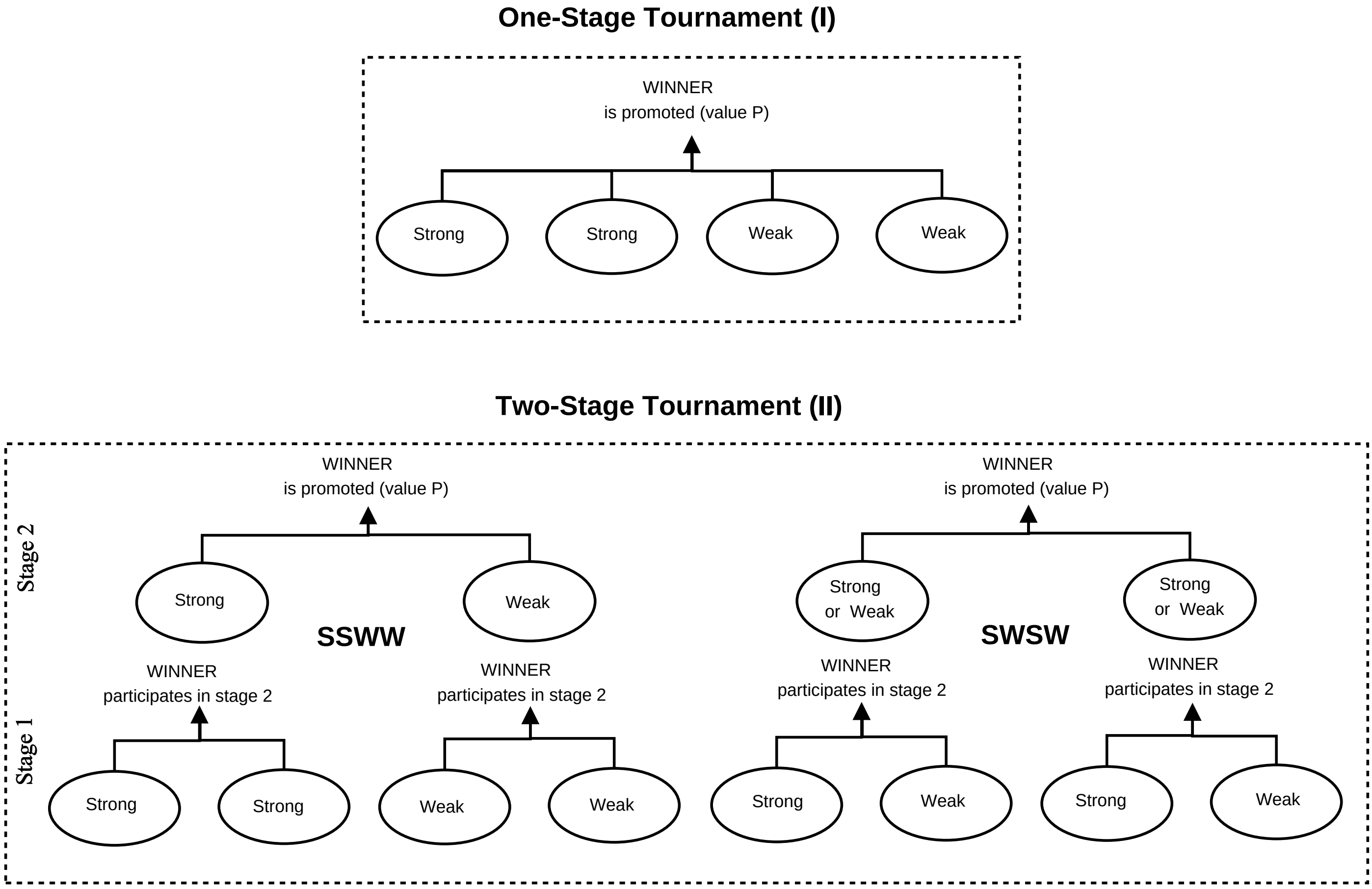


Figure 1: Design options available to the tournament designer

Work effort by employees determines output and profits of corporations. Since effort is often costly for the workers and non-contractible at the same time, explicit incentives for effort provision are needed. Therefore, the provision of incentives is an important goal for any corporation, and the prospect of being promoted to a better paid, more attractive position can be used to motivate and incentive workers. Selection performance is usually equally important, however, since the 'up or out' character of the competition implies that only promoted employees stay within the corporation. As able workers are certainly better suited for positions with more responsibilities, institutions intend to promote (and keep) productive employees. This does even hold if the losers of the promotion competition are allowed to stay, since they are certainly discouraged, such that many of them will apply at different companies, i.e., they will leave voluntarily. Thus, promoting the “wrong” worker is costly, and avoiding this cost by implementing a tournament format with optimal selection properties is a natural second objective.

The goal of our analysis is to find out whether the careful design of structural parameters by the principal can ensure that the promotion tournament performs well in both performance dimensions. In particular, we make two comparisons. First, we compare incentive and selection properties of a static (one-stage) tournament and a dynamic (two-stage) pairwise elimination tournament, i.e., we determine the effect the introduction of an additional (intermediate) hierachy level by the principal on incentive provision and selection performance in the promotion tournament. The two different tournament formats are depicted in Figure 1, which also shows that two different constellations are possible in the dynamic specification: Either a strong worker competes against another strong worker (and a weak worker against another weak worker) in the parallel stage-1 interactions (setting `SSWW`); or both stage-1 interactions are mixed in terms of the productivity of the competing workers (setting `SWSW`). In the comparison of the static and the dynamic tournament format, we assume that workers' types are not observable. Therefore, the seeding in stage 1 is random if the principal decides in favor of the dynamic format; setting `SSWW` occurs with probability 1/3, setting `SWSW` with probability 2/3.[10] Second, we compare the performance of settings `SSWW` and `SWSW` with respect to incentives and selection. Even though the principal needs to know the worker's types for this structural variation, the selection performance of the promotion tournament is still important, since tournaments have a commitment property. Therefore, the winner of the tournament must be promoted, independent of his type. One might certainly argue that a promotion tournament is not the optimal mechanism to assure incentive provision and selection if types are known. However, our approach is positive rather than normative; it is a well known fact that promotion tournaments are widely used, even though theory sometimes suggests other mechanisms.[11] Alternatively, one may argue that the principal's belief about workers' types is distorted, such that tournament outcomes provide a signal to update this beliefs.

2.2 Equilibrium Behavior by Workers

Workers face a trade-off in the one-stage, as well as in each pair-wise interaction of the two-stage tournament: Ceteris paribus, increasing own effort leads to higher costs as well as a higher probability of winning. In equilibrium, workers choose their efforts such that the marginal cost of effort provision equals the expected marginal gain in terms of a higher probability of being promoted. To introduce heterogeneity, the effort costs of strong workers, $c_{\mathtt{S}}$, are assumed to be lower than the effort costs $c_{\mathtt{W}}$ of weak workers ($c_{\mathtt{S}} \leq c_{\mathtt{W}}$). Intuitively, effort costs are used as an inverse measure for ability.[12]

2.2.1 One-Stage Tournament

The one-stage tournament model we consider, denoted `I` in the sequel, is a special case of the model developed by (and extensively discussed in) Stein (2002). It is a simultaneous move game, the natural

[10] After the first worker has been chosen randomly from the pool of four workers, the probability that the next worker drawn from the pool of the remaining three workers is of the same type is 1/3 (since only one of the remaining workers is of the same type), while the probability that the next worker is of the other type is 2/3 (because two of the three remaining workers are of the other type).

[11] See Baker, Jensen, and Murphy (1988), or Gibbs and Hendricks (2004), for example.

[12] Modeling heterogeneity in terms of effort cost is without loss of generality. Proofs are available from the authors upon request.

solution concept is therefore Nash Equilibrium (NE). In a NE, each worker i with constant marginal effort costs c_i maximizes his expected payoff $\Pi_i(\mathtt{I})$ by choosing optimal effort $x_i \geq 0$, taking the total effort of all other workers X as given. Formally, the optimization problem of worker i reads as follows:

$$\max_{x_i \geq 0} \Pi_i(x_i, X) = \frac{x_i}{x_i + X} P - c_i x_i.$$

The formal expressions for individual equilibrium efforts of strong and weak workers, $x^*_{\mathtt{S}}(\mathtt{I})$ and $x^*_{\mathtt{W}}(\mathtt{I})$, respectively, are provided in equation (A2) in the Appendix. Equilibrium efforts determine both the incentive provision and the selection performance of the promotion tournament. Our measure for the incentive provision performance in the one-stage tournament, denoted $\mathcal{E}(\mathtt{I})$, is defined as the sum of individual equilibrium efforts. Since two workers are strong and weak, respectively, we obtain

$$\mathcal{E}(\mathtt{I}) = 2x^*_{\mathtt{S}} + 2x^*_{\mathtt{W}}. \tag{2}$$

While the incentive provision measure depends on the absolute value of equilibrium efforts, winning probabilities depend on the ratio of $x^*_{\mathtt{S}}(\mathtt{I})$ and $x^*_{\mathtt{W}}(\mathtt{I})$. To determine the selection performance $\mathcal{S}(\mathtt{I})$, i.e., the probability that a strong worker wins, the equilibrium winning probability of a strong worker must be multiplied by two, since two strong workers participate in the promotion tournament. Thus,

$$\mathcal{S}(\mathtt{I}) = \frac{2x^*_{\mathtt{S}}}{x^*_{\mathtt{S}} + x^*_{\mathtt{W}}}. \tag{3}$$

2.2.2 Two-Stage Tournament

Subgame Perfect Nash Equilibrium is the relevant solution concept for the two-stage tournament, since this structure is a sequential game. Therefore, the equilibrium is obtained through backward induction. First, all possible stage-2 interactions that occur in setting SSWW or SWSW, respectively, must be solved. With four workers of two types, there are three potential stage-2 games, namely SS (both workers are strong), WW (both workers are weak), or SW (one strong and one weak worker). The formal optimization problem of some worker i with effort cost c_i who competes with worker j in stage 2 reads

$$\max_{x_{i2} \geq 0} \Pi_{i2}(x_{i2}, x_{j2}) = \frac{x_{i2}}{x_{i2} + x_{j2}} P - c_i x_{i2},$$

where x_{i2} and x_{i2} are individual efforts by workers i and j, respectively. A detailed solution of all stage-2 games is provided in section B1 of the Appendix. Note, however, that the equilibrium effort of each worker depends both on his own and on the type of the opponent: $x^*_{\mathtt{S}2}(\mathtt{SS})$ is the equilibrium effort of a strong worker in interaction SS, $x^*_{\mathtt{W}2}(\mathtt{WW})$ the optimal choice of weak workers in interaction WW, while $x^*_{\mathtt{S}2}(\mathtt{SW})$ and $x^*_{\mathtt{W}2}(\mathtt{SW})$ are the equilibrium efforts of strong and weak workers, respectively, in the mixed stage-2 configuration $\texttt{SW}$.[13] Since stage-2 equilibrium efforts solve the last stage of the game, we can move forward to stage 1.

Setting $\texttt{SSWW}$. The stage-1 interactions in setting $\texttt{SSWW}$ ensure, as Figure 1 shows, that one strong and one weak worker reach stage 2 with certainty.[14] Consequently, $\texttt{SW}$ is the only possible constellation on stage 2. This implies that both strong workers know that, conditional on reaching stage 2, a weak worker will be the opponent, while weak workers anticipate that they will interact with a strong worker if they reach stage 2. The only reward for winning stage 1 is the participation in stage 2, in which workers may then receive the promotion of value P. Thus, the expected equilibrium payoffs of stage-2 interaction $\texttt{SW}$ for strong and weak workers, $\Pi^*_{\texttt{S}2}(\texttt{SW})$ and $\Pi^*_{\texttt{W}2}(\texttt{SW})$, respectively, determine the continuation values for which workers compete in stage 1. This becomes clear when considering the optimization problem of some strong worker i, who competes with the second strong worker j:

$$\max_{x_{i1}\geq 0} \Pi_i(\texttt{SSWW}) = \frac{x_{i1}}{x_{i1}+x_{j1}}\Pi^*_{\texttt{S}2}(\texttt{SW}) - c_{\texttt{S}} x_{i1}.$$

Worker i chooses stage-1 effort x_{i1} to increase the probability to participate in stage 2, which is worth $\Pi^*_{\texttt{S}2}(\texttt{SW})$ in equilibrium. Similarly, the two weak workers compete for participation in stage 2, which is worth $\Pi^*_{\texttt{W}2}(\texttt{SW})$ for them. Let $x^*_{\texttt{S}1}(\texttt{SSWW})$ and $x^*_{\texttt{W}1}(\texttt{SSWW})$ be the stage-1 equilibrium efforts in setting $\texttt{SSWW}$ by strong and weak workers, respectively, which are determined in the Appendix.[15] Then, the incentive measure in setting $\texttt{SSWW}$ of the two-stage tournament format, denoted $\mathcal{E}(\texttt{SSWW})$, is defined as follows:

$$\mathcal{E}(\texttt{SSWW}) = \underbrace{2[x^*_{\texttt{S}1}(\texttt{SSWW}) + x^*_{\texttt{W}1}(\texttt{SSWW})]}_{\text{stage 1 effort}} + \underbrace{x^*_{\texttt{S}2}(\texttt{SW}) + x^*_{\texttt{W}2}(\texttt{SW})}_{\text{stage 2 effort}}. \tag{4}$$

Total effort provision $\mathcal{E}(\texttt{SSWW})$ amounts to individual efforts by two strong and two weak workers in stage 1, and one strong and one weak worker in stage 2. The selection measure, i.e., the probability that a strong worker receives the promotion, is determined by relative effort provision of the stage-2 participants. As mentioned previously, one strong and one weak worker compete in stage 2, independent of stage 1 outcomes. Therefore, the selection measure $\mathcal{S}(\texttt{SSWW})$ depends on the ratio of stage-2 equilibrium efforts $x^*_{\texttt{S}2}(\texttt{SW})$ and $x^*_{\texttt{W}2}(\texttt{SW})$:

$$\mathcal{S}(\texttt{SSWW}) = \frac{x^*_{\texttt{S}2}(\texttt{SW})}{x^*_{\texttt{S}2}(\texttt{SW}) + x^*_{\texttt{W}2}(\texttt{SW})}. \tag{5}$$

Setting $\texttt{SWSW}$. Since both stage-1 interations are mixed in setting $\texttt{SWSW}$, the type configuration in stage 2 is uncertain; any of the three stage-2 games $\texttt{SS}$, $\texttt{WW}$, and $\texttt{SW}$ is possible, as Figure 1 clearly shows. As a

[13]For formal expressions of equilibrium efforts, see equations (B1), (B3), and (B5), respectively.
[14]Note that Stein and Rapoport (2004) considers a very similar model.
[15]Formal expressions for equilibrium efforts are provided in equations (B8) and (B9).

consequence, the solution of this setting is complicated by the fact that stage-1 continuation values are endogenously determined.[16] To illustrate this complication, assume that some strong worker i and an arbitrary weak worker j compete for the right to participate in stage 2. Simultaneously, strong worker k and weak worker l compete for the remaining stage-2 slot in the other stage-1 interaction. Then, the formal optimization problems of workers i and j are as follows:

$$\max_{x_{i1}\geq 0} \Pi_i(\texttt{SWSW}) = \frac{x_{i1}}{x_{i1}+x_{j1}} \underbrace{\left[\frac{x_{k1}}{x_{k1}+x_{l1}}\pi^*_{\texttt{S2}}(\texttt{SS}) + \frac{x_{l1}}{x_{k1}+x_{l1}}\pi^*_{\texttt{S2}}(\texttt{SW})\right]}_{\equiv P_i(x_{k1},x_{l1})} - c_{\texttt{S}} x_{i1}$$

$$\max_{x_{j1}\geq 0} \Pi_j(\texttt{SWSW}) = \frac{x_{j1}}{x_{i1}+x_{j1}} \underbrace{\left[\frac{x_{k1}}{x_{k1}+x_{l1}}\pi^*_{\texttt{W2}}(\texttt{SW}) + \frac{x_{l1}}{x_{k1}+x_{l1}}\pi^*_{\texttt{W2}}(\texttt{WW})\right]}_{\equiv P_j(x_{k1},x_{l1})} - c_{\texttt{W}} x_{j1} \, .$$

Interestingly, the continuation values $P_i(x_{k1}, x_{l1})$ and $P_j(x_{k1}, x_{l1})$ of workers i and j, respectively, depend on the behavior of workers k and l in the other stage-1 interaction. The reason is that expected equilibrium payoffs for workers differ across the three potential stage-2 interactions SS, WW, and SW.[17] Intuitively, the same holds for the continuation values $P_k(x_{i1}, x_{j1})$ and $P_l(x_{i1}, x_{j1})$ of workers k and l in the second stage-1 interaction. Thus, the two heterogeneous stage-1 interactions are linked through endogenously determined continuation values. This interesting technical complication is relegated to the Appendix, which also provides closed-form solutions for stage-1 equilibrium efforts $x^*_{\texttt{S1}}(\texttt{SWSW})$ and $x^*_{\texttt{W1}}(\texttt{SWSW})$ of strong and weak workers, respectively.[18] Using individual equilibrium efforts, we can compute aggregate effort provision, i.e., the incentive measure $\mathcal{E}(\texttt{SWSW})$, as follows:

$$\mathcal{E}(\texttt{SWSW}) = \underbrace{2[x^*_{\texttt{S1}}(\texttt{SWSW}) + x^*_{\texttt{W1}}(\texttt{SWSW})]}_{\text{stage 1 effort}} + \underbrace{2\{\pi^2 x^*_{\texttt{S2}}(\texttt{SS}) + (1-\pi)^2 x^*_{\texttt{W2}}(\texttt{WW}) + \pi(1-\pi)[x^*_{\texttt{S2}}(\texttt{SW}) + x^*_{\texttt{W2}}(\texttt{SW})]\}}_{\text{stage 2 effort}} \, . \quad (6)$$

where $\pi = \frac{x^*_{\texttt{S1}}(\cdot)}{x^*_{\texttt{S1}}(\cdot)+x^*_{\texttt{W1}}(\cdot)}$ is the probability that a strong worker wins against the weak opponent in stage 1; this probability determines the likelyhood for a particular stage-2 configuration: The stage-2 participants are both strong with probability π^2, both weak ith probability $(1-\pi)^2$, or of different types with probability $2\pi(1-\pi)$. The probability π that a strong worker wins in stage 1 is also relevant

[16] In contrast to the one-stage tournament and setting SSWW, the analysis of setting SWSW is novel; a closed-form solution of this model has not been presented in the existing literature. Note, however, that Groh, Moldovanu, Sela, and Sunde (2011) derive a closed-form solution for this setting in an all-pay auction framework, i.e., for the case of a perfectly discriminating contest-success function where the ordinal signal which the principal receives is not distorted by random noise. Rosen (1986) considers exactly the same specification as analyzed here, but provides only numerical simulations and then conjectures (without analytical proof) that certain properties of numerical simulations should hold in general. Finally, Harbaugh and Klumpp (2005) solve a very similar model, but make use of the simplifying assumption that total effort by each participant over both stages of the contest is equal to some constant. In other words, they derive optimal behavior when strong and weak participants face the same binding effort endowment (which has no intrinsic value), and then discuss how the endowment is distributed across the two stages; the effort choice is unrestricted in our model.

[17] Conditional on reaching stage 2, workers of both types have a higher expected payoff from meeting a weak rather than a strong opponent, since $\Pi^*_{\texttt{W2}}(\texttt{WW}) > \Pi^*_{\texttt{W2}}(\texttt{SW})$ and $\Pi^*_{\texttt{S2}}(\texttt{SW}) > \Pi^*_{\texttt{S2}}(\texttt{SS})$. Details are provided in the Appendix.

[18] See (B13) and (B14) for details.

for the selection performance of setting SWSW, measured by $\mathcal{S}(\texttt{SWSW})$. It is defined as

$$\mathcal{S}(\texttt{SWSW}) \quad = \quad \pi^2 + 2\pi(1-\pi)\frac{x^*_{\texttt{S2}}(\texttt{SW})}{x^*_{\texttt{W2}}(\texttt{SW}) + x^*_{\texttt{S2}}(\texttt{SW})}. \tag{7}$$

Intuitively, a strong worker is promoted if either both strong workers win their stage-1 interactions, which happens with probability π^2, or if only one strong worker wins in stage 1, and subsequently also in stage 2.

Random Seeding. If the principal decides in favor of the dynamic format and seeding of types in stage 1 is random, setting SSWW occurs with probability 1/3; the probability of the complementary event that setting SWSW realizes is 2/3. Consequently, the expected incentive provision measure for the two-stage promotion tournament, denoted $\mathcal{E}(\texttt{II})$, is a weighted average of total effort provision in the two settings. Formally,

$$\mathcal{E}(\texttt{II}) = \frac{\mathcal{E}(\texttt{SSWW}) + 2 \cdot \mathcal{E}(\texttt{SWSW})}{3}. \tag{8}$$

Conceptually, the same holds for the selection measure $\mathcal{S}(\texttt{II})$, which is a weighted average of $\mathcal{S}(\texttt{SSWW})$ and $\mathcal{S}(\texttt{SWSW})$. Formally,

$$\mathcal{S}(\texttt{II}) = \frac{\mathcal{S}(\texttt{SSWW}) + 2 \cdot \mathcal{S}(\texttt{SWSW})}{3} \ . \tag{9}$$

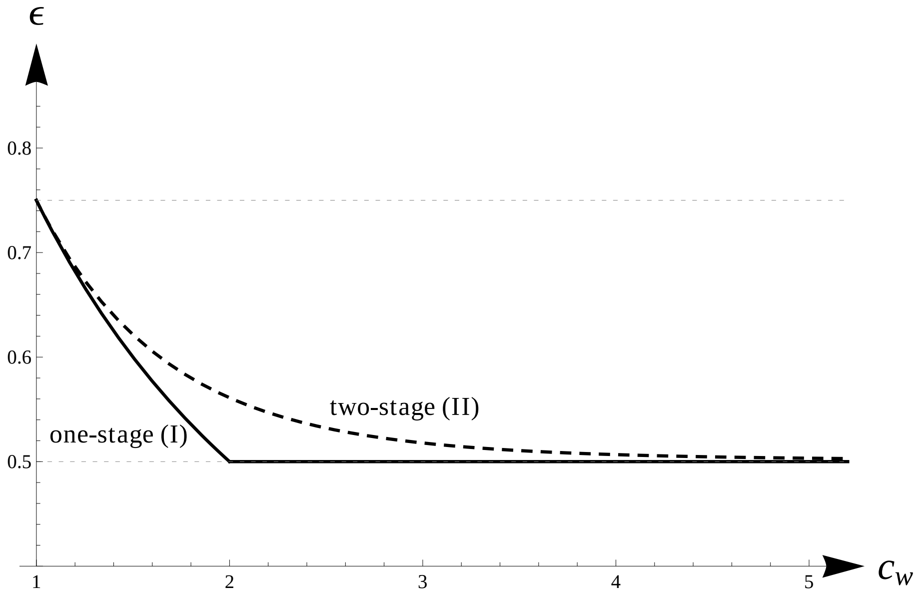
2.3 Designing the Promotion Tournament

Using results on equilibrium behavior of workers in the different promotion tournament specifications, we will now investigate how structural modifications by the principal affect incentive provision and selection performance. We start with the comparison of one- and two-stage tournaments (I versus II), before the two possible settings of the two-stage tournament, SSWW and SWSW are compared.

2.3.1 One-Stage vs. Two-Stages

Incentive provision and selection performance are identical in the one-stage and the two-stage tournament if all workers are of the same type. The equality in terms of selection performance follows automatically from the homogeneity assumption: Either, all workers are weak, and the probability that a strong worker wins is always zero, or, all workers are strong, and the probability that a strong worker wins must be one. That the tournament structure does not affect aggregate effort provision is less obvious. However, as Gradstein and Konrad (1999) established, this holds for the specification we consider. Consequently, the comparison of these two structures also allows us to investigate whether heterogeneity differently affects workers' behavior in one-stage and two-stage tournaments. A formal comparison of the incentive measures $\mathcal{E}(\texttt{I})$ and $\mathcal{E}(\texttt{II})$, and the selection measures $\mathcal{S}(\texttt{I})$ and $\mathcal{S}(\texttt{II})$, respectively, delivers the following Proposition:

Figure 2: Incentive Provision in One-Stage and Two-Stage Tournaments



Notes: The figure plots expressions (2) and (8) with $c_S = 1$ and $P = 1$.

Proposition 1. *When the cost of effort is strictly higher for weak than for strong agents ($c_W > c_S$),*

(a) aggregate effort is strictly higher in the two- than in one-stage tournament, i.e.,

$$\mathcal{E}(I) < \mathcal{E}(II) \text{ for all } c_W > c_S.$$

(b) the probability that a strong agent receives the promotion is strictly higher in the one- than in the two-stage tournament, i.e.,

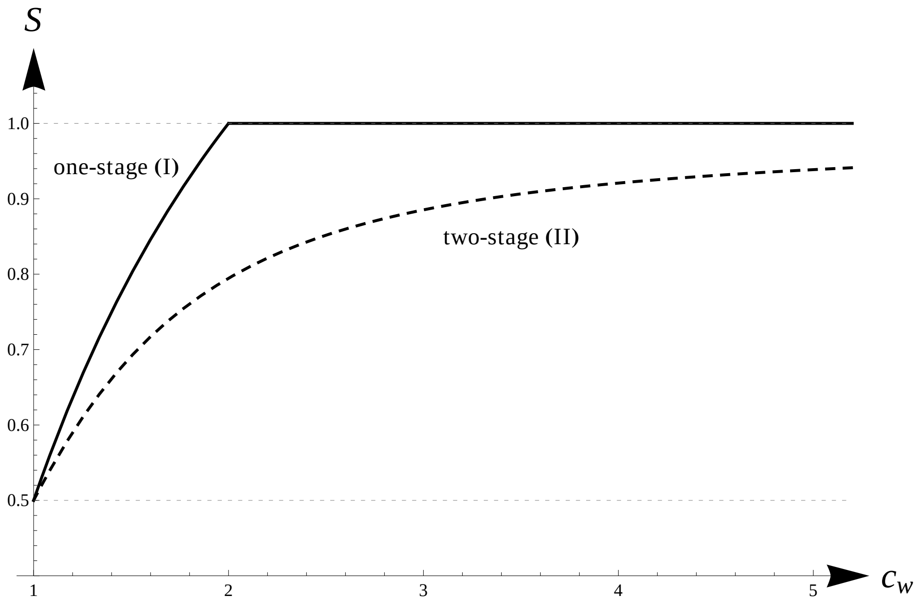
$$\mathcal{S}(I) > \mathcal{S}(II) \text{ for all } c_W > c_S.$$

Proof. See Appendix. □

Figure 2 plots the incentive measures of both tournament formats, $\mathcal{E}(\texttt{I})$ and $\mathcal{E}(\texttt{II})$, as a function of the effort costs of weak workers, c_W; effort costs for strong types and the value of the promotion are normalized to one, i.e., $c_S = 1$ and $P = 1$. The figure shows that the dotted line for aggregate effort provision in the two-stage tournament is always above the solid line for overall effort provision in the one-stage tournament, as part (a) of Proposition 1 suggests. The difference is highest at the kink of the one-stage incentive measure for $c_W = 2$ (where weak workers drop-out voluntarily, see Appendix A for details), and decreases subsequently. For extremely high values of c_W, aggregate effort provision approaches 0.5 in both tournament formats. The selection performance of both tournament formats is illustrated in Figure 3, which plots $\mathcal{S}(\texttt{I})$ and $\mathcal{S}(\texttt{II})$, i.e., the probability that a strong worker wins.[19] The figure shows that the one-stage dominates the two-stage tournament in terms of its selection

[19]The dependend variable is once again c_W, and effort costs of strong workers are again normalized to one.

Figure 3: Selection in One-Stage and Two-Stage Tournaments



Notes: The figure plots expressions (5) and (7) with $c_S = 1$.

performance; the probability that a strong worker is promoted is strictly higher in I than in II when workers are heterogeneous, i.e., if $c_W > 1$, as suggested by part (b) of Proposition 1. This difference is particularly pronounced for relatively low degrees of heterogeneity, since the curve of $\mathcal{S}(\texttt{I})$ is much steeper initially than the one for $\mathcal{S}(\texttt{II})$. Only after the kink of the one-stage selection measure at $c_W = 2$ the difference is reduced. However, even if the costs of effort are five times as high for weak than for strong workers, the probability that a strong worker wins is still almost ten percentage points higher in the one-stage than in the two-stage tournament; only when $c_W \to \infty$, both $\mathcal{S}(\texttt{I})$ and $\mathcal{S}(\texttt{II})$ approach one.[20]

2.3.2 Setting SSWW vs. Setting SWSW

As in the previous comparison, both the incentive provision and the selection performance are identical in settings SSWW and SWSW if workers are homogeneous. Intuitively, it does not matter how types are seeded if they are all equally talented. We start with a formal comparison of the incentive measures $\mathcal{E}(\texttt{SSWW})$ and $\mathcal{E}(\texttt{SWSW})$, and the selection measures $\mathcal{S}(\texttt{SSWW})$ and $\mathcal{S}(\texttt{SWSW})$, respectively. The results are summarized in the following Proposition:

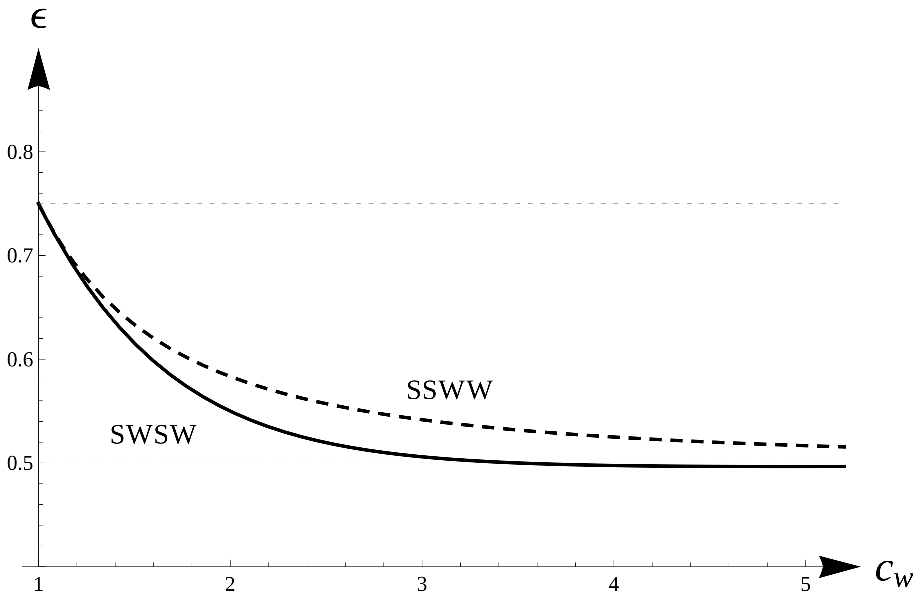
Proposition 2. *When the cost of effort is strictly higher for weak than for strong agents ($c_W > c_S$),*

(a) aggregate effort is strictly higher in setting SSWW than in setting SWSW, i.e.,

$$\mathcal{E}(\texttt{SSWW}) > \mathcal{E}(\texttt{SWSW}) \quad \text{for all} \quad c_W > c_S.$$

[20]This cannot be seen in Figure 3, but it can be formally derived. Details available upon request.

Figure 4: Incentive Provision in Two-Stage Tournaments by Setting



Notes: The figure plots expressions (4) and (6) with $c_{\mathtt{S}} = 1$ and $P = 1$.

(b) the probability that a strong agent receives the promotion is strictly higher in setting SWSW than in setting SSWW, i.e.,

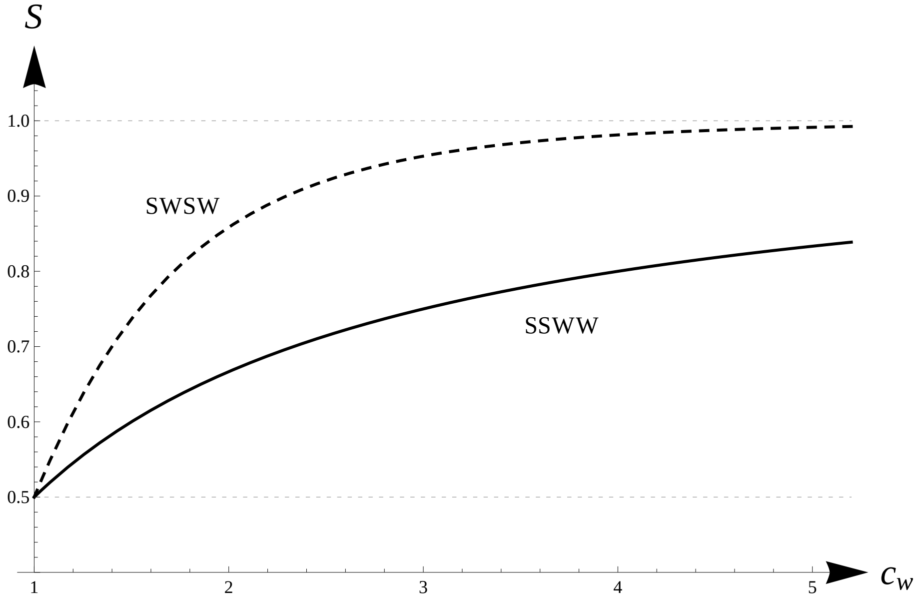
$$\mathcal{S}(\mathit{SSWW}) < \mathcal{S}(\mathit{SWSW}) \text{ for all } c_W > c_S.$$

Proof. See Appendix. □

Figure 4 graphically illustrates part (a) of Proposition 2 by plotting aggregate (expected) effort, i.e., $\mathcal{E}(\mathtt{SSWW})$ and $\mathcal{E}(\mathtt{SWSW})$, as a function of the constant marginal cost of effort for the weak workers, $c_{\mathtt{W}}$; the effort cost of the strong type and the value of a promotion are normalized to one ($c_{\mathtt{S}} = 1$ and $P = 1$). The figure shows that incentives for effort provision are strictly higher in setting SSWW than in setting SWSW. Moreover, one can see that the difference between the two settings is most pronounced for intermediate values of $c_{\mathtt{W}}$, since aggregate effort provision in both settings converges towards that of a tournament with strong workers only if $c_{\mathtt{W}} \to c_{\mathtt{S}}$, while both measures approach 0.5 if $c_{\mathtt{W}} \to \infty$. The selection measures $\mathcal{S}(\mathtt{SSWW})$ and $\mathcal{S}(\mathtt{SWSW})$ are plotted as a function of $c_{\mathtt{W}}$ in Figure 5; as in previous figures, the effort cost of the strong types are normalized to one. The promotion probability for strong workers is clearly higher in setting SWSW than in setting SSWW, as established in part (b) of Proposition 2. The difference between the two settings is sizable, both for low and for comparably high degrees of heterogeneity. Only for the limiting case $\frac{c_{\mathtt{W}}}{c_{\mathtt{S}}} \to \infty$, the selection performance of both settings becomes identical and converges to one.[21] This convergence is much faster in setting SWSW than in SSWW, however.

[21]Details of the proof for this claim are available from the authors upon request.

Figure 5: Selection in Two-Stage Tournaments by Setting



Notes: The figure plots expressions (5) and (7) with $c_S = 1$.

3 Discussion of Results

The previous analysis has shown that no structure is optimal both with respect to incentive provision and selection performance. Clearly, any tournament structure provides some incentives for effort and some sorting of types. However, it was shown that modifications which improve the performance in one deteriorate the performance in the other dimension: The two-stage tournament with random seeding dominates the one-stage format in terms of incentive provision, whereas the opposite holds for selection performance (Proposition 1). Similarly, the incentive provision properties of setting `SSWW` are better than they are in setting `SWSW`; yet, setting `SWSW` dominates with respect to selection performance (Proposition 2). Taken together, these results suggest that the two objectives incentive provision and selection are incompatible.

To explain this finding, one has to distinguish *absolute* and *relative* incentives for effort provision. The ratio of the workers' efforts determines the selection performance: The lower equilibrium efforts of weak workers are *relative* to equilibrium efforts of strong workers, the better is the selection performance of a tournament. In other words, the selection performance is increasing in the degree of heterogeneity between workers, which is also graphically illustrated in Figures 3 and 5. The higher the effort costs of weak workers are relative to effort costs of strong workers (which are normalized to one), the better does selection work in any tournament format. By contrast, it is well known that heterogeneity reduces the incentives for effort provision in tournaments: *Absolute* incentives, i.e., the sum of workers' efforts, are decreasing in the degree of heterogeneity. In all tournament formats considered in this paper, total effort provision is lower the higher heterogeneity between types, i.e., the higher the effort costs of weak workers are relative to effort costs of strong workers, as Figures 2 and 4 show. Summing up, it seems that tournament structures which amplify the degree of heterogeneity between strong and weak workers perform well in terms of selection, as heterogeneity discourages weak workers relatively more than it induces strong workers to slack off. At the same time, the more a tournament accommodates heterogeneity between types, the better is its performance in the incentive dimension, since heterogeneity decreases the incentives for effort provision for both strong and weak workers in absolute terms. Essentially, the formal analysis has shown that structural variations of tournaments with heterogeneous workers have similar effects as strategic handicaps. Lazear and Rosen (1981) showed that incentives for effort provision are maximized if strong participants of a tournament are handicapped in such a way that equilibrium winning probabilities are equalized across types. This result already indicated a conflict between the two goals incentive provision and selection performance. This paper shows that structural variation do not solve this problem. If, for example, a one-stage rather than a two-stage tournament is used to fill a vacancy, this modification works like an inverse handicap: Weak workers are discouraged, since they now compete with two strong workers simultaneously, rather than against one opponent at a time in pair-wise interaction of the two-stage structure; this reduces total effort provision and improves selection. Alternatively, using setting `SSWW` rather than `SWSW` essentially handicaps strong workers: Whereas it is faily easy for strong workers to reach stage 2 in setting `SWSW` due to the weak stage-1 opponent, it is equally hard for workers of both types to reach stage-2 in setting `SSWW`. Thus, incentives for effort provision are now higher, and selection performance is lower due to structural handicapping of strong types.

4 Concluding Remarks

We investigated whether the performance dimensions 'incentive provision' and 'accuracy in selection' are compatible in tournaments with heterogeneous workers. Comparing static one-stage and dynamic two-stage promotion tournaments, as well as two different seeding variants of two-stage promotion tournaments, our results suggest that they are incompatible. Even though any tournament with heterogeneous participants provides some incentives for effort and some sorting of types, modifications which improve the performance in one will deteriorate the performance in the other dimension, i.e., tournament formats that perform better in terms of incentive provision do worse in terms of selecting the best participant, and vice versa. The reason is that structural variations of tournaments with heterogeneous workers have similar effects as strategic handicaps. Intuitively, tournament structures which amplify the degree of heterogeneity between strong and weak workers perform well in terms of selection, as heterogeneity discourages weak workers relatively more than it induces strong workers to slack off. At the same time, the more a tournament accommodates heterogeneity between types, the better is its performance in the incentive dimension, since heterogeneity decreases the incentives for effort provision for both strong and weak workers in absolute terms. Therefore, multiple instruments should be used whenever two two goals are equally important, since a promotion tournament cannot be designed in such a way that it is optimal along both dimensions.

Our results are also important for applications where tournaments between heterogeneous participants are only used as a means for incentive provision. It is well known that heterogeneity is problematic in such settings. Existing solutions to this problem, such as handicapping à la Lazear and Rosen (1981), or wage discrimination à la Gürtler and Kräkel (2010), require that the principal knows workers' types, which is not always the case. In these cases, structural variations of the tournament structure may be the best solution. Therefore, we believe that the comparison of different tournament structures with heterogeneous participants is a promising topic for future work.

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Appendix

A Solution of the One-Stage Tournament

Due to symmetry, it suffices to solve the optimization problem of one strong and one weak worker. Without loss of generality, we consider the strong worker i and the weak worker k and obtain

$$\begin{aligned}\max_{x_i \geq 0} \Pi_i(\mathtt{I}) &= \frac{x_i}{X} P - c_{\mathtt{S}} x_i, \\ \max_{x_k \geq 0} \Pi_k(\mathtt{I}) &= \frac{x_k}{X} P - c_{\mathtt{W}} x_k,\end{aligned}$$

where $X = \sum_{m=1}^{4} x_m$ as before. This leads to the two first-order optimality conditions

$$(X - x_i)P = c_{\mathtt{S}} X^2 \quad \text{and} \quad (X - x_k)P = c_{\mathtt{W}} X^2.$$

Combining these conditions with symmetry (implying $X = 2x_i + 2x_k$) reveals that the relation

$$x^*_{\mathtt{W}} = \frac{2c_{\mathtt{S}} - c_{\mathtt{W}}}{2c_{\mathtt{W}} - c_{\mathtt{S}}} x^*_{\mathtt{S}} \tag{A1}$$

holds in an interior NE. Since the equilibrium efforts cannot be negative, a corner solution (with $x^*_{\mathtt{W}} = 0$) applies for $c_{\mathtt{W}} \geq 2c_{\mathtt{S}}$. In other words, weak workers drop out from the competition voluntarily for large differences in productivity (for $c_{\mathtt{W}} \geq 2c_{\mathtt{S}}$), leaving the two strong workers as the only contenders for the prize.[22] Taking these considerations into account, the equilibrium efforts of strong and weak workers are given by

$$x^*_{\mathtt{S}}(\mathtt{I}) = \begin{cases} \frac{3(2c_{\mathtt{W}} - c_{\mathtt{S}})}{4(c_{\mathtt{S}} + c_{\mathtt{W}})^2} P & \text{if } \frac{c_{\mathtt{W}}}{c_{\mathtt{S}}} < 2 \\ \frac{1}{4c_{\mathtt{S}}} P & \text{if } \frac{c_{\mathtt{W}}}{c_{\mathtt{S}}} \geq 2 \end{cases} \quad \text{and} \quad x^*_{\mathtt{W}}(\mathtt{I}) = \begin{cases} \frac{3(c_{\mathtt{W}} - 2c_{\mathtt{S}})}{4(c_{\mathtt{S}} + c_{\mathtt{W}})^2} P & \text{if } \frac{c_{\mathtt{W}}}{c_{\mathtt{S}}} < 2 \\ 0 & \text{if } \frac{c_{\mathtt{W}}}{c_{\mathtt{S}}} \geq 2 \end{cases}. \tag{A2}$$

B Solution of the Two-Stage Tournament

B1 Solution for Stage 2

(1) SS: If two strong (type S) workers i and j compete against each other on stage 2, they both face the same maximization problem. Without loss of generality, we consider the optimization by worker i, who maximizes his stage-2 payoff $\pi_{i2}(\mathtt{SS})$ by choosing an optimal level of effort x_{i2}, while taking the effort of his opponent x_{j2} as given.[23] Formally, this maximization problem reads

$$\max_{x_{i2} \geq 0} \pi_{i2}(\mathtt{SS}) = \frac{x_{i2}}{x_{i2} + x_{j2}} P - c_{\mathtt{S}} x_{i2}\,.$$

As shown by Cornes and Hartley (2005), any pairwise tournament has a unique interior equilibrium when the lottery CSF is used. Consequently, it suffices to consider first-order conditions which are both necessary and sufficient. Using the first-order condition for worker i ($x_{j2}P - c_{\mathtt{S}}(x_{i2} + x_{j2})^2 = 0$)

[22] See also Stein (2002) for details.

[23] Throughout the paper the first subscript of the variables π and x indicates the player, while the second subscript indicates the stage. The particular tournament environment considered (that is, SS, SW, WW, SSWW, or SWSW) is in parentheses – as in $\pi_{i2}(\mathtt{SS})$ – or is omitted when there is no risk of confusion.

and invoking symmetry ($x_{i2}^* = x_{j2}^*$) delivers equilibrium efforts

$$x_{\mathtt{S}2}^*(\mathtt{SS}) = x_{i2}^*(\mathtt{SS}) = x_{j2}^*(\mathtt{SS}) = \frac{P}{4c_{\mathtt{S}}}. \tag{B1}$$

Inserting optimal actions in the objective function gives the payoff that a strong worker can expect in equilibrium if he meets another strong worker on stage 2. Since the expected payoff is the same for workers i and j, the indices can be replaced by S (indicating strong workers). Equilibrium payoffs then read

$$\pi_{\mathtt{S}2}^*(\mathtt{SS}) = \frac{P}{4}\,. \tag{B2}$$

(2) WW: Suppose now that two weak (type W) workers k and l compete with each other on stage 2. Without loss of generality, we consider the optimization problem of worker k: $\max_{x_{k2}\geq 0} \pi_{k2}(\mathtt{WW}) = \frac{x_{k2}}{x_{k2}+x_{l2}}P - c_{\mathtt{W}}x_{k2}$. The same steps as in the solution of interaction SS deliver equilibrium efforts

$$x_{\mathtt{W}2}^*(\mathtt{WW}) = x_{k2}^*(\mathtt{WW}) = x_{l2}^*(\mathtt{WW}) = \frac{P}{4c_{\mathtt{W}}}. \tag{B3}$$

When inserting these efforts in the objective function, the expected equilibrium payoff for a weak worker in a stage 2 interaction WW is given by

$$\pi_{\mathtt{W}2}^*(\mathtt{WW}) = \frac{P}{4}\,. \tag{B4}$$

(3) SW: Finally, consider the situation where a strong worker S meets a weak worker W on stage 2. The optimization problems are as follows:

$$\begin{aligned}
\max_{x_{\mathtt{S}2}\geq 0} \pi_{\mathtt{S}2}(\mathtt{SW}) &= \frac{x_{\mathtt{S}2}}{x_{\mathtt{S}2}+x_{\mathtt{W}2}}P - c_{\mathtt{S}}x_{\mathtt{S}2}\,,\\
\max_{x_{\mathtt{W}2}\geq 0} \pi_{\mathtt{W}2}(\mathtt{SW}) &= \frac{x_{\mathtt{W}2}}{x_{\mathtt{S}2}+x_{\mathtt{W}2}}P - c_{\mathtt{W}}x_{\mathtt{W}2}\,.
\end{aligned}$$

First order conditions are necessary as well as sufficient in *heterogeneous* pairwise interactions (see Nti, 1999, or Cornes and Hartley, 2005). The combination of first-order conditions implies equilibrium efforts

$$x_{\mathtt{S}2}^*(\mathtt{SW}) = \frac{c_{\mathtt{W}}}{(c_{\mathtt{S}}+c_{\mathtt{W}})^2}P \quad \text{and} \quad x_{\mathtt{W}2}^*(\mathtt{SW}) = \frac{c_{\mathtt{S}}}{(c_{\mathtt{S}}+c_{\mathtt{W}})^2}P, \tag{B5}$$

respectively. Inserting optimal actions in the two objective functions gives the expected payoffs for strong and weak workers in a stage 2 interaction SW:

$$\pi_{\mathtt{S}2}^*(\mathtt{SW}) = \frac{c_{\mathtt{W}}^2}{(c_{\mathtt{S}}+c_{\mathtt{W}})^2}P\,, \tag{B6}$$

$$\pi_{\mathtt{W}2}^*(\mathtt{SW}) = \frac{c_{\mathtt{S}}^2}{(c_{\mathtt{S}}+c_{\mathtt{W}})^2}P. \tag{B7}$$

B2 Solution for Stage 1

Setting SSWW. Due to symmetry of the optimization problems, it suffices to solve the optimization problem of one strong worker (i or j), and one weak worker (k or l). Without loss of generality, we consider the maximization problems of workers i and k,

$$\max_{x_{i1}\geq 0} \Pi_i(\texttt{SSWW}) = \frac{x_{i1}}{x_{i1}+x_{j1}}\pi^*_{\texttt{S}2}(\texttt{SW}) - c_{\texttt{S}}x_{i1} ,$$
$$\max_{x_{k1}\geq 0} \Pi_k(\texttt{SSWW}) = \frac{x_{k1}}{x_{k1}+x_{l1}}\pi^*_{\texttt{W}2}(\texttt{SW}) - c_{\texttt{W}}x_{k1} .$$

Note that the optimization problem for strong workers is identical to the one considered in stage-2 interaction SS, the only difference is the expected prize, which now amounts to $\pi^*_{\texttt{S}2}(\texttt{SW})$ rather than P. Analogously, weak workers face the same situation as in stage-2 interaction WW with a different prize ($\pi^*_{\texttt{W}2}(\texttt{SW})$ instead of P). Consequently, first-order and symmetry conditions deliver stage 1 equilibrium efforts

$$x^*_{\texttt{S}1}(\texttt{SSWW}) \equiv x^*_{i1}(\texttt{SSWW}) = x^*_{j1}(\texttt{SSWW}) = \frac{c_{\texttt{W}}^2}{4c_{\texttt{S}}(c_{\texttt{S}}+c_{\texttt{W}})^2}P \tag{B8}$$
$$x^*_{\texttt{W}1}(\texttt{SSWW}) \equiv x^*_{k1}(\texttt{SSWW}) = x^*_{l1}(\texttt{SSWW}) = \frac{c_{\texttt{S}}^2}{4c_{\texttt{W}}(c_{\texttt{S}}+c_{\texttt{W}})^2}P. \tag{B9}$$

Setting SWSW. We assume (without loss of generality) that workers i and k are strong, whereas workers j and l are weak, and that the two pairwise stage-1 interactions are between workers i and j, and between workers k and l, respectively. We start by considering the decision problem of strong worker i and weak worker j. Both workers choose their optimal stage-1 effort, given equilibrium behavior in any potential stage-2 interaction. The optimization problems are

$$\max_{x_{i1}\geq 0} \Pi_i(\texttt{SWSW}) = \frac{x_{i1}}{x_{i1}+x_{j1}} \underbrace{\left[\frac{x_{k1}}{x_{k1}+x_{l1}}\pi^*_{\texttt{S}2}(\texttt{SS}) + \frac{x_{l1}}{x_{k1}+x_{l1}}\pi^*_{\texttt{S}2}(\texttt{SW})\right]}_{\equiv P_i(x_{k1},x_{l1})} - c_{\texttt{S}}x_{i1}$$
$$\max_{x_{j1}\geq 0} \Pi_j(\texttt{SWSW}) = \frac{x_{j1}}{x_{i1}+x_{j1}} \underbrace{\left[\frac{x_{k1}}{x_{k1}+x_{l1}}\pi^*_{\texttt{W}2}(\texttt{SW}) + \frac{x_{l1}}{x_{k1}+x_{l1}}\pi^*_{\texttt{W}2}(\texttt{WW})\right]}_{\equiv P_j(x_{k1},x_{l1})} - c_{\texttt{W}}x_{j1} .$$

The continuation values $P_i(x_{k1}, x_{l1})$ and $P_j(x_{k1}, x_{l1})$ of workers i and j, respectively, depend on the behavior of workers k and l in the other stage-1 interaction. Similarly, the continuation values $P_k(x_{i1}, x_{j1})$ and $P_l(x_{i1}, x_{j1})$ of workers k and l depend on the behavior of workers i and j, as their formal optimization problems show:

$$\max_{x_{k1}\geq 0} \Pi_k(\texttt{SWSW}) = \frac{x_{k1}}{x_{k1}+x_{l1}} \underbrace{\left[\frac{x_{i1}}{x_{i1}+x_{j1}}\pi^*_{\texttt{S}2}(\texttt{SS}) + \frac{x_{j1}}{x_{i1}+x_{j1}}\pi^*_{\texttt{S}2}(\texttt{SW})\right]}_{\equiv P_k(x_{i1},x_{j1})} - c_{\texttt{S}}x_{k1},$$
$$\max_{x_{l1}\geq 0} \Pi_l(\texttt{SWSW}) = \frac{x_{l1}}{x_{k1}+x_{l1}} \underbrace{\left[\frac{x_{i1}}{x_{i1}+x_{j1}}\pi^*_{\texttt{W}2}(\texttt{SW}) + \frac{x_{j1}}{x_{i1}+x_{j1}}\pi^*_{\texttt{W}2}(\texttt{WW})\right]}_{\equiv P_l(x_{i1},x_{j1})} - c_{\texttt{W}}x_{l1}.$$

Therefore, the two stage-1 interactions are linked through endogenously determined continuation values. The reason is that expected equilibrium payoffs for workers differ across the three potential stage-2 interactions SS, WW, and SW. Conditional on reaching stage 2, workers of both types have a higher expected payoff from meeting a weak rather than a strong opponent, since $\pi^*_{\texttt{W}2}(\texttt{WW}) > \pi^*_{\texttt{W}2}(\texttt{SW})$ and $\pi^*_{\texttt{S}2}(\texttt{SW}) > \pi^*_{\texttt{S}2}(\texttt{SS})$. However, each worker takes the probability that the opponent is of a certain type as given, since it is determined in the parallel stage-1 interaction. The first-order conditions for the interaction between workers i and j read

$$x_{j1}P_i(x_{k1},x_{l1}) - c_{\mathtt{S}}(x_{i1}+x_{j1})^2 = 0 \quad \text{and} \quad x_{i1}P_k(x_{k1},x_{l1}) - c_{\mathtt{W}}(x_{i1}+x_{j1})^2 = 0.$$

The respective conditions for the other stage-1 interaction between workers k and l are

$$x_{l1}P_j(x_{i1},x_{j1}) - c_{\mathtt{S}}(x_{k1}+x_{l1})^2 = 0 \quad \text{and} \quad x_{k1}P_l(x_{i1},x_{j1}) - c_{\mathtt{W}}(x_{k1}+x_{l1})^2 = 0\,.$$

Combining the four conditions, we obtain two expressions that define a relation between equilibrium effort choices of workers *within* each interaction, namely

$$\frac{x_{i1}}{x_{j1}} = \frac{c_{\mathtt{W}}}{c_{\mathtt{S}}}\frac{P_i(x_{k1},x_{l1})}{P_j(x_{k1},x_{l1})} \quad \text{and} \quad \frac{x_{k1}}{x_{l1}} = \frac{c_{\mathtt{W}}}{c_{\mathtt{S}}}\frac{P_k(x_{i1},x_{j1})}{P_l(x_{i1},x_{j1})}, \tag{B10}$$

respectively. These expressions show that each stage-1 interaction is a tournament between workers with different costs and endogenously different valuations of winning. While the costs of effort differ by construction, the difference of the value for winning is a result of the tournament structure: Reaching stage 2 is more valuable for strong than for weak workers.

We proceed now to the solution of the problem, which comprises two heterogeneous participants with regard to their effort costs and their valuation. As mentioned previously, any tournament with two heterogeneous participants has a unique, interior equilibrium for the chosen contest success function (Cornes and Hartley 2005, Nti 1999). Consequently, each of the two pairwise stage-1 interactions has a unique equilibrium for each pair of continuation values. What remains to be shown is that the two expressions in (B10) can be satisfied *jointly* such that both stage-1 interactions are satisfied simultaneously in equilibrium. Inserting the expressions for the continuation values in (B10) and simplifying gives

$$\frac{x_{i1}}{x_{j1}} = \frac{c_{\mathtt{W}}}{c_{\mathtt{S}}}\frac{(c_{\mathtt{S}}+c_{\mathtt{W}})^2\frac{x_{k1}}{x_{l1}} + 4c_{\mathtt{W}}^2}{4c_{\mathtt{S}}^2\frac{x_{k1}}{x_{l1}} + (c_{\mathtt{S}}+c_{\mathtt{W}})^2} \quad \text{and} \quad \frac{x_{k1}}{x_{l1}} = \frac{c_{\mathtt{W}}}{c_{\mathtt{S}}}\frac{(c_{\mathtt{S}}+c_{\mathtt{W}})^2\frac{x_{i1}}{x_{j1}} + 4c_{\mathtt{W}}^2}{4c_{\mathtt{S}}^2\frac{x_{i1}}{x_{j1}} + (c_{\mathtt{S}}+c_{\mathtt{W}})^2}\,. \tag{B11}$$

System (B11) consists of two equations in the two unknowns $\frac{x_{i1}^*}{x_{j1}^*}$ and $\frac{x_{k1}^*}{x_{l1}^*}$, respectively. Note that both equations are symmetric, since the two workers in each of the two stage-1 interactions face identical optimization problems. This implies that the conditions $x_{\mathtt{S}1}^* \equiv x_{i1}^* = x_{k1}^*$ and $x_{\mathtt{W}1}^* \equiv x_{j1}^* = x_{l1}^*$ do hold in the symmetric equilibrium.[24] Imposing this condition on (B11) gives a quadratic equation in $x_{\mathtt{S}1}^*$ and $x_{\mathtt{W}1}^*$:

$$\begin{aligned}
\frac{x_{\mathtt{S}1}^*}{x_{\mathtt{W}1}^*} &= \frac{c_{\mathtt{W}}}{c_{\mathtt{S}}}\frac{(c_{\mathtt{S}}+c_{\mathtt{W}})^2\frac{x_{\mathtt{S}1}^*}{x_{\mathtt{W}1}^*} + 4c_{\mathtt{W}}^2}{4c_{\mathtt{S}}^2\frac{x_{\mathtt{S}1}^*}{x_{\mathtt{W}1}^*} + (c_{\mathtt{S}}+c_{\mathtt{W}})^2} \\
\Leftrightarrow \quad 0 &= 4c_{\mathtt{S}}^2\left[\frac{x_{\mathtt{S}1}^*}{x_{\mathtt{W}1}^*}\right]^2 + \left(1-\frac{c_{\mathtt{W}}}{c_{\mathtt{S}}}\right)(c_{\mathtt{S}}+c_{\mathtt{W}})^2\left[\frac{x_{\mathtt{S}1}^*}{x_{\mathtt{W}1}^*}\right] - 4\frac{c_{\mathtt{W}}^3}{c_{\mathtt{S}}} \\
\Leftrightarrow \quad \frac{x_{\mathtt{S}1}^*}{x_{\mathtt{W}1}^*} &= F^*(c_{\mathtt{S}},c_{\mathtt{W}}),
\end{aligned}$$

[24]The symmetric equilibrium exists for any degree of heterogeneity and is unique. Intuitively, one must show that the graphs of the two relations in (B11) have a unique intersection in the domain defined by $\frac{x_{j1}^*}{x_{i1}^*} \in [0,1]$ and $\frac{x_{l1}^*}{x_{k1}^*} \in [0,1]$. It suffices to consider this domain, since the assumption of lower costs of effort and the resulting higher value of winning of strong workers imply that $x_{i1}^* \geq x_{j1}^*$ and $x_{k1}^* \geq x_{l1}^*$, respectively. This follows from (B10). Details and a complete formal proof are available from the authors upon request.

where

$$F^*(c_{\mathtt{S}}, c_{\mathtt{W}}) = \frac{(c_{\mathtt{W}} - c_{\mathtt{S}})(c_{\mathtt{S}} + c_{\mathtt{W}})^2 + \sqrt{64 c_{\mathtt{W}}^3 c_{\mathtt{S}}^3 + (c_{\mathtt{S}} - c_{\mathtt{W}})^2 (c_{\mathtt{S}} + c_{\mathtt{W}})^4}}{8 c_{\mathtt{S}}^3}. \tag{B12}$$

$F^*(c_{\mathtt{S}}, c_{\mathtt{W}})$ is the ratio of stage-1 efforts of the two worker types, which is directly proportional to heterogeneity in costs and continuation values, as equation (B10) shows. Therefore, $F^*(c_{\mathtt{S}}, c_{\mathtt{W}})$ can be interpreted as a measure for both the exogenous heterogeneity in effort costs between strong and weak workers and the endogenous heterogeneity between types that is due to different continuation values in stage 1.

The expression $F^*(c_{\mathtt{S}}, c_{\mathtt{W}})$ allows us to disentangle and solve analytically the two interdependent stage-1 interactions. We start by considering the continuation values which satisfy

$$\begin{aligned} P_i(x^*_{\mathtt{S}1}, x^*_{\mathtt{W}1}) = P_k(x^*_{\mathtt{S}1}, x^*_{\mathtt{W}1}) &= \frac{(c_{\mathtt{S}} + c_{\mathtt{W}})^2 F^*(c_{\mathtt{W}}, c_{\mathtt{S}}) + 4 c_{\mathtt{W}}^2}{4(c_{\mathtt{S}} + c_{\mathtt{W}})^2 [1 + F^*(c_{\mathtt{S}}, c_{\mathtt{W}})]} P\,, \\ P_j(x^*_{\mathtt{S}1}, x^*_{\mathtt{W}1}) = P_l(x^*_{\mathtt{S}1}, x^*_{\mathtt{W}1}) &= \frac{(c_{\mathtt{S}} + c_{\mathtt{W}})^2 + 4 c_{\mathtt{S}}^2 F^*(c_{\mathtt{W}}, c_{\mathtt{S}})}{4(c_{\mathtt{S}} + c_{\mathtt{W}})^2 [1 + F^*(c_{\mathtt{S}}, c_{\mathtt{W}})]} P. \end{aligned}$$

Note that $P_i(x^*_{\mathtt{S}1}, x^*_{\mathtt{W}1}) = P_k(x^*_{\mathtt{S}1}, x^*_{\mathtt{W}1})$ and $P_j(x^*_{\mathtt{S}1}, x^*_{\mathtt{W}1}) = P_l(x^*_{\mathtt{S}1}, x^*_{\mathtt{W}1})$ due to symmetry. Given these continuation values, stage 1 equilibrium efforts can be determined as

$$x^*_{\mathtt{S}1}(\mathtt{SWSW}) \equiv x^*_{i1}(\mathtt{SWSW}) = x^*_{k1}(\mathtt{SWSW}) = \frac{(c_{\mathtt{S}} + c_{\mathtt{W}})^2 F^*(c_{\mathtt{W}}, c_{\mathtt{S}})^2 + 4 c_{\mathtt{W}}^2 F^*(c_{\mathtt{W}}, c_{\mathtt{S}})}{4 c_{\mathtt{S}} (c_{\mathtt{S}} + c_{\mathtt{W}})^2 [1 + F^*(c_{\mathtt{S}}, c_{\mathtt{W}})]^3} P \tag{B13}$$

$$x^*_{\mathtt{W}1}(\mathtt{SWSW}) \equiv x^*_{j1}(\mathtt{SWSW}) = x^*_{l1}(\mathtt{SWSW}) = \frac{(c_{\mathtt{S}} + c_{\mathtt{W}})^2 F^*(c_{\mathtt{W}}, c_{\mathtt{S}}) + 4 c_{\mathtt{S}}^2 F^*(c_{\mathtt{W}}, c_{\mathtt{S}})^2}{4 c_{\mathtt{W}} (c_{\mathtt{S}} + c_{\mathtt{W}})^2 [1 + F^*(c_{\mathtt{S}}, c_{\mathtt{W}})]^3} P\,. \tag{B14}$$

C Proofs

Lemma 1. *Assume without loss of generality that $c_W \geq c_S = 1$ and define $f(c_W) = \frac{5c_W^3 + 2c_W^2 + c_W}{c_W^2 + 2c_W + 5}$. Then, the relation $F^*(1, c_W) > f(c_W)$ does hold for all $c_W > 1$, where $F^*(1, c_W)$ is defined as in* (B12)*. Furthermore, for $c_W = 1$ it holds that $F^*(1, c_W) = f(c_W)$.*

Proof. From equation (B10), we know that $\frac{x_{i1}}{x_{k1}} = \frac{c_{\mathtt{W}}}{c_{\mathtt{S}}} \frac{P_i(x_{j1}, x_{l1})}{P_k(x_{j1}, x_{l1})}$. Further, equation (B12) tells us that $\frac{x^*_{i1}}{x^*_{k1}} = F^*(c_{\mathtt{S}}, c_{\mathtt{W}})$. Consequently, using the assumption that $c_{\mathtt{W}} \geq c_{\mathtt{S}} = 1$, it must hold that

$$F^*(1, c_{\mathtt{W}}) = c_{\mathtt{W}} \frac{P_i(x_{j1}, x_{l1})}{P_k(x_{j1}, x_{l1})} = \frac{4 c_{\mathtt{W}}^3 + c_{\mathtt{W}} (1 + c_{\mathtt{W}})^2 \times \frac{x_{j1}}{x_{l1}}}{(1 + c_{\mathtt{W}})^2 + 4 \times \frac{x_{j1}}{x_{l1}}}.$$

Note that

$$\frac{\partial F^*(1, c_{\mathtt{W}})}{\partial \frac{x_{j1}}{x_{l1}}} = \frac{(1 + c_{\mathtt{W}})^4 - 16 c_{\mathtt{W}}^2}{[(1 + c_{\mathtt{W}})^2 + 4 \times \frac{x_{j1}}{x_{l1}}]^2} > 0$$

if $c_{\mathtt{W}} > 1$. Further, recall that player l has both higher cost ($c_{\mathtt{W}} > 1$) and a lower continuation value ($P_j > P_l$), such that $x_{j1} > x_{l1}$ does hold. Therefore, assuming $x_{j1} = x_{l1}$ underestimates $F^*(1, c_{\mathtt{W}})$. Since

$$f(c_{\mathtt{W}}) = \frac{5 c_{\mathtt{W}}^3 + 2 c_{\mathtt{W}}^2 + c_{\mathtt{W}}}{c_{\mathtt{W}}^2 + 2 c_{\mathtt{W}} + 5}$$

is the expression we derive from $F^*(1, c_{\mathtt{W}})$ under this assumption, we have proven $F^*(1, c_{\mathtt{W}}) > f(c_{\mathtt{W}})$. If we assume $c_{\mathtt{W}} = 1$, all players are perfectly symmetric, such that $x_{j1} = x_{l1}$ does hold. Consequently, the relation $F^*(1, c_{\mathtt{W}}) = f(c_{\mathtt{W}})$ does hold for $c_{\mathtt{W}} = 1$. □

Lemma 2. *Assume without loss of generality that $c_W \geq c_S = 1$ and define $f_{\text{low}}(c_W) = 2c_W - 1$. Then, the relation $F^*(1, c_W) < f_{\text{low}}(c_W)$ does hold for all $c_W > 1$. Furthermore, for $c_W = 1$, it holds that $f(c_W) = f_{\text{low}}(c_W)$.*

Proof. We start with the relation that we want to prove, namely:

$$\begin{aligned} f(c_W) &> f_{\text{low}}(c_W) \\ \Leftrightarrow 5c_W^3 + 2c_W^2 + c_W &> (2c_W - 1)(c_W^2 + 2c_W + 5) \\ \Leftrightarrow 3c_W^3 - c_W^2 - 7c_W + 5 &> 0 \end{aligned}$$

We now have to prove that $\phi(c_W) \equiv 3c_W^3 - c_W^2 - 7c_W + 5 > 0$ does always hold for $c_W > 1$. To see this, note that $\phi(\cdot)$ is a cubic function that has a local minimum at $c_W = 1$, and a local maximum at $c_W = -7/9$. Furthermore, $\phi(1) = 0$, which implies that $\phi(c_W) > 0$ for all $c_W > 1$. □

Lemma 3. *Assume without loss of generality that $c_W \geq c_S = 1$ and define $f_{\text{high}}(c_W) = \frac{c_W^3 + 2c_W^2 + c_W}{4}$. Then, the relation $F^*(1, c_W) < f_{\text{high}}(c_W)$ does hold for all $c_W > 1$. Furthermore, for $c_W = 1$, it holds that $F^*(1, c_W) = f_{\text{high}}(c_W)$.*

Proof. From equation (B10), we know that $\frac{x_{i1}}{x_{k1}} = \frac{c_W}{c_S} \frac{P_i(x_{j1}, x_{l1})}{P_k(x_{j1}, x_{l1})}$. Further, equation (B12) tells us that $\frac{x_{i1}^*}{x_{k1}^*} = F^*(c_S, c_W)$. Consequently, using the assumption that $c_W \geq c_S = 1$, it must hold that

$$F^*(1, c_W) = c_W \frac{P_i(x_{j1}, x_{l1})}{P_k(x_{j1}, x_{l1})} = \frac{4c_W^3 \times \frac{x_{l1}}{x_{j1}} + c_W(1 + c_W)^2}{(1 + c_W)^2 \times \frac{x_{l1}}{x_{j1}} + 4}.$$

Note that

$$\frac{\partial F^*(1, c_W)}{\partial \frac{x_{l1}}{x_{j1}}} = -\frac{(c_W - 1)^2 c_W (c_W^2 + 6c_W + 1)}{[(1 + c_W)^2 \times \frac{x_{l1}}{x_{j1}} + 4]^2} < 0$$

if $c_W > 1$. Further, recall from the main text that player l will never drop out in a pairwise competition for any finite degree of heterogeneity in terms of costs and continuation value, such that $x_{l1} > 0$ does hold. Therefore, assuming $x_{l1} = 0$ (which implies $\frac{x_{l1}}{x_{j1}} = 0$) overestimates $F^*(1, c_W)$, since this expression is decreasing in $\frac{x_{l1}}{x_{j1}}$. Since

$$f_{\text{high}}(c_W) = \frac{c_W^3 + 2c_W^2 + c_W}{4}$$

is the expression we derive from $F^*(1, c_W)$ under this assumption, we have proven $F^*(1, c_W) < f_{\text{high}}(c_W)$. If we assume $c_W = 1$, all players are perfectly symmetric, such that $x_{l1} = x_{j1}$ does hold. When inserting this relation in $F^*(1, c_W)$, we see that the relation $F^*(1, c_W) = f(c_W)$ does hold for $c_W = 1$. □

Proposition 1: *When the cost of effort is strictly higher for weak than for strong agents ($c_{\mathtt{W}} > c_{\mathtt{S}}$),*

(a) aggregate effort is strictly higher in the two- than in one-stage tournament, i.e.,

$$\mathcal{E}(\mathtt{I}) < \mathcal{E}(\mathtt{II}) \text{ for all } c_{\mathtt{W}} > c_{\mathtt{S}}.$$

(b) the probability that a strong agent receives the promotion is higher in the one- than in the two-stage tournament, i.e.,

$$\mathcal{S}(\mathtt{I}) > \mathcal{S}(\mathtt{II}) \text{ for all } c_{\mathtt{W}} > c_{\mathtt{S}}.$$

Proof. We will separately prove parts (a) and (b) of Proposition 1. We start with part (a).

(a): To prove the relation $\mathcal{E}(\mathtt{II}) > \mathcal{E}(\mathtt{I})$ for all $c_{\mathtt{W}} > c_{\mathtt{S}}$, we assume without loss of generality that $c_{\mathtt{W}} > c_{\mathtt{S}} = 1$. Recall from (2) that $\mathcal{E}(\mathtt{I})$ is defined stepwise, i.e., $\mathcal{E}(\mathtt{I}) = \max\{\frac{3}{2+2c_{\mathtt{W}}}P, \frac{1}{2}P\}$. First, we will consider the range $1 < c_{\mathtt{W}} \leq 2$, where $\mathcal{E}(\mathtt{I}) = \frac{3}{2+2c_{\mathtt{W}}}P$. In the second part of this proof, we will devote attention to $c_{\mathtt{W}} > 2$ and $\mathcal{E}(\mathtt{I}) = \frac{1}{2}P$.

(i) We consider the range $1 < c_{\mathtt{W}} \leq 2$ and want to prove that

$$\begin{aligned} \mathcal{E}(\mathtt{I}) &< \mathcal{E}(\mathtt{II}) \\ \Leftrightarrow \mathcal{E}(\mathtt{I}) &< \frac{2}{3}\mathcal{E}(\mathtt{SWSW}) + \frac{1}{3}\mathcal{E}(\mathtt{SSWW}). \end{aligned}$$

Recall from the proof of Proposition 1 that the formal expression for $\mathcal{E}(\mathtt{SWSW})$ is fairly complicated, in particular due to the $F^*(1, c_{\mathtt{W}})$-function. To simplify the subsequent analysis, we will therefore make use again of Lemmata 1/2 and replace $F^*(1, c_{\mathtt{W}})$ by $f_{\text{low}}(c_{\mathtt{W}}) = 2c_{\mathtt{W}} - 1$. This is without loss of generality, since $\mathcal{E}(\mathtt{SWSW})$ is strictly increasing in $F^*(1, c_{\mathtt{W}})$:

$$\frac{\partial \mathcal{E}(\mathtt{SWSW})}{\partial F^*(1, c_{\mathtt{W}})} = \frac{(2c_{\mathtt{W}}^3 - c_{\mathtt{W}}^2 - 4c_{\mathtt{W}} + 7)F^*(1, c_{\mathtt{W}}) + 3c_{\mathtt{W}}^2 + 2c_{\mathtt{W}} - 1}{2c_{\mathtt{W}}(1 + c_{\mathtt{W}})^2(1 + F^*(1, c_{\mathtt{W}}))^3} > 0.$$

Note that the denominator is always greater than zero, since we know from Lemma 1 that (a) $\frac{\partial F^*(1,c_{\mathtt{W}})}{\partial c_{\mathtt{W}}} > 0$ and (b) $F^*(1) = 1$; this implies that the sign of the derivative is determined by the numerator, which is also greater than zero for all $1 < c_{\mathtt{W}} \leq 2$. Consequently, effort $\mathcal{E}(\mathtt{SWSW})$ is underestimated through the replacement of $F^*(1, c_{\mathtt{W}})$ by $f_{\text{low}}(c_{\mathtt{W}})$. Inserting $f_{\text{low}}(c_{\mathtt{W}})$ and simplifying leaves us with the sufficient condition

$$Q(c_{\mathtt{W}}) \equiv \frac{(c_{\mathtt{W}} - 1)^2(6c_{\mathtt{W}}^3 + 2c_{\mathtt{W}}^2 - 9c_{\mathtt{W}} + 4)}{12c_{\mathtt{W}}^3(1 + c_{\mathtt{W}})^2} = \frac{(c_{\mathtt{W}} - 1)^2 q(c_{\mathtt{W}})}{12c_{\mathtt{W}}^3(1 + c_{\mathtt{W}})^2} > 0.$$

In the relevant range $1 < c_{\mathtt{W}} \leq 2$, the expression $(c_{\mathtt{W}} - 1)^2$ in the numerator as well as the denominator $12c_{\mathtt{W}}^3(1 + c_{\mathtt{W}})^2$ are always greater than zero, such that the sign of $Q(c_{\mathtt{W}})$ is determined by the expression $q(c_{\mathtt{W}}) \equiv 6c_{\mathtt{W}}^3 + 2c_{\mathtt{W}}^2 - 9c_{\mathtt{W}} + 4$. Note that $q(1) = 3$ and $q(2) = 42$. Since

$$\frac{\partial q(c_{\mathtt{W}})}{\partial c_{\mathtt{W}}} = 18c_{\mathtt{W}}^2 + 4c_{\mathtt{W}} - 9 > 0$$

for all $c_{\mathtt{W}} > 1$, it holds that $q(c_{\mathtt{W}}) > 0$ for all $1 < c_{\mathtt{W}} \leq 2$, which immediately implies that $Q(c_{\mathtt{W}}) > 0$. This completes the first part of the proof.

(ii) When $c_{\mathtt{W}} > 2$, it holds that $\mathcal{E}(\mathtt{I}) = \frac{1}{2}P$. We have to prove that the relation $\mathcal{E}(\mathtt{II}) > \mathcal{E}(\mathtt{I})$ is satisfied.

Inserting the respective expressions for $\mathcal{E}(\texttt{II})$ and $\mathcal{E}(\texttt{I})$ gives the condition:

$$\begin{aligned} \frac{(3c_{\texttt{W}}^3+6c_{\texttt{W}}^2+4c_{\texttt{W}}+9)F^*(1,c_{\texttt{W}})^2+(2c_{\texttt{W}}^3+14c_{\texttt{W}}^2+16c_{\texttt{W}}+4)F^*(1,c_{\texttt{W}})+c_{\texttt{W}}^3+4c_{\texttt{W}}^2+6c_{\texttt{W}}+3}{3c_{\texttt{W}}(c_{\texttt{W}}+1)^2(1+F^*(1,c_{\texttt{W}}))^2} &> 1 \\ \Leftrightarrow B(F^*(1,c_{\texttt{W}}),c_{\texttt{W}}) \equiv (c_{\texttt{W}}+9)F^*(1,c_{\texttt{W}})^2-(4c_{\texttt{W}}^3-2c_{\texttt{W}}^2-10c_{\texttt{W}}-4)F^*(1,c_{\texttt{W}})-(2c_{\texttt{W}}^2-3)(c_{\texttt{W}}+1) &> 0 \end{aligned}$$

Note that $B(\cdot)$ is minimized for $F^*(1,c_{\texttt{W}})_{\min} = \frac{2c_{\texttt{W}}^3-c_{\texttt{W}}^2-5c_{\texttt{W}}-2}{9+c_{\texttt{W}}}$, since

$$\frac{\partial B(\cdot)}{\partial F^*(1,c_{\texttt{W}})} = (18+2c_{\texttt{W}})F^*(1,c_{\texttt{W}})-2c_{\texttt{W}}^3+2c_{\texttt{W}}^2+10c_{\texttt{W}}+4 \quad \text{and} \quad \frac{\partial^2 B(\cdot)}{\partial[F^*(1,c_{\texttt{W}})]^2} = 18+2c_{\texttt{W}} > 0.$$

Moreover, note that $F^*(1,c_{\texttt{W}})_{\min} > 0$ for all $c_{\texttt{W}} > 2$, which implies that $B(\cdot)$ is increasing in $F^*(1,c_{\texttt{W}})$ in the range which is relevant for this proof. Consequently, when solving the relation $B(\cdot) > 0$ for $F^*(1,c_{\texttt{W}})$, we know that $F^*(1,c_{\texttt{W}})$ must not be in the range between the two roots, as $B(\cdot)$ is negative here. We obtain

$$\begin{gathered} B(F^*(1,c_{\texttt{W}}),c_{\texttt{W}}) > 0 \\ \Leftrightarrow F^*(1,c_{\texttt{W}})^2 - \frac{(4c_{\texttt{W}}^3-2c_{\texttt{W}}^2-10c_{\texttt{W}}-4)}{(c_{\texttt{W}}+9)}F^*(1,c_{\texttt{W}}) - \frac{(2c_{\texttt{W}}^2-3)(c_{\texttt{W}}+1)}{(c_{\texttt{W}}+9)} > 0 \\ \Leftrightarrow F^*(1,c_{\texttt{W}}) < \frac{2c_{\texttt{W}}^3-c_{\texttt{W}}^2-5c_{\texttt{W}}-2-\sqrt{K(c_{\texttt{W}})}}{9+c_{\texttt{W}}} \;\vee\; F^*(1,c_{\texttt{W}}) > \frac{2c_{\texttt{W}}^3-c_{\texttt{W}}^2-5c_{\texttt{W}}-2+\sqrt{K(c_{\texttt{W}})}}{9+c_{\texttt{W}}}, \end{gathered}$$

where

$$K(c_{\texttt{W}}) = 4c_{\texttt{W}}^6-4c_{\texttt{W}}^5-17c_{\texttt{W}}^4+22c_{\texttt{W}}^3+44c_{\texttt{W}}^2-10c_{\texttt{W}}-23.$$

We do only have to consider the second relation, since the first one is always below one for $c_{\texttt{W}} > 2$, while $F^*(1,c_{\texttt{W}}) \geq 1$ for all $c_{\texttt{W}} \geq 1$.[25] To complete the proof, we have to show that

$$F^*(1,c_{\texttt{W}}) > \frac{2c_{\texttt{W}}^3-c_{\texttt{W}}^2-5c_{\texttt{W}}-2+\sqrt{K(c_{\texttt{W}})}}{9+c_{\texttt{W}}}$$

for all $c_{\texttt{W}} > 2$. Inserting the equilibrium relation $F^*(1,c_{\texttt{W}})$ from (B12) gives:

$$\begin{aligned} \frac{(c_{\texttt{W}}-1)(1+c_{\texttt{W}})^2+\sqrt{64c_{\texttt{W}}^3+(1-c_{\texttt{W}})^2(1+c_{\texttt{W}})^4}}{8} &> \frac{2c_{\texttt{W}}^3-c_{\texttt{W}}^2-5c_{\texttt{W}}-2+\sqrt{K(c_{\texttt{W}})}}{9+c_{\texttt{W}}} \\ \Leftrightarrow (c_{\texttt{W}}-1)(1+c_{\texttt{W}})^2(9+c_{\texttt{W}})+(9+c_{\texttt{W}})\sqrt{64c_{\texttt{W}}^3+(1-c_{\texttt{W}})^2(1+c_{\texttt{W}})^4} &> 16c_{\texttt{W}}^3-8c_{\texttt{W}}^2-40c_{\texttt{W}}-16+8\sqrt{K(c_{\texttt{W}})}. \end{aligned}$$

Rearranging and simplifying gives the condition

$$H(c_{\texttt{W}}) \equiv \underbrace{c_{\texttt{W}}^4-6c_{\texttt{W}}^3+16c_{\texttt{W}}^2+30c_{\texttt{W}}-7}_{\mu(c_{\texttt{W}})}+\underbrace{(9+c_{\texttt{W}})\sqrt{64c_{\texttt{W}}^3+(1-c_{\texttt{W}})^2(1+c_{\texttt{W}})^4}}_{\gamma(c_{\texttt{W}})}-\underbrace{8\sqrt{K(c_{\texttt{W}})}}_{\zeta(c_{\texttt{W}})} > 0.$$

$H(c_{\texttt{W}})$ consists of three parts $\mu(c_{\texttt{W}})$, $\gamma(c_{\texttt{W}})$, and $\zeta(c_{\texttt{W}})$. Close inspection of $\mu(c_{\texttt{W}})$ reveals that $\mu(c_{\texttt{W}})$ is strictly increasing and greater than zero for all $c_{\texttt{W}} > 2$.[26] Consequently, it is a sufficient condition for

[25] Note that $F^*(1,1) = 1$; also, we know from Lemma 1 that $\frac{\partial F^*(1,c_{\texttt{W}})}{\partial c_{\texttt{W}}} > 0$. Therefore, $F^*(1,c_{\texttt{W}}) \geq 1$ for all $c_{\texttt{W}} \geq 1$.

[26] $\mu(2) = 85$, and $\mu'(c_{\texttt{W}}) = 4c_{\texttt{W}}^3-18c_{\texttt{W}}^2+32c_{\texttt{W}}+30$.

$H(c_{\mathtt{W}}) > 0$ to show that $\gamma(c_{\mathtt{W}}) > \zeta(c_{\mathtt{W}})$ in the range $c_{\mathtt{W}} > 2$:

$$\begin{aligned}(9 + c_{\mathtt{W}})\sqrt{64c_{\mathtt{W}}^3 + (1 - c_{\mathtt{W}})^2(1 + c_{\mathtt{W}})^4} &> 8\sqrt{K(c_{\mathtt{W}})} \\ \Leftrightarrow (9 + c_{\mathtt{W}})^2[64c_{\mathtt{W}}^3 + (1 - c_{\mathtt{W}})^2(1 + c_{\mathtt{W}})^4] &> 64K(c_{\mathtt{W}}) \\ \Leftrightarrow c_{\mathtt{W}}^8 + 20c_{\mathtt{W}}^7 - 140c_{\mathtt{W}}^6 + 460c_{\mathtt{W}}^5 + 2086c_{\mathtt{W}}^4 + 3436c_{\mathtt{W}}^3 - 2860c_{\mathtt{W}}^2 + 820c_{\mathtt{W}} + 1553 &> 0\end{aligned}$$

A sufficient condition for the above relation to hold is

$$\begin{aligned}20c_{\mathtt{W}}^7 - 140c_{\mathtt{W}}^6 + 460c_{\mathtt{W}}^5 + 2086c_{\mathtt{W}}^4 + 3436c_{\mathtt{W}}^3 - 2860c_{\mathtt{W}}^2 &> 0 \\ \Leftrightarrow c_{\mathtt{W}}^2[20c_{\mathtt{W}}^5 - 140c_{\mathtt{W}}^4 + 460c_{\mathtt{W}}^3 + 2086c_{\mathtt{W}}^2 - 2860] &> 0.\end{aligned}$$

Since $c_{\mathtt{W}} > 2$ by assumption, we are left with

$$20c_{\mathtt{W}}^5 - 140c_{\mathtt{W}}^4 + 460c_{\mathtt{W}}^3 + 2086c_{\mathtt{W}}^2 - 2860 > 0.$$

For $c_{\mathtt{W}} > 2$, it must hold that $2086c_{\mathtt{W}}^2 - 2860 > 0$, such that we can drop those two expressions without loss of generality. We get

$$\begin{aligned}20c_{\mathtt{W}}^5 - 140c_{\mathtt{W}}^4 + 460c_{\mathtt{W}}^3 &> 0 \\ \Leftrightarrow c_{\mathtt{W}}^3[20c_{\mathtt{W}}^2 - 140c_{\mathtt{W}} + 460] &> 0 \\ \Leftrightarrow c_{\mathtt{W}}^2 - 7c_{\mathtt{W}} + 23 &> 0,\end{aligned}$$

which is greater than zero for all $c_{\mathtt{W}} > 2$. This completes part (a) of the proof.

(b): Recall from part (b) of Proposition 2 that selection in setting SSWW is always dominated by selection in SSWW. Consequently, it is sufficient to show that $\mathcal{S}(\mathtt{I}) > \mathcal{S}(\mathtt{SWSW})$ to prove part (b) of Proposition 1, since $\mathcal{S}(\mathtt{II})$ is a composite measure of $\mathcal{S}(\mathtt{SWSW})$ and $\mathcal{S}(\mathtt{SSWW})$. We start with the relation which we want to prove:

$$\begin{aligned}\mathcal{S}(\mathtt{I}) &> \mathcal{S}(\mathtt{SWSW}) \\ \Leftrightarrow \min\left\{\frac{2c_{\mathtt{W}} - c_{\mathtt{S}}}{c_{\mathtt{S}} + c_{\mathtt{W}}}, 1\right\} &> \frac{(c_{\mathtt{S}} + c_{\mathtt{W}})F^*(1, c_{\mathtt{W}})^2 + 2c_{\mathtt{W}}F^*(1, c_{\mathtt{W}})}{(c_{\mathtt{S}} + c_{\mathtt{W}})[1 + F^*(1, c_{\mathtt{W}})]^2}\end{aligned}$$

Since $\mathcal{S}(\mathtt{I})$ is defined stepwise, we have to proceed in two steps. First, we start with the case where $1 < c_{\mathtt{W}} \leq 2$ such that $\mathcal{S}(\mathtt{I}) = \frac{2c_{\mathtt{W}} - c_{\mathtt{S}}}{c_{\mathtt{S}} + c_{\mathtt{W}}}$, before we consider $c_{\mathtt{W}} > 2$ and $\mathcal{S}(\mathtt{I}) = 1$.

(i) We assume without loss of generality that $c_{\mathtt{S}} = 1$ and consider the range $1 < c_{\mathtt{W}} \leq 2$. Then, we get

$$\begin{aligned}\mathcal{S}(\mathtt{I}) &> \mathcal{S}(\mathtt{SWSW}) \\ \Leftrightarrow \frac{2c_{\mathtt{W}} - 1}{1 + c_{\mathtt{W}}} &> \frac{(1 + c_{\mathtt{W}})F^*(1, c_{\mathtt{W}})^2 + 2c_{\mathtt{W}}F^*(1, c_{\mathtt{W}})}{(1 + c_{\mathtt{W}})[1 + F^*(1, c_{\mathtt{W}})]^2} \\ \Leftrightarrow (2c_{\mathtt{W}} - 1)[1 + F^*(1, c_{\mathtt{W}})]^2 &> (1 + c_{\mathtt{W}})F^*(1, c_{\mathtt{W}})^2 + 2c_{\mathtt{W}}F^*(1, c_{\mathtt{W}})\end{aligned}$$

Rearranging gives the condition

$$N(c_{\mathtt{W}}) = (c_{\mathtt{W}} - 2)[F^*(1, c_{\mathtt{W}})]^2 + 2(c_{\mathtt{W}} - 1)F^*(1, c_{\mathtt{W}}) + 2c_{\mathtt{W}} - 1 > 0$$

Recall from equation (B12) that the expression for $F^*(1, c_{\mathtt{W}})$ is faily complicated. To simplify the subsequent analysis, we make use of Lemma 3, where we established that $F^*(1, c_{\mathtt{W}}) < f_{\text{high}}(c_{\mathtt{W}})$ for all $c_{\mathtt{W}} > 1$. Since $\mathcal{S}(\mathtt{SWSW})$ is strictly increasing in $F^*(1, c_{\mathtt{W}})$, it is sufficient for the proof if we use the much simpler expression $f_{\text{high}}(c_{\mathtt{W}})$, as this tends to reduce the difference between the one-stage and the two-stage tournament in terms of selection:

$$\frac{\partial \mathcal{S}(\mathtt{SWSW})}{\partial F^*(1, c_{\mathtt{W}})} = \frac{2(c_{\mathtt{W}} + F^*(1, c_{\mathtt{W}})}{(1 + c_{\mathtt{W}})[1 + F^*(1, c_{\mathtt{W}})]^3} > 0.$$

This leaves us with

$$\begin{aligned}
\bar{N}(c_{\mathtt{W}}) &= (c_{\mathtt{W}} - 2)\left[\frac{c_{\mathtt{W}}^3 + 2c_{\mathtt{W}}^2 + c_{\mathtt{W}}}{4}\right]^2 + 2(c_{\mathtt{W}} - 1)\frac{c_{\mathtt{W}}^3 + 2c_{\mathtt{W}}^2 + c_{\mathtt{W}}}{4} + 2c_{\mathtt{W}} - 1 \\
&= \frac{(c_{\mathtt{W}} - 2)\left[c_{\mathtt{W}}^3 + 2c_{\mathtt{W}}^2 + c_{\mathtt{W}}\right]^2 + 8(c_{\mathtt{W}} - 1)(c_{\mathtt{W}}^3 + 2c_{\mathtt{W}}^2 + c_{\mathtt{W}}) + 32c_{\mathtt{W}} - 16}{16} \\
&= \frac{(c_{\mathtt{W}} - 1)[(c_{\mathtt{W}} - 1)(c_{\mathtt{W}} + 2)(c_{\mathtt{W}}(c_{\mathtt{W}} - 1)^2 + 4)c_{\mathtt{W}} + 16]}{16}.
\end{aligned}$$

Recall that we must show that $\bar{N}(c_{\mathtt{W}}) > 0$ holds for all $1 < c_{\mathtt{W}} \leq 2$. Note that $\bar{N}(1) = 0$ and $\bar{N}(2) = 12$. Therefore, the proof is complete if we can show that $\bar{N}(c_{\mathtt{W}})$ is strictly increasing in the relevant range. Since $c_{\mathtt{W}} > 1$, the factor $(c_{\mathtt{W}} - 1)$ in the expression of $\bar{N}(c_{\mathtt{W}})$ is always positive and can be disregarded in the subsequent analysis of the slope. Subsequently, we use the simpler expression

$$\hat{N}(c_{\mathtt{W}}) = \frac{(c_{\mathtt{W}} - 1)(c_{\mathtt{W}} + 2)(c_{\mathtt{W}}(c_{\mathtt{W}} - 1)^2 + 4)c_{\mathtt{W}} + 16}{16}.$$

When computing the first derivative of $\bar{N}(c_{\mathtt{W}})$ with respect to $c_{\mathtt{W}}$, we obtain

$$\frac{\partial \hat{N}(c_{\mathtt{W}})}{\partial c_{\mathtt{W}}} = \frac{6c_{\mathtt{W}}^5 + 15c_{\mathtt{W}}^4 + 4c_{\mathtt{W}}^3 + 3c_{\mathtt{W}}^2 + 4c_{\mathtt{W}} - 8}{16},$$

which is clearly positive for all values in the range $1 < c_{\mathtt{W}} \leq 2$. This proves the first part of the Proposition.

(ii) We assume without loss of generality that $c_{\mathtt{S}} = 1$. Then, a comparision of $\mathcal{S}(\mathtt{I})$ and $\mathcal{S}(\mathtt{SWSW})$ in the range $c_{\mathtt{W}} > 2$ gives

$$\begin{aligned}
\mathcal{S}(\mathtt{I}) &> \mathcal{S}(\mathtt{SWSW}) \\
\Leftrightarrow \quad 1 &> \frac{(1 + c_{\mathtt{W}})F^*(1, c_{\mathtt{W}})^2 + 2c_{\mathtt{W}}F^*(1, c_{\mathtt{W}})}{(1 + c_{\mathtt{W}})[1 + F^*(1, c_{\mathtt{W}})]^2} \\
\Leftrightarrow (1 + c_{\mathtt{W}})[1 + F^*(1, c_{\mathtt{W}})]^2 &> (1 + c_{\mathtt{W}})F^*(1, c_{\mathtt{W}})^2 + 2c_{\mathtt{W}}F^*(1, c_{\mathtt{W}})
\end{aligned}$$

Rearranging gives the condition

$$M(c_{\mathtt{W}}) = 2F^*(1, c_{\mathtt{W}}) + c_{\mathtt{W}} + 1 > 0.$$

As in the first part of this proof, we substitute $f_{\text{high}}(c_{\mathtt{W}})$ for $F^*(1, c_{\mathtt{W}})$, which gives

$$\begin{aligned}
\bar{M}(c_{\mathtt{W}}) &= 2\frac{c_{\mathtt{W}}^3 + 2c_{\mathtt{W}}^2 + c_{\mathtt{W}}}{4} + c_{\mathtt{W}} + 1 \\
&= \frac{c_{\mathtt{W}}^3 + 2c_{\mathtt{W}}^2 + 3c_{\mathtt{W}} + 2}{2}.
\end{aligned}$$

$\bar{M}(c_{\mathtt{W}})$ is clearly positive for all $c_{\mathtt{W}} > 2$, which proves the second part of the Proposition. □

Proposition 2: *When the cost of effort is strictly higher for weak than for strong agents ($c_W > c_S$),*

(a) aggregate effort is strictly higher in setting **SSWW** *than in setting* **SWSW**, *i.e.,*

$$\mathcal{E}(\texttt{SSWW}) > \mathcal{E}(\texttt{SWSW}) \text{ for all } c_{\texttt{W}} > c_{\texttt{S}}.$$

(b) the probability that a strong agent receives the promotion is strictly higher in setting **SWSW** *than in setting* **SSWW**, *i.e.,*

$$\mathcal{S}(\texttt{SSWW}) < \mathcal{S}(\texttt{SWSW}) \text{ for all } c_{\texttt{W}} > c_{\texttt{S}}.$$

Proof. We will separately prove parts (a) and (b) of Proposition 2. We start with part (a) below.

(a): To prove the relation $\mathcal{E}(\texttt{SSWW}) > \mathcal{E}(\texttt{SWSW})$ for all $c_{\texttt{W}} > c_{\texttt{S}}$, we assume without loss of generality that $c_{\texttt{W}} > c_{\texttt{S}} = 1$. In the proof, we will proceed in two steps. First, we derive a necessary and sufficient condition in terms of the function $F^*(1, c_{\texttt{W}})$ for the relation $\mathcal{E}(\texttt{SSWW}) > \mathcal{E}(\texttt{SWSW})$ to hold. Second, we prove that the equilibrium function $F^*(1, c_{\texttt{W}})$, which was derived in (B12), indeed satisfies this condition. We start with the relation which we want to prove:

$$\begin{aligned} \mathcal{E}(\texttt{SSWW}) &> \mathcal{E}(\texttt{SWSW}) \\ \Leftrightarrow \frac{c_{\texttt{W}}^3 + 2c_{\texttt{W}}(1+c_{\texttt{W}}) + 1}{2c_{\texttt{W}}(1+c_{\texttt{W}})^2} &> \frac{(1+c_{\texttt{W}})^2[1+[1+F^*(1,c_{\texttt{W}})]c_{\texttt{W}}F^*(1,c_{\texttt{W}})] + 4c_{\texttt{W}}[c_{\texttt{W}}^2 + (1+c_{\texttt{W}})F^*(1,c_{\texttt{W}})]}{2c_{\texttt{W}}(1+c_{\texttt{W}})^2[1+F^*(1,c_{\texttt{W}})]^2} \end{aligned}$$

Multiplying both sides by $2c_{\texttt{W}}(1+c_{\texttt{W}})^2[1+F^*(1,c_{\texttt{W}})]^2$ and rearranging gives

$$F^*(1,c_{\texttt{W}})^2 + \frac{c_{\texttt{W}}^3 - 2c_{\texttt{W}}^2 - c_{\texttt{W}} + 2}{c_{\texttt{W}}+1} F^*(1,c_{\texttt{W}}) - \frac{3c_{\texttt{W}}^3 - c_{\texttt{W}}^2}{c_{\texttt{W}}+1} > 0$$

Solving for $F^*(1, c_{\texttt{W}})$ gives us two conditions:

$$F^*(1,c_{\texttt{W}}) < \frac{-c_{\texttt{W}}^3 + 2c_{\texttt{W}}^2 + c_{\texttt{W}} - 2 - R(c_{\texttt{W}})}{2c_{\texttt{W}}+2} \quad \vee \quad F^*(1,c_{\texttt{W}}) > Z(c_{\texttt{W}}) \equiv \frac{-c_{\texttt{W}}^3 + 2c_{\texttt{W}}^2 + c_{\texttt{W}} - 2 + R(c_{\texttt{W}})}{2c_{\texttt{W}}+2},$$

where

$$R(c_{\texttt{W}}) = \sqrt{c_{\texttt{W}}^6 - 4c_{\texttt{W}}^5 + 14c_{\texttt{W}}^4 + 16c_{\texttt{W}}^3 - 11c_{\texttt{W}}^2 - 4c_{\texttt{W}} + 4}.$$

We do only have to consider the second relation, since the first one is below one for some values of $c_{\texttt{W}}$, while $F^*(1, c_{\texttt{W}}) \geq 1$ for all $c_{\texttt{W}} \geq 1$.[27] This completes the first part of the proof. We now have to prove that

$$F^*(1,c_{\texttt{W}}) > Z(c_{\texttt{W}}) \equiv \frac{-c_{\texttt{W}}^3 + 2c_{\texttt{W}}^2 + c_{\texttt{W}} - 2 + R(c_{\texttt{W}})}{2c_{\texttt{W}}+2} \tag{C1}$$

for all $c_{\texttt{W}} > 1$. From Lemmata 1 and 2 we know that $F^*(1, c_{\texttt{W}}) > f_{\text{low}}(c_{\texttt{W}})$. Consequently, a sufficient condition for (C1) is given by $f_{\text{low}}(c_{\texttt{W}}) > Z(c_{\texttt{W}})$. Rearranging this condition gives

$$c_{\texttt{W}}^3 + 2c_{\texttt{W}}^2 + c_{\texttt{W}} > R(c_{\texttt{W}}).$$

Squaring both sides leaves us with[28]

$$\begin{aligned} 2c_{\texttt{W}}^5 - 2c_{\texttt{W}}^4 - 3c_{\texttt{W}}^3 + 3c_{\texttt{W}}^2 + c_{\texttt{W}} - 1 &> 0 \\ \Leftrightarrow 2(c_{\texttt{W}}-1)^2(c_{\texttt{W}}+1)(c_{\texttt{W}} - \frac{1}{\sqrt{2}})(c_{\texttt{W}} + \frac{1}{\sqrt{2}}) &> 0. \end{aligned}$$

[27] Note that $F^*(1,1) = 1$; also, we know from Lemma 1 that $\frac{\partial F^*(1,c_{\texttt{W}})}{\partial c_{\texttt{W}}} > 0$. Therefore, $F^*(1, c_{\texttt{W}}) \geq 1$ for all $c_{\texttt{W}} \geq 1$.

[28] Note that squaring is without loss of generality here, since we are only interested in solutions for $c_{\texttt{W}} > 1$.

This relation is always satisfied if $c_{\texttt{W}} > 1$, which completes part (a) of this proof.

(b): In part (b) of this proof, we first derive a necessary and sufficient condition which assures that the relation $\mathcal{S}(\texttt{SSWW}) < \mathcal{S}(\texttt{SWSW})$ does hold in terms of the function $F^*(1, c_{\texttt{W}})$. Then, we prove that the equilibrium function $F^*(1, c_{\texttt{W}})$ satisfies this condition.

(i) As previously, we assume that $c_{\texttt{W}} > c_{\texttt{S}} = 1$ does hold without loss of generality. Consequently, we can use the expressions in equations (5) and (7) in what follows. We start with the relation which we want to prove:

$$
\begin{aligned}
\mathcal{S}(\texttt{SWSW}) &> \mathcal{S}(\texttt{SSWW}) \\
\Leftrightarrow (1 + c_{\texttt{W}})F^*(1, c_{\texttt{W}})^2 + 2c_{\texttt{W}}F^*(1, c_{\texttt{W}}) &> c_{\texttt{W}}F^*(1, c_{\texttt{W}})^2 + 2c_{\texttt{W}}F^*(1, c_{\texttt{W}}) + c_{\texttt{W}} \\
\Leftrightarrow F^*(1, c_{\texttt{W}})^2 &> c_{\texttt{W}} \\
\Leftrightarrow F^*(1, c_{\texttt{W}}) < -\sqrt{c_{\texttt{W}}} &\vee \quad F^*(1, c_{\texttt{W}}) > \sqrt{c_{\texttt{W}}}
\end{aligned}
$$

Note that it is sufficient to show that $F^*(1, c_{\texttt{W}}) > c_{\texttt{W}}$, since $c_{\texttt{W}} > \sqrt{c_{\texttt{W}}}$ for $c_{\texttt{W}} > 1$.

(ii) From Lemma 1, we know that $F^*(1, c_{\texttt{W}}) > f(c_{\texttt{W}})$. We will now prove that $f(c_{\texttt{W}}) > c_{\texttt{W}}$ for $c_{\texttt{W}} > 1$ to complete the proof. $f(c_{\texttt{W}}) > c_{\texttt{W}}$ implies that

$$
\frac{5c_{\texttt{W}}^3 + 2c_{\texttt{W}}^2 + c_{\texttt{W}}}{c_{\texttt{W}}^2 + 2c_{\texttt{W}} + 5} > c_{\texttt{W}}
$$

does hold. Rearranging gives

$$
\begin{aligned}
5c_{\texttt{W}}^3 + 2c_{\texttt{W}}^2 + c_{\texttt{W}} &> c_{\texttt{W}}^3 + 2c_{\texttt{W}}^2 + 5c_{\texttt{W}} \\
\Leftrightarrow c_{\texttt{W}}(c_{\texttt{W}}^2 - 1) &> 0 \\
\Leftrightarrow c_{\texttt{W}} > 1 &\vee \quad -1 < c_{\texttt{W}} < 0
\end{aligned}
$$

This proves the claim $\mathcal{S}(\texttt{SWSW}) > \mathcal{S}(\texttt{SSWW})$ for all $c_{\texttt{W}} > 1$. □