How the Value of Information Shapes the Value of Commitment
Or:
Why the Value of Commitment Does Not Vanish*

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Abstract

This paper challenges recent results on the fragility of the value of commitment. It introduces a specific notion of the 'value of information' for a later-moving player about the action choice of a previously-moving player, gives conditions under which this value is positive and shows that a positive value of information for the later-moving player is sufficient for a positive value of commitment for the previously-moving player. It then argues that the value of information for a later-moving player is unlikely to vanish in real-world applications, implying that the value of commitment for the previously-moving player does not vanish either.

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1 Introduction

The idea that the power to pre-commit confers a strategic advantage is central in economics and beyond. The basic insight dates back at least to Von Stackelberg (1934), who demonstrated that a firm can increase its profit in a quantity-setting duopoly by pre-committing to a larger output than would be optimal in a simultaneous interaction. Schelling (1960) generalized this point in his famous book *The Strategy of Conflict*, emphasizing in particular the value of commitment in military conflicts and in social interactions. The idea of strategic pre-commitment to affect the future has since then been applied in almost all fields of economics, in particular, in industrial organization, corporate finance, international trade and political economy. For instance, in industrial organisation it has been shown that in a competitive environment, in which firms choose their locations (or product characteristics) in the first stage and compete in prices in the second stage, firms have an incentive to strategically pre-commit to extreme locations (characteristics) in the first stage in order to soften price competition in the second stage (see D'Aspremont et al. 1979, or Economides 1986, for instance). Similarly, in a setting in which firms choose their cost-reduction efforts in the first stage and compete in quantities (prices) in the second stage, firms have an incentive to overinvest (underinvest) in cost reduction in order to appear as more (less) aggressive competitors in the market interaction (see Brander and Spencer 1983 and Fudenberg and Tirole 1984). Other prominent examples of strategic pre-commitment in an earlier stage to affect behavior at a later stage include countries choosing their trade subsidies strategically (as in Brander and Spencer 1985, or Eaton and Grossmann 1986), firms distorting the mix between equity and debt financing to improve their competitive position (as in Brander and Lewis 1986, Bolton and Scharfstein 1990 and Showalter 1995), and owners delegating decision rights strategically (see e.g. Fershtmann and Judd 1987, Fershtmann and Gneezy 2001, or Lambertini and Primavera 2001).

In those and in many other cases the logic of pre-commitment requires perfect observability at no cost of the action choices at earlier stages by the players moving at later stages. Indeed, as several contributions in the past twenty years have shown, the value of (pre-) commitment completely vanishes when the action choices at earlier stages are only imperfectly observable for the players moving at later stages, or if observability involves a cost. The first who has challenged the value of commitment in a formal framework was Bagwell (1995).[1] The author studies a two-player two-stage game, in which a first mover (FM, he) has the ability to commit to a choice in stage 1 and to communicate the choice to a second mover (SM, she) who then decides in stage 2. Bagwell shows that the value of commitment for the FM (defined in the usual way as the additional equilibrium payoff the FM receives in the "sequential-move" version of the game as compared to the "simultaneous-move" version of the game) is eliminated in pure-strategy equilibrium when there is even a slight amount of noise associated with the observation of the FM's choice by the SM. A second strand of literature on the 'fragility' of the

[1] Informally the idea that commitment requires observability was around much longer – indeed, already Schelling (1960) discusses the issue.

value of commitment considers two-player sequential-move games in which the SM faces a cost for observing the FM's action choice. The seminal article in this branch is Vardy (2004) who investigates a game in which the SM gets to observe what the FM has done if and only if she pays some (arbitrary small) amount of money (otherwise, she receives no information at all). He shows that, irrespective of the size of the amount, the value of commitment for the FM is lost completely in all pure-strategy equilibria.

Those results are challenging because in real life applications imperfect observability of action choices and information costs seem to be more the rule than the exception. Does this mean that in real life applications there is no value of commitment and therewith no strategic incentive to affect the future? The present article argues that this conclusion is premature, as the results about the fragility of the value of commitment are themselves fragile – in the sense that they are unlikely to hold in realistic settings. Specifically, we argue that for the value of commitment for previously-moving players to disappear, the value of information for later-moving players about the action choices of previously-moving players has to disappear. Since in realistic scenarios there is arguably always at least a small amount of uncertainty about the behaviour of a previously-moving player –for instance, because there is uncertainty about his rationality, his preferences or his beliefs– there is arguably always a positive willingness to pay for a later-moving player for observing the choices of a previously-moving player and therewith room for a positive value of commitment for the previously-moving player. This is the central message of the present paper.

An important step in formalizing this message is the introduction of an appropriate notion of the value of information for a later-moving player on the action-choice of a previously-moving player. Here, it is important to isolate the effect of information on the payoffs of the later-moving player by keeping 'everything else' –and in particular, the previously-moving player's information about whether the later-moving player is informed or not– constant. So, a key step in our analysis will be the introduction of a notion of the value of information for a later-moving player that has this property.

To keep things simple we follow much of the rest of the literature in concentrating on the simplest class of games where the central point of the present paper can sensibly be made, on the class of two-player two-stage games, in which a FM has the ability to commit to a choice in stage 1 which is observed (possibly with some noise or only at a cost) by a SM before she decides in stage 2. To isolate the effect of information about the action choice of the FM on the payoff of the SM in this class of games we construct a meta game in which nature first chooses whether the SM will be informed (possibly with some noise) about the FM's action choice or not. The SM is then privately informed about Nature's choice before she makes her own move. This meta game allows to analyse the value of additional information to the SM about the action choice of the FM while keeping the level of information of the FM constant: The value of information is simply the difference in the expected payoffs of the two incarnations of the SM in the same equilibrium of the same meta game. In the sequel we refer to this specific comparison when we use the term 'value of information for the SM' and we will abbreviate it as the

'*VOI for the SM*'.

We first show that in a pure-strategy equilibrium of the complete-information version of the meta game the SM's equilibrium payoff is independent of whether she is informed about the FM's action choice or not. Thus, the VOI for the SM is zero in this case. Only in cases where either (i) the SM has incomplete information about the preferences of the FM, or (ii) the FM plays a mixed strategy can the VOI for the SM be positive. The intuition for this result is quite simple: In an equilibrium the SM has correct beliefs about the strategy choice of the FM. In a pure-strategy equilibrium of the complete-information version of the game the FM's strategy choice is an action choice implying that in equilibrium the SM's belief is a point belief. Thus, the SM does not learn anything from observing (a possibly imperfect signal of) the actual move of the FM. So, the VOI for her is simply zero in that case. Only in cases where the FM either possesses some private information, or plays a non-degenerate mixed-strategy can the VOI for the SM be positive because only then observation of the FM's actual move allows for updating of beliefs.

While this intuition is entirely straightforward within the equilibrium logic of game theory, it also suggests that it is unlikely to extend to reality. The reason is that the logic of the argument requires that agents do not make a distinction between equilibrium beliefs and observed facts. Specifically, the result that the VOI for the SM is zero in any pure strategy equilibrium of a complete information game requires that the SM does not have the slightest uncertainty about the behaviour of the FM and is therefore not interested in observing his choice. Absolute certainty about the behaviour of another person without actually observing it is arguably a harsh assumption which is likely to be violated in most real-world application. However, when there is even the slightest degree of uncertainty about the behaviour of the FM and if the behaviour of the FM has an impact on the optimal choice of the SM then there is necessarily a positive VOI for the SM (in our sense). And, whenever there is a positive VOI for the SM (in our sense) there is room for a positive value of commitment for the FM (in the usual sense). This conclusion emerges from our discussion in Section 4, where we explain the connection between our results on the VOI for the SM and existing results on the value of commitment for the FM (abbreviated as the *VOC for the FM*).

Literally speaking the paper does not contain any new results regarding the value of commitment. Our main innovation in this respect is rather the different perspective we take on the issue. Our main point is that the results challenging the value of commitment all require that later-moving players do not have the slightest uncertainty about the behaviour of previously-moving players and are therefore not interested in observing that behaviour – an assumption that is arguably violated in most real-world applications. We consider this point as important, simply because strategic incentives to affect the future are so central in applied game theory. While there is quite some literature exploring the generality of the results on the fragility of the value of commitment (see, for instance, van Damme and Hurkens 1997, Güth et al. 1998, Maggi 1999, Oechssler and Schlag 2000, or Bhaskar 2009), this literature remains technical and the results obtained are often model-specific, or they rely on specific equilibrium selection criteria and therewith on ad hoc restrictions on beliefs off the equilibrium path. Our point is more general and it holds independently of whether players behave in accordance with an equilibrium or not: We argue, that for the value of commitment for a previously-moving player to disappear, the value of information for the later-moving player on the action choice of the previously-moving player has to disappear, which is unlikely to be the case in any application.

Besides its relation to the literature on the value of commitment in strategic games, the present paper is also related to the literature on the value of information in games. First, there is a literature dealing with the value of information in zero-sum games. Important entries in that strand of literature are Gossner and Mertens (2001), Lehrer and Rosenberg (2006 and 2010) and De Meyer et al. (2010), and an important finding is that the value of information in zero-sum games is always positive for the player who gets it; that is, in this class of games the equilibrium payoff of a player cannot decrease as a result of receiving more information. That this finding does not extend to non-constant-sum games is well known and has been highlighted by several authors. Bassan et al. (1997), for instance, show by examples that in a two-player game "[...] almost every situation is conceivable: Information can be beneficial for all the players, or only for the one who receives it, or, less intuitively, just for the one who does not receive it, or it could be bad for both." An important difference to this strand of literature is our focus on the value of *information about the action choice* of a FM, while in this literature the focus is on the value of *information about the preferences* (payoffs) of the players in the game.[2] More importantly, in those papers the 'value of information' is calculated as the difference in the equilibrium payoffs in different games while our notion of the VOI for the SM is motivated by Neyman (1991) and involves the comparison of the equilibrium payoffs of different types of the same player within the same (meta) game.

The remainder of this article is organized as follows: Section 2 presents a simple motivating example which illustrates the main ideas. In Section 3 we look at a class of abstract games and derive the main results on the VOI for the SM. In Section 4 we discuss how our results on the VOI for the SM relate to recent results on the VOC for the FM. Section 5 concludes.

2 A Stackelberg-Cournot Example

To motivate our notion of the value of information consider the textbook example of two symmetric firms competing in quantities of some homogeneous good. For this context it is quite clear that a firm who is able to commit to an action first and to perfectly communicate its choice to the other firm can realize a positive 'value of commitment'; and it is also clear that more information harms the firm who possesses it when the

[2]An exception is a paper by Kamien et al. (1990) who consider a situation in which an agent possesses information on payoffs or action choices relevant to the players in a game in which he is not a participant. By contrast, our focus is on the value of information for an active player in a game.

rival is aware of the information she holds: In the simultaneous-move (or "Cournot") version of the game, where no firm is able to commit and no firm is able to observe the market decision of its rival, both firms choose the same quantity and both get the same payoff in the unique Nash (and Subgame Perfect) Equilibrium; in the sequential-move (or "Stackelberg") version of the game, where the Stackelberg leader (the FM) is able to pre-commit to an output quantity and to perfectly communicate his choice to the Stackelberg follower (SM), the FM produces a higher quantity and earns a higher profit while the SM produces a lower quantity and earns a lower profit in the unique Subgame Perfect Equilibrium compared to the corresponding equilibrium values in the simultaneous-move version of the game. So, the FM is better off in the version of the game where he has the ability to move first implying a positive value of commitment; and the SM is worse off in the version of the game where she has more information (because she can observe the action choice of the FM) implying a negative 'value of information' to the player who holds it in this game.

The notion of the value of information used in the previous paragraph relies on the comparison of the equilibrium payoffs of one and the same player in two different games: a simultaneous-move game in which no player is able to observe the action choice of the rival and a sequential-move game, where the FM is able to pre-commit to an action choice which is perfectly observed by the SM before she has to decide. However, as Neyman (1991) has pointed out, by comparing two games – and assuming that the game to be played is common knowledge among players – we are not changing the information of a single player (keeping the information of rivals constant), but rather change the information of all players involved. So, what hurts the SM in the Stackelberg-Cournot example is not the fact that she knows more but rather the fact that the rival knows that she knows more.

To hold the FM's information constant we embed the simultaneous-move (or Cournot) version of the game and its sequential-move counterpart in the same Bayesian meta game. In the meta game the SM has two incarnations, one in which she can (possibly with some noise) observe the action choice of the FM, and one in which she cannot. The SM knows her incarnation, while the FM knows only the ex ante probability of each incarnation.[3] This meta game allows to analyse the value of additional information to the SM about the action choice of the FM while keeping the level of information of the FM constant: The value of information is simply the difference in the expected payoffs of the two incarnations of the SM in the same Perfect Bayesian Equilibrium of the same meta game. Below we illustrate the construction for a simplified version of the quantity-competition game, one where both players (firms) have only two available actions (quantities).

[3]To motivate this information structure within our quantity-competition example think of a market where the first moving firm has incomplete information about the ability of the rival to observe its quantity choice.

2.1 The Game

Consider a two-players, two-stages, two-actions game with the FM as the leader and the SM as the follower. The action set of the FM is $A_1 = \{C, L\}$ and that of the SM is $A_2 = \{C, F\}$, where the label L is mnemonic for "Leader" action, F is mnemonic for "Follower" action, and C is mnemonic for "Cournot" action. The FM chooses his action first. The SM either observes the FM's action choice, or she does not, and then chooses her own action. Payoffs are as given in the bi-matrix in Figure 1. If the SM does not observe the FM's action choice and if this fact is common knowledge (as in the Cournot version of the motivating example), then the SM's strategy space corresponds to her action space and the game has a unique Nash Equilibrium (which, of course, is also the unique Subgame Perfect Equilibrium) at $s^* = (s_1^*, s_2^*) = (C, C)$. By contrast, if the SM can perfectly observe the FM's action choice and if this fact is common knowledge (as in the Stackelberg version of the motivating example), then the SM's strategy space is $\{CC, CF, FC, FF\}$ and the game has a unique Subgame Perfect Equilibrium at $s^* = (s_1^*, s_2^*) = (L, CF)$.[4] The associated equilibrium outcomes are (C, C) in the simultaneous- and (L, F) in the sequential-move version of the game, and the equilibrium payoffs are such that the FM receives (with 7) a higher and the SM (with 2) a lower payoff in the sequential- than in the simultaneous-move version of the game (where both get 6).

		Player 2	
		C	F
Player 1	C	$6, 6$	$10, 4$
	L	$4, 1$	$7, 2$

Figure 1: Material Payoffs

To assess the VOI for the SM (as defined in the introduction) we now embed those two games in one meta game, where nature decides which of the two constituent games is played. The resulting Bayesian game is shown in Figure 2. In this game nature first decides whether the SM will be able to observe the FM's action choice (in this part of the game tree the SM has type t_{2i}), or not (in this case the SM has type t_{2n}). The ex ante probability of the former event is $p \in (0, 1)$, while the latter event occurs with probability $(1 - p)$. The FM does not observe nature's move but rather has to decide knowing only the ex ante probabilities. The SM observes nature's move. When she is of type t_{2i} she also (perfectly) observes the FM's action choice, while in the incarnation t_{2n} she does not. Then the SM decides and the game ends.

Since the game tree is common knowledge, the level of information is constant for the FM, and type t_{2i} of the SM has more information (on the action choice of the FM) than type t_{2n}. The superior information of t_{2i} of the SM means that this type has a finer information partition than t_{2n} and therefore more available strategies. Specifically, the

[4] Here and below the first entry in the strategy of the informed SM corresponds to $s_2^*(C)$ and the second entry corresponds to $s_2^*(L)$.

set of pure strategies of type t_{2i} of the SM is $S_{2i} = \{CC, CF, FC, FF\}$, with generic element s_{2i}, whereas the set of pure strategies of type t_{2n} is $S_{2n} = A_2 = \{C, F\}$, with generic element s_{2n}. The available pure strategies of player 1 correspond to the available actions: $S_1 = A_1 = \{C, L\}$, with generic element s_1.

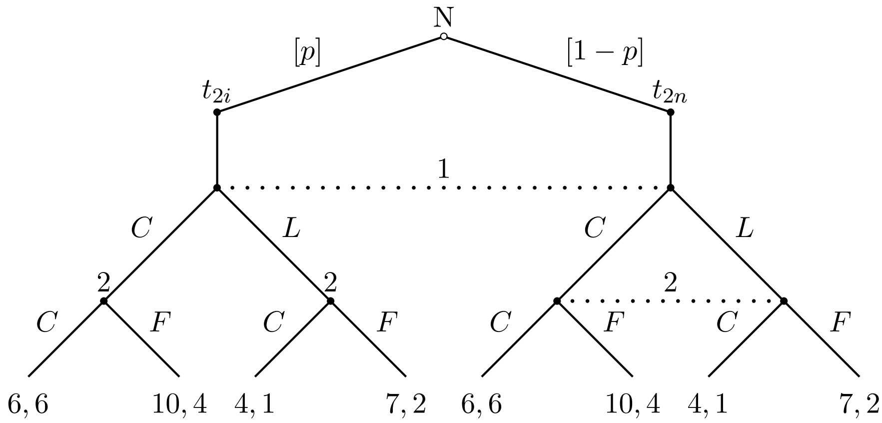

Figure 2: Meta Game

2.2 Equilibria and the Value of Information

The solution concept used to solve the meta game is that of a Perfect Bayesian Equilibrium (PBE).[5] We first look for **pure-strategy PBEs** and get the following result:

Result 1 (Pure-Strategy Equilibria in Stackelberg-Cournot Example). *For each value of p the Stackelberg-Cournot meta game in Figure 2 has at most one pure-strategy PBE:*

- *For $p \leq \frac{2}{3}$ the game has exactly one pure-strategy PBE, equilibrium strategies are $s^* = (s_1^*, (s_{2i}^*, s_{2n}^*)) = (C, (CF, C))$ and equilibrium payoffs are $u_1^* = 6$, $u_{2i}^* = 6$ and $u_{2n}^* = 6$.*
- *For $p \in (\frac{2}{3}, \frac{3}{4})$ the game has no pure-strategy PBE.*
- *For $p \geq \frac{3}{4}$ the game has exactly one pure-strategy PBE, equilibrium strategies are $s^* = (s_1^*, (s_{2i}^*, s_{2n}^*)) = (L, (CF, F))$ and equilibrium payoffs are $u_1^* = 7$, $u_{2i}^* = 2$ and $u_{2n}^* = 2$.*

Proof: In any PBE beliefs are consistent with equilibrium strategies, which have to be optimal given beliefs and given the strategy of the opponent. For type t_{2i} beliefs are

[5] In the class of games considered here there is no difference between PBE and Sequential Equilibrium.

trivial (since she perfectly observes the FM's action choice; so her info sets are singletons) and the only strategy consistent with equilibrium is CF. For type t_{2n} the beliefs in a pure-strategy equilibrium are point beliefs and she has to react optimally to them. Thus, she has to react with C to the expectation that the FM plays C and with F to the expectation that the FM plays L. It follows that only the constellations $(C,(CF,C))$ and $(L,(CF,F))$ qualify as equilibrium candidates. It is then straightforward to derive the bounds in the result. □

An immediate implication of Result 1 is:

Implication 1 (VOI in Pure-Strategy Equilibrium of Stackelberg-Cournot Example). *In a pure-strategy PBE of the Stackelberg-Cournot meta game in Figure 2 the VOI for the SM is zero.*

The intuition for this result is quite simple: In equilibrium both types of the SM have correct beliefs about the strategy choice of the FM. In a pure-strategy PBE the FM's strategy choice is an action choice and type t_{2n}'s belief is a point belief. Given the point belief type t_{2n} has exactly the same information and behaves in exactly the same way as type t_{2i} along the equilibrium path. Since both types have also the same payoff function it follows that both obtain the same equilibrium payoffs, implying that the VOI for the SM (about the action choice of the FM) is zero.

Next we search for **mixed-strategy PBEs.** First note that neither the simultaneous-move nor the sequential-move game admits mixed-strategy equilibria. However, there are such equilibria in the Stackelberg-Cournot meta game in Figure 2. In the sequel we use the following notation for mixed strategies:

$$\begin{aligned}
\lambda_1 &\equiv \Pr(a_1 = C);\\
\lambda_{2iC} &\equiv \Pr(a_2 = C \mid t_{2i}, a_1 = C);\\
\lambda_{2iL} &\equiv \Pr(a_2 = C \mid t_{2i}, a_1 = L);\\
\lambda_{2n} &\equiv \Pr(a_2 = C \mid t_{2n}).
\end{aligned}$$

First note that in any non-degenerate mixed-strategy PBE we must have $\lambda = (\lambda_1, \lambda_{2iC}, \lambda_{2iL}, \lambda_{2n}) = (x, 1, 0, y)$, with $x \in (0,1)$. That is, in a mixed-strategy equilibrium the FM necessarily plays a strictly mixed strategy, while type t_{2i} of the SM necessarily plays the pure strategy CF. Why? Because if the FM plays a pure strategy we necessarily end up in one of the pure-strategy equilibria discussed in the previous paragraph (by the arguments given there) and because type t_{2i} of the SM has a unique best reply at each of her trivial information sets. An immediate consequence of this observation is, that only the mutually exclusive constellations

(i) $\lambda = (x,1,0,0)$ with $x \in (0,1)$,
(ii) $\lambda = (x,1,0,y)$ with $x \in (0,1)$ and $y \in (0,1)$, and
(iii) $\lambda = (x,1,0,1)$ with $x \in (0,1)$

qualify as candidates for strategy combinations in a non-degenerate mixed-strategy PBE. Closer inspection reveals that PBEs of type (i) and type (iii) only exist at a single point, while PBEs of type (ii) exist for a non-trivial range of values for p.[6] Specifically we have:

Result 2 (Mixed-Strategy Equilibria in Stackelberg-Cournot Example). *The Stackelberg-Cournot meta game in Figure 2 generically has at most one non-degenerate mixed-strategy PBE:*

- *For $p < \frac{2}{3}$ and for $p > \frac{3}{4}$ the game has no non-degenerate mixed-strategy PBE.*
- *For $p \in (\frac{2}{3}, \frac{3}{4})$ the game has exactly one mixed-strategy PBE, equilibrium strategies are $\lambda^* = (\lambda_1^*, \lambda_{2iC}^*, \lambda_{2iL}^*, \lambda_{2n}^*) = (\frac{1}{3}, 1, 0, y)$ with $y = \frac{3-4p}{1-p}$ and equilibrium payoffs are $u_1^* = 12p - 2$, $u_{2i}^* = \frac{10}{3}$ and $u_{2n}^* = \frac{8}{3}$.*

Proof: Straightforward and therefore omitted. □

An immediate implication of Result 2 is:

Implication 2 (VOI in Mixed-Strategy Equilibrium of Stackelberg-Cournot Example). *In a mixed-strategy PBE of the Stackelberg-Cournot meta game in Figure 2 the VOI for the SM is strictly positive.*

The intuition for this result is quite simple: In the mixed-strategy equilibria of Result 2 the FM randomizes between his two actions. Thus, by observing the FM's actual move, type t_{2i} learns the realization of the mixed-strategy of the FM. By contrast, type t_{2n} only has correct expectations on the probability distribution over moves. Thus, type t_{2i} of the SM can tailor her best reply to the actual move of the FM while type t_{2n} has to content herself with a reply that fits well only on average. This fact reflects itself in the equilibrium payoffs. While type t_{2i} earns $\frac{10}{3}$, on average, type t_{2n} has to settle for $\frac{8}{3}$, on average.

Summing up we conclude that our Stackelberg-Cournot example generically has just one equilibrium. For low (high) values of p it is in pure strategies, the equilibrium outcome corresponds to that of the simultaneous-move (sequential-move) game and the VOI for the SM is zero. For intermediate values of p the unique PBE is in mixed strategies and the VOI for the SM is strictly positive.[7]

3 A More General Framework

We now extend the results above in various directions. First, instead of looking at a parametric example we look at an arbitrary finite two-player two-stage game. Second, instead of assuming that the SM either perfectly observes the FM's action or observes

[6] Type (i) equilibria only exist at $p = 3/4$ and type (iii) equilibria only exist at $p = 2/3$.

[7] For completeness we mention that at the borders between the ranges we have both, a pure-strategy PBE as described in Result 1 and a mixed-strategy PBE as described in Result 2. Furthermore, everything in between is also part of a PBE. Specifically, at $p = 2/3$ any strategy combination $\lambda^* = (x, 1, 0, 1)$ with $x \geq 1/3$ is consistent with PBE. Similarly, at $p = 3/4$ any strategy combination $\lambda^* = (x, 1, 0, 1)$ with $x \leq 1/3$ is consistent with PBE.

nothing at all, we now allow for imperfect observability of the FM's action choice in the case where the SM gets more information. Specifically, we consider two cases, one in which the SM can perfectly observe the FM's action choice and one in which she only observes an imperfect signal. We refer to the former case as the "*perfect observability case*" and to the latter as the "*imperfect observability case*". Third, instead of assuming that the SM has perfect information about the preferences (payoffs) of the FM we now allow for incomplete information on the side of the SM on the payoffs of the FM for each given profile of actions. Specifically, we consider two cases, one in which the SM is perfectly informed about the preferences of the FM and one where she is not. We refer to the former as the "*complete information case*" and to the latter as the "*incomplete information case*".

3.1 Complete Information with Perfect and Imperfect Observability

We start by considering a dynamic Bayesian game Γ^c in which

- there are two players, indexed by $i \in N = \{1, 2\}$;
- player 1 (FM, he) moves first and player 2 (SM, she) moves second;
- player i chooses an action a_i from some finite set of available actions A_i;
- player 1 has a single incarnation ($t_1 \in T_1 = \{t_1\}$), while player 2 has two ($t_2 \in T_2 = \{t_{2i}, t_{2n}\}$);
- player 2 knows her incarnation, while player 1 does not; he assigns probability $p(t_2)$ to the event that player 2 is of type $t_2 \in T_2$ with $\sum_{t_2} p(t_2) = 1$;
- type t_{2i} of player 2 observes a (possibly imperfect) signal $\phi \in \Phi = A_1$ about the action choice of player 1; defining $f(\phi|a_1)$ as the probability of signal ϕ given that the FM's actual choice is a_1 we consider two polar cases: while in the "*perfect observability case*" $f(\phi|a_1)$ is 1 for $\phi = a_1$ and 0 otherwise, in the "*imperfect observability case*" $f(\phi|a_1) > 0$ for all $\phi \in \Phi$ and $a_1 \in A_1$ just as in Bagwell (1995);
- player i's preferences are represented by a von Neumann-Morgenstern utility function $u_i(a; t)$, where $a = (a_1, a_2)$ and $t = (t_1, t_2)$; since t_1 has a single realization we suppress it in the sequel and represent each t by the associated t_2;
- utility functions $u_i(a; t)$ satisfy (i) $u_i(a; t_{2i}) = u_i(a; t_{2n})$ for all $i \in N = \{1, 2\}$ and all $a \in A = A_1 \times A_2$, and (ii) if $(a_1, a_2) \neq (a_1', a_2')$ then $u_i(a_1, a_2; t) \neq u_i(a_1', a_2'; t)$ for all $i \in N = \{1, 2\}$ and all $t \in T$; that is, (i) payoffs depend only on the actions chosen by the players and not on the type of the SM, and (ii) for each point expectation regarding the action choice of the opponent each player has a unique best reply.

The time and information structure in the game is (except for the new signal structure) as in the example studied in the previous section: At the beginning nature chooses t_{2i} with probability $p(t_{2i})$, and t_{2n} with probability $p(t_{2n})$. The FM does not observe nature's move, he only knows the ex ante probabilities of the events $t_2 = t_{2i}$ and $t_2 = t_{2n}$. On the basis of this information he chooses an a_1 from A_1. The SM observes nature's move. When she is of type t_{2i} she observes the signal ϕ about the FM's action choice, while in the incarnation t_{2n} she does not. Then the SM chooses an a_2 from A_2 and the game ends.

Strategies: The time and information structure implies that the FM's set of pure strategies corresponds to his action set $S_1 = A_1$. The set of pure strategies of the SM depends on her type: For type t_{2i} a pure strategy is a function that assigns an element of A_2 to each signal value ϕ; we denote the set of such functions by S_{2i}. For type t_{2n} the set of pure strategies coincides with the action set of the SM; thus, $S_{2n} = A_2$. For mixed strategies we use the following notation: For the FM a mixed-strategy $\lambda_1(.)$ is an element of $\Delta(A_1)$, where $\lambda_1(a_1) \geq 0$ denotes the probability that the FM chooses action $a_1 \in A_1$, with $\sum_{a_1} \lambda_1(a_1) = 1$. A mixed-strategy of type t_{2i} of the SM is a map that assigns a probability distribution on A_2 (that is, an element of $\Delta(A_2)$) to each element of $\Phi = A_1$. We let $\lambda_{2i}(a_2|\phi) \geq 0$ denote the probability that the SM chooses a_2 in response to the signal ϕ, with $\sum_{a_2} \lambda_{2i}(a_2|\phi) = 1$ for each $\phi \in \Phi$.[8] For type t_{2n} a mixed-strategy $\lambda_{2n}(.)$ is an element of $\Delta(A_2)$, with the usual properties.

Equilibrium: A PBE of the game consists of a profile of strategies $\lambda^* = (\lambda_1^*(\cdot), ((\lambda_{2i}^*(\cdot|\phi)_{\phi\in\Phi}, \lambda_{2n}^*(\cdot))$ and a system of beliefs μ^*. For the informed type of the SM, t_{2i}, beliefs are determined by the FM's equilibrium strategy and by the signal structure. In the perfect observability case the information sets of t_{2i} are singletons and beliefs therefore trivial. For the imperfect observability case each signal value leads to an information set of t_{2i} which contains a node for each possible action a_1 of the FM. In the sequel we let $\mu_{2i}(a_1|\phi)$ denote the probability type t_{2i} of the SM assigns to the event that the FM has chosen a_1 when she observes the signal ϕ, with $\sum_{a_1} \mu_{2i}(a_1|\phi) = 1$ for each signal $\phi \in \Phi$. The belief of the uninformed type of the SM is a probability distribution over the actions of the FM and we denote it by $\mu_{2n}(.)$, with the interpretation that $\mu_{2n}(a_1)$ is the probability type t_{2n} of the SM assigns to the event that the FM has chosen a_1. The requirements for a PBE now are (i) that $\lambda_1^*(\cdot)$ maximizes the FM's expected utility given $((\lambda_{2i}^*(\cdot|\phi)_{\phi\in\Phi}, \lambda_{2n}^*(\cdot))$ and $f(\cdot|\cdot)$, and given his belief that type t_{2i} is realized with probability $p(t_{2i})$ and t_{2n} with probability $p(t_{2n})$; (ii) that $\lambda_{2i}^*(\cdot|\phi)$ maximizes the expected utility of type t_{2i} for each ϕ given $\mu_{2i}^*(.|.)$; (iii) that $\lambda_{2n}^*(\cdot)$ maximizes the expected utility of type t_{2n} given $\mu_{2n}^*(.)$; (iv) that $\mu_{2i}^*(\cdot|\cdot)$ is consistent with $\lambda_1^*(\cdot)$ and $f(\cdot|\cdot)$ in the sense that $\mu_{2i}^*(a_1|\phi) = \lambda_1(a_1)f(\phi|a_1)/\sum_{a_1'} \lambda_1(a_1')f(\phi|a_1')$ for all information sets on the equilibrium path; and (v) that $\mu_{2n}^*(.)$ is consistent with $\lambda_1^*(\cdot)$ in the sense that $\mu_{2n}^*(a_1) = \lambda_1^*(a_1)$ for each $a_1 \in A_1$.

We are now in the position to prove the following result regarding the VOI for the SM:

[8]Technically this is a behavior strategy, of course. We will not distinguish between mixed strategies and behavior strategies in the sequel.

Proposition 1. ***(VOI in Pure-Strategy Equilibrium of Complete Info Game)*** *In a pure-strategy PBE of* Γ^c *the VOI for the SM is zero.*

Proof: In a pure-strategy PBE of game Γ^c we have $\lambda_1^*(a_1^+) = 1$ for some $a_1^+ \in A_1$. Thus, $\mu_{2i}^*(a_1^+|\phi) = 1$ for all signal values $\phi \in \Phi$ on the equilibrium path by requirement (iv) in the definition of a PBE. Furthermore, from requirement (v) we get that $\mu_{2n}^*(a_1^+) = 1$. By requirement (ii) type t_{2i} must choose a best reply for every signal; since she assigns probability 1 to the event that the FM has chosen a_1^+ at any info set on the equilibrium path, she chooses a best reply to a_1^+ at any such info set. By requirement (iii) type t_{2n} must also choose a best reply to $\mu_{2n}^*(.)$ and since $\mu_{2n}^*(a_1^+) = 1$ to a_1^+. Thus, both types behave exactly the same way along the equilibrium path which –together with the assumption that both types have the same payoff functions– implies that both get the same equilibrium payoff. □

Proposition 1 is an immediate consequence of the consistency requirement inherent in any equilibrium concept. With pure strategies type t_{2i} ignores the signal she observes because she already knows what the FM does. The belief of type t_{2n} is also a point belief leading to the same assessment about the action chosen by the FM. Sequential rationality then requires that t_{2i} and t_{2n} choose the same action along the equilibrium path. Thus, with pure strategies getting information about the FM's actual choice does not hurt but it is of no avail either.

Whereas the result for pure-strategy equilibria does not depend on the signal precision, we distinguish between the perfect and the imperfect observability case for mixed-strategy equilibria. In the former case the informed incarnation of the SM necessarily has a unique best reply to each action choice of the FM. Therefore, we can state the following result:

Proposition 2a. ***(VOI in Mixed-Strategy Equilibrium of Complete Info Game with Perfect Observability)*** *In a mixed-strategy PBE of the perfect observability version of* Γ^c *the VOI for the SM is strictly positive iff the support of the mixed-strategy of the FM contains at least two actions for which the best replies of the SM differ.*

Proof: Straightforward and therefore omitted. □

The positive VOI for the SM here stems from the fact that the less informed type t_{2n} can only choose an average best response to the mixed-strategy of the FM whereas type t_{2i} is able to react to the actual action choice of the FM. When the signal is noisy we need a somewhat stronger requirement since the informed incarnation of the SM might have multiple best replies in that case:

Proposition 2b. ***(VOI in Mixed-Strategy Equilibrium of Complete Info Game with Imperfect Observability)*** *In a mixed-strategy PBE of* Γ^c *the VOI for the SM is strictly positive iff there is at least one* $\phi \in \Phi$ *for which the informed SM strictly prefers an* a_2 *that differs from the one chosen by the uninformed SM.*

Proof: Straightforward and therefore omitted. □

3.2 Incomplete Information with Perfect and Imperfect Observability

Now we extend the game to the case where the FM has more than one incarnation: $t_{1j} \in T_1$ with $T_1 = \{t_{11}, t_{12}..., t_{1J}\}$ and $J \geqslant 2$. Type $t_{1j} \in T_1$ is realized with probability $p(t_{1j})$ with $\sum_j p(t_{1j}) = 1$. Each player observes the realization of the own but not the realization of the rival's type – for the latter a player only knows the ex-ante probabilities. The utility functions are again denoted by $u_i(a, t)$ for all i, where $a \in A = A_1 \times A_2$ and $t \in T = T_1 \times T_2$. The type of the FM has only an impact on the own payoff but not on the payoff of the SM.[9] As in the complete information game the type of the SM has neither an impact on the FM's nor an impact on the SM's payoff. To keep the notation simple we assume that for each $a_1 \in A_1$ the probability $f(\phi|a_1)$ does not depend on the type of the FM, meaning that the signal precision is the same regardless of which type player 1 is. The rest is as in the complete information case. We denote this version of the game by Γ^i.

Strategies: A pure strategy of the FM now is a function that assigns an element of A_1 to each type in T_1 and a mixed strategy for him is a map that assigns a probability distribution on A_1 (that is, an element of $\Delta(A_1)$) to each element of T_1. We let $\lambda_{1j}(a_1) \geq 0$ denote the probability type t_{1j} of player 1 assigns to action $a_1 \in A_1$. The notation and meaning of pure and mixed strategies for the SM is the same as in the complete-information case.

Equilibrium: A PBE of the game consists of a profile of strategies $\lambda^* = ((\lambda^*_{1j}(\cdot))_{j\in\{1,..,J\}}, (\lambda^*_{2i}(\cdot|\cdot))_{\phi\in\Phi}, \lambda^*_{2n}(\cdot))$ and a system of beliefs μ^*. The beliefs of the FM about the type of the SM are trivial – for each $t_{1j} \in T_1$ they correspond to the prior probabilities. The belief of the informed incarnation of the SM, t_{2i}, is now a map that assigns a probability distribution over $(a_1, t_{1j}) \in A_1 \times T_1$ for each signal. We let $\mu_{2i}(a_1, t_1|\phi)$ denote the probability type t_{2i} of the SM assigns to the event that type t_{1j} of the FM has chosen action a_1 when the signal is ϕ. In equilibrium it is determined by the probability $p(t_{1j})$ that the FM is of type $t_{1j} \in T_1$, the FM's equilibrium strategy $(\lambda^*_{1j}(\cdot))_{j\in\{1,..,J\}}$ and the signaling technology $f(\cdot|\cdot)$ by applying Bayes' rule. The uninformed incarnation of the SM does not get a signal, so her belief is simply a probability distribution over $(a_1, t_{1j}) \in A_1 \times T_1$ and it is denoted by $\mu_{2n}(a_1, t_{1j})$. In equilibrium it is derived from the FM's equilibrium strategy and the ex ante probabilities of the different types. The requirements for a PBE are the same as before, except for the fact that they have to be extended to all types of player 1. Each type of each player has to maximize its expected utility given beliefs. For this incomplete information version of the game we get the following result:

Proposition 3. ***(VOI in Pure-Strategy Equilibrium of Incomplete Info Game)*** *In a pure-strategy equilibrium of Γ^i the VOI for the SM is zero, when the FM plays a pooling strategy; when the FM plays a (semi-) separating strategy instead then the VOI*

[9] To motivate this payoff structure within the quantity competition example of Section 2, think of a market in which the later moving firm has incomplete information on whether the previously-moving firm has high or low unit cost of production.

for the SM is positive iff there is at least one $\phi \in \Phi$ for which the informed SM strictly prefers an a_2 that differs from the one chosen by the uninformed SM.

Proof: Straightforward and therefore omitted. $\square$

If the FM plays a pooling strategy then we are essentially back to the complete-information result: In this case each incarnation of the SM has a point belief about the action choice of the FM implying that there is no room for updating of beliefs along the equilibrium path. Things are different when the FM plays a (semi-) separating strategy (meaning that at least two types of the FM choose different actions in equilibrium). Then observing a signal of the FM's actual action choice may benefit the informed SM even in a pure-strategy equilibrium because she can use her observation to update her beliefs about the FM's type and thereby about the FM's action choice (which is a function of the type). By contrast, the uninformed SM, although she has perfect information on the FM's equilibrium strategy, remains uninformed on his actual move. Now, if the differences in beliefs imply different best replies then there is a positive VOI for the SM in pure-strategy equilibrium.

4 How the VOI for the SM Shapes the VOC for the FM

In Section 3 we have identified conditions under which the value of information about the action choice of the FM is positive for the SM in a meta game in which the information of the FM is held constant. We have referred to this specific meaning of the value of information as the *VOI for the SM.* In this section we will clarify how the fact whether the VOI for the SM is positive or zero influences the value of commitment for the FM in the constituent game. As before the value of commitment for the FM (referred to as the *VOC for the FM*) is defined as the additional equilibrium payoff the FM gets in the "sequential-move" version of the game (where he is able to commit to an action choice first which is then observed by the SM - possibly with some noise - before she has to decide), as compared to the equilibrium payoff he receives in the "simultaneous-move" version of the game (where he lacks this ability). Thus, while our notion of the VOI for the SM involves comparing the equilibrium payoffs of the two incarnations of the SM in a meta game in which the sequential-move version is played with probability $p \in (0, 1)$ and the simultaneous-move version with probability $1 - p$ (and in which the FM is uninformed on which version is played), our notion of the VOC for the FM (which corresponds to the standard notion in the literature) involves comparing the equilibrium payoffs of the FM in the two underlying games, the one obtained by setting p equal to 1 and the one obtained by setting p equal to 0.

At first glance, there is no obvious connection between the VOI for the SM and the VOC for the FM. For instance, in the Stackelberg-Cournot example in Section 2 there is always a positive VOC for the FM while the VOI for the SM is only positive if the FM mixes but zero otherwise. Below we argue that if the FM's action choice is either only imperfectly observable or only observable at a cost then the existence of a positive VOI for the SM in our meta game is necessary for a positive VOC for the FM in the constituent game. Our discussion is motivated by two strands of recent literature that both stress the non-robustness of the VOC for the FM against introducing small frictions in the transmission of information. On the one hand, there is a bunch of articles – starting with the seminal paper by Bagwell (1995) – arguing that the VOC for the FM may vanish if the FM's action is observed by the SM with some (arbitrary small) noise. On the other hand, there are several contributions – starting with Várdy (2004) – showing that the VOC for the FM is often lost when observing the FM's action choice involves some (arbitrary small) cost. We discuss these two strands of literature in turn, the former under the heading '*imperfect observability*' and the latter under the heading '*costly observability*'– and show that the main results obtained in both strands are intimately related to our results on the VOI for the SM. Subsequently – under the heading *'stochastic observability'* – we will discuss how our framework and results relate to a third strand of literature studying still another kind of friction in the transmission of information, one that has been termed "errors in communication" – by Güth et al. (2006), for instance.

4.1 VOC with Imperfect Observability

Bagwell (1995) considers a standard two-player two-stage game augmented by the presence of some noise in the observation of the FM's action by the SM and shows that –under a non-moving support assumption– the set of pure-strategy equilibrium outcomes of the sequential-move game coincides exactly with the set of pure-strategy equilibrium outcomes of the associated simultaneous-move game.[10] He concludes, that "the [positive VOC for the FM] is eliminated when there is even a *slight* amount of noise associated with the observation of the [FM]'s selection" (Bagwell 1995, Abstract; emphasis in original). The intuition for this striking result is that the SM disregards an imperfect signal about the action choice of the FM since she already "knows" what the FM does in any pure-strategy equilibrium. This is implied by our Proposition 1 where we have shown that in a pure-strategy equilibrium of the complete-information game receiving a (perfect or imperfect) signal about the action choice of the FM has no value for the SM because she already knows the FM's action choice from knowing the equilibrium. Indeed, along the equilibrium path Bayes rule dictates that the SM believes that the FM has chosen his equilibrium action no matter what the signal says. Anticipating that the SM will disregard any information contained in the signal about his action choice, the FM has an incentive to choose a best reply to his expectation about the SM's action choice. But, this is exactly what he also does in the simultaneous-move version of the game. Thus, the VOC for the FM (in the usual definition) is completely lost in any pure-strategy equilibrium *because* there is no VOI for the SM (as defined here).

For a specific 2x2 game Bagwell (1995) also shows that it is possible to recover a strictly positive VOC for the FM from a mixed-strategy equilibrium of the 'noisy leader game'

[10]Bagwell's non-moving support condition requires that the support of the signal observed by the SM is independent of the action choice of the FM.

when observability is close to perfect.[11] This is perfectly in line with our findings on the VOI for the SM: From Proposition 2b we know that a (noisy) signal about the action choice of the FM is potentially valuable for the SM when the FM chooses a mixed-strategy. The reason is that with mixed strategies common knowledge of equilibrium only dictates that the SM correctly predicts the probability distribution over the FM's choices. Thus, observing a signal about the actual choice of the FM now has value for the SM if it allows her to respond more accurately to the FM's actual choice. But, if the FM expects the SM to respond to the signal, he has an incentive to influence her behavior, so a positive VOC for the FM reappears exactly *because* there is a positive VOI for the SM.[12]

Maggi (1999) reconsiders the VOC in a slightly different context where in addition to the imperfect observation of the FM's action choice there is private information on the side of the FM on a parameter that is not directly payoff-relevant for the SM. He shows that the VOC for the FM is restored in this context even in pure-strategy equilibria. Specifically, he observes that "[...] if the noise in the observation of the [FM]'s action is small, any equilibrium outcome must be close to the Stackelberg outcome, except for a possible interval of pooling" (Maggi 1999, p. 556). This result is again easily understood given the results in the previous section: With private information on the side of the FM the SM is potentially able to realize a positive VOI in pure-strategy equilibria because unlike to the complete-information case common knowledge of equilibrium now only dictates that the SM correctly predicts the FM's contingent plan but not his actual action choice. Thus, observation of a signal about the actual choice of the FM (potentially) allows for updating of beliefs about the FM's type, and better information about the FM's type is valuable for the SM if different types behave differently and her optimal behavior depends on the behavior of the FM. But, if the FM expects that the SM reacts to the signal she observes then he has again an incentive to influence her behavior. So the VOC for the FM again reappears, *because* there is a positive VOI for the SM.

4.2 VOC with Costly Observability

The second strand of literature on the 'fragility' of the VOC for the FM takes a somewhat different route by considering games in which the SM faces a cost for observing the FM's action choice. The seminal article in this branch is Várdy (2004) who considers a class

[11] Van Damme and Hurkens (1997) generalize this result by showing that each 'noisy leader game' has a mixed-strategy equilibrium that generates an outcome that converges to the "Stackelberg outcome" of the game with perfect observability when the noise vanishes. Moreover, this mixed-strategy equilibrium is picked by 'plausible' equilibrium selection theories, implying a strictly positive VOC for the FM in the 'most focal equilibrium'. By contrast, Oechssler and Schlag (2000) find that almost all evolutionary and learning dynamics they consider lead to the "Cournot outcome", implying a VOC of zero for the FM in the 'most stable equilibrium'.

[12] Huck and Müller (2000) and Müller (2001) investigate Bagwell's 2x2 game experimentally and show that the VOC for the FM is not lost for low noise levels: SMs tend to ignore small levels of noise and play a best reply against the observed signal; FMs tend to exploit this by frequently playing the Stackelberg leader quantity. Thus, for low levels of noise observed play in experiments seems to converge to the VOC-preserving mixed-strategy equilibrium discussed in the main text.

of discrete two-player sequential-move games in which the SM gets to observe what the FM has done if and only if he pays some (arbitrary small) amount of money. If she pays, the FM's action is perfectly revealed to her; otherwise, she receives no information at all. Várdy shows that, irrespective of the size of the amount, the VOC for the FM is lost completely in all pure-strategy equilibria. Again, this is easily understood given our results. Proposition 1 implies that acquiring additional information on the action choice of the FM at a cost is never rational for the SM in a pure-strategy equilibrium because the VOI is zero for her. This is so because she already knows what the FM does by knowing his equilibrium strategy.[13] Anticipating that the SM will not get to know what he has done and will therefore not react to his action choice, the FM chooses a best reply to the SM's action just as in the simultaneous-move version of the game. Thus, the VOC for the FM is again lost *because* there is no VOI for the SM.

Várdy (2004) also observes that for sufficiently low levels of the observation cost his game also admits a mixed-strategy equilibrium that fully preserves the VOC for the FM. Again, this in line with our Proposition 2a which shows that in a mixed-strategy equilibrium there is room for a positive VOI for the SM. For the Várdy (2004) framework this implies that the SM might be prepared to pay a strictly positive amount to observe the action choice of the FM which restores the VOC for the latter.[14]

Morgan and Várdy (2007 and 2011) investigate the VOC for the FM in a similar framework as Várdy (2004) does and show that when the SM faces a small cost of observing the FM's action then the VOC is completely destroyed in *all* equilibria. That the result must hold for pure-strategy equilibria follows from the same arguments as the Várdy (2004) result; and the extension to *all* equilibria is simply a consequence of the facts that (i) Morgan and Várdy (2007 and 2011) study games with continuous action spaces, and (ii) no mixed-strategy equilibria exist with a continuous action space under certain regularity assumptions. Thus, the results in Morgan and Várdy (2007 and 2011) are completely in line with those obtained in the other papers on the fragility of the VOC: if the SM is free to decide whether to obtain or use the information about the action choice of the FM then there is no VOC for the FM without a VOI for the SM. Our discussion in the previous subsection also suggests that the results obtained in the literature on games with costly observability are not robust against the introduction of private information on the side of the FM: Maggi's trick –initially 'invented' to restore the VOC in the Bagwell world– has power also in the "costly observability" framework because it potentially introduces a positive VOI for the SM.

[13]Here note that although Várdy assumes that the SM makes her decision on whether to acquire the information only after the FM has made his decision, his model is strategically equivalent to a game where the SM makes this decision already at the start of the game but the FM does not observe how the SM has decided. Thus, the Várdy game can be regarded as a version of our meta game where the initial move is not made by nature but rather endogenously by the SM.

[14]Morgan and Várdy (2004) investigate games with costly observability experimentally and find that the VOC for the FM is preserved when observation costs are low but lost if they are high. Thus, the qualitative findings here are similar to those found in experiments investigating games with imperfect observability (see one of the previous footnotes).

4.3 VOC with Stochastic Observability

There is a small number of papers studying a third kind of friction in the transmission of information – 'errors in communication'. Examples are Chakravorti and Spiegel (1993), Güth et al. (2006) and Poulsen and Poulsen (2008). In contrast to the literature discussed in the previous two subsections these papers do not challenge the VOC for the FM. The formal models investigated are similar to our meta game with complete information and perfect observability – with given probability $p \in (0, 1)$ the SM observes the FM's action choice perfectly, with complementary probability nothing is observed.[15] A typical finding for environments with continuous (large discrete) action spaces is that the VOC for the FM increases continuously (monotonically) in the probability with which the SM observes the FM's choice – see Chakravorti and Spiegel (1993), and Güth et al. (2006) for details. This result is easily understood by noting that the authors use a slightly different notion of the VOC for the FM than we used so far: It involves comparing the equilibrium payoff of the FM in our meta game with the equilibrium payoff he gets in the simultaneous-move version of the game. Thus, the results in those papers are simply the continuous counterparts of the results we obtained from the Stackelberg-Cournot example studied in Section 2.[16]

5 Conclusion

The article has introduced a specific notion of the value of information for the second mover (SM, she) about the action choice of the first mover (FM, he) in the context of a two-player two-stage game. It has then used this specific notion of the value of information for the SM to shed new light on some recent results on the fragility of the value of commitment for the FM. By doing so, this article has connected two seemingly unrelated strands of recent literature, the literature on the value of information in games and the literature on the value of commitment in games.

Specifically, we have introduced a definition of the value of information for the SM about the action choice of the FM that holds the FM's information constant. We have referred to this specific notion of the value of information for the SM as the '*VOI for the SM*'. For this specific meaning of the VOI for the SM we have first shown that it is necessarily zero in a pure-strategy equilibrium of any two-stage game in which the SM is informed about the preferences of the FM: In a pure-strategy equilibrium of a two-stage game in which the SM has complete information about the preferences of the FM the strategy of the FM is simply an action choice and common knowledge of equilibrium dictates that the SM correctly predicts this choice. Thus, there is no room for Bayesian

[15]The 2x2 game studied by Poulsen and Poulsen (2008) is slightly different to our baseline model with perfect observability because it allows the two actions of the FM to be observed with different probabilities by the SM while in our framework the observation probability p does not depend on the action chosen by the FM.

[16]Güth et al. (2006) and Fischer et al. (2006) investigate games with stochastic observability experimentally and find that FMs indeed enjoy a (weakly) larger VOC when p increases, roughly as theory predicts.

updating and therefore also no room for a positive VOI for the SM along the predicted paths. Things are different when either mixed-strategy equilibria or games with private information in the hands of the FM are considered. Then, correct expectations about the strategy choice of the opponent only dictates correct expectations about the probability distribution over moves. Thus, information about the actual choice of the FM allows for updating of beliefs and this typically leads to a strictly positive VOI for the FM.

We have then used our results on the VOI for the SM to shed new light on some recent results on the fragility of the value of commitment in sequential-move games. As usual the value of commitment for the FM (referred to as the '*VOC for the FM*') is defined as the additional equilibrium payoff he gets in the "sequential-move" version of the game (where he is able to commit to an action choice first which is then observed by the SM before she has to decide), as compared to the equilibrium payoff he receives in the "simultaneous-move" version of the game (where he lacks this ability). Specifically, we have shown that in cases where the SM has the *freedom to decide* whether to take the information she receives into account or not (the imperfect-observability case pioneered by Bagwell 1995), or whether to observe the FM's action choice or not (the costly observability case studied by Várdy 2004, among others), a positive VOI for the SM is needed for a positive VOC for the FM: Without an advantage out of the additional information the SM just ignores it, or does not acquire it in the first place, implying that the FM has an incentive to behave just as in the simultaneous-move version of the game. By contrast, in cases where the SM is *forced to observe* the FM's choice for sure (the perfect-observability case studied in the earlier literature on the VOC for the FM) or with strictly positive probability (the stochastic observability case studied in a recent strand of literature under the heading "errors in communication") the VOC for the FM can be positive even when the VOI for the SM is zero: In such games the FM knows that the SM must take his action choice into account (at least with positive probability). Therefore, he has an incentive to strategically influence the SM's choice when the observation probability is sufficiently high. Thus, the crucial point to preserve a positive VOC for the FM is that the SM either must observe the FM's action choice (at least with positive probability), or has an incentive to observe it (because there is a positive VOI for her).

Our overall conclusion is that in most realistic scenarios a later-moving player has at least a small degree of uncertainty about the behaviour of a previously-moving player, for instance because there is uncertainty about his rationality, his preferences or his beliefs. With uncertainty about the behavior of the previously-moving player there is room for a positive VOI for the later-moving player and therewith room for a positive VOC for the previously-moving one. Thus, the recent results about the fragility of the value of commitment seem themselves fragile in the sense that they are unlikely to hold in realistic settings. This conclusion receives support from recent experimental studies – by Huck and Müller (2000), Müller (2001), and Morgan and Várdy (2001), for instance – which show that play in the lab tends to converge to the VOC-preserving equilibrium when the frictions in the transmission of information (the level of noise, or the cost of observing) are not too large.

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