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Fairness and Efficiency in a Subjective Claims Problem*

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Abstract

In a subjective claims problem several agents have contributed to the production of a cake which is to be divided among them. Since contributions are difficult to compare and the production function is nonlinear, agents' subjective evaluations of claims are likely to be conflicting. In a large-scale experimental study we compare the performance of three mechanisms which use agents' reports to resolve the subjective claims problem. The mechanisms differ with respect to the information they process and they are compared in terms of efficiency and perceived allocative and procedural fairness.

Keywords: Fair Division, Subjective Claims, Experiment, Mechanisms

JEL Classification: D63, C91, D61

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1 Introduction

In the oil and gas industry, a single oil pool often underlies the land parcels of several owners. If owners act non-cooperatively, each drilling his own well, oil reserves are depleted at an excessive rate, resulting in large efficiency losses (see Libecap and Wiggins 1984). An obvious solution to this 'tragedy of the commons' problem is unitization, that is, treating the oil pool as a production unit controlled by a single firm. Unitization, however, brings in the new problem of how the proceeds of the oil field should be divided among the multiple owners. Due to physical characteristics of oil and gas deposits, individual contributions to the joint profit are difficult to compare. For example, a simple division according to the size of contributed land might be considered unfair, since some parcels are located over the centre of the oil field while others are located in the periphery, thus oil pressure differs across parcels. There is no other simple measure of each involved party's contribution to the measurable output (see Cramer et al. 2009). As a result, parties typically have conflicting subjective perceptions about the relative contribution of their own property to the joint profit and thus also about what constitutes a fair allocation.[1] An important question then is how to aggregate these conflicting subjective perceptions into a fair division of the joint profit.

The motivation for the present study came from another example where parties' contributions to a joint surplus are difficult to compare: In Tyrol, Austria, a new water power plant will be built in a location that has common borders with three municipalities. The company who owns and operates the power plant pays a certain amount of money to the three municipalities in exchange for the right to use land (i.e. the valley that would need to be flooded in order to operate the power plant) and water. The amount to be paid is no longer an issue as the decision to build the plant has already been made.[2] Now the municipalities have to decide how to distribute the money amongst themselves, and the question arises what

[1]Indeed, conflicting subjective perceptions about what constitutes a fair allocation of the joint profit and the resulting failure of negotiations about unitization has led major oil producing states in the US to provide legal mechanisms by which an oil pool can be unitized with less than unanimous consent of the right owners. See, for example, Rule 530 in Colodrado's Oil and Gas Conservation Act.

[2]The fact that the municipalities agreed to give up their resources at a point in time where they did not know what each of them gets in exchange might seem strange to an economist. But this is exactly what happened in this case.

would be the fair share for each of them. The question is non-trivial because the municipalities give up different amounts of land and water rights, and it is unclear how land and water should be evaluated on a common scale.

In order to evaluate fair shares, one could consider estimating the contribution of a given partner by removing him from the partnership. However, the individual market values of the individual rights are fairly small compared to the combined value of the resources. Hence, this procedure of estimating the contribution of a partner is of limited help. Another way of resolving the division problem would be to ask an outside observer as impartial arbitrator to assign shares to the prospective partners. However, such an external agent is typically in a worse position to discern the partners' relative contributions and their fairness perceptions than the partners are themselves, as his information might be incomplete, or biased if provided from the partners' side. The partners may thus prefer to rely on their own subjective perceptions on how the profit should be divided fairly and they may find it useful to use a rule or procedure to reach a consensus. The obvious question then is which rule this should be. The present paper investigates this question by comparing the performance of three cake division mechanisms which use agents' reports to resolve the subjective claims problem in a large-scale experiment with more than 600 participants.

Our two motivating examples incorporate two different sources of potential inefficiencies: In the petroleum-industry example the inefficiency of the non-cooperative solution is due to the over-exploration of the joint resource, as captured in the familiar 'tragedy of the commons' problem: Multiple owners are each endowed with the right to use the common resource, and no one has the right to exclude anybody. By contrast, in the power plant example the source of the inefficiency of the stand-alone solution is the under-exploration of the common resource due to what Heller (1998) has termed the 'tragedy of the anticommons' problem: To use the resource, many individual rights would have to be assembled; the owners of these rights each may block their coordinated use, and these rights are almost useless individually. Several of Heller's applications fit into the present framework, for instance, a technological innovation that requires the accumulation of many patents held by diverse private parties, or a rundown neighbourhood where the value of the consolidated properties greatly exceeds the sum of the individual properties in their current uses.

Our aim is to take a unifying approach for the study of such problems by focussing on their similarities, which are the subjective perceptions of involved parties' fair shares of the common resource and the inefficiency of the non-cooperative solution. To this aim, a stylized version of the problem, which shall be referred to as the *subjective claims problem*, is formulated as follows: Several agents – the partners – have contributed inputs to a joint project whose final value is a fixed sum of money denoted by S. Inputs have a stand-alone value of zero.[3] Once S is produced, the partners have to divide it amongst themselves. Since inputs are difficult to compare, partners typically have conflicting subjective perceptions about what constitutes a fair division of the joint profit S. In the following, we shall refer to an allocation of S that is considered fair by partner i as her *subjective evaluation of claims*. In the case of n partners such a subjective evaluation of claims is a vector with n entries summing up to S.[4] This formulation implicitly assumes that the partners are interested not only in their own material payoff or share of S, but also in the fairness of the allocation. The subjective claims problem is then to find an allocation $s = (s_1, \ldots s_n)$, where $\sum_i s_i \leq S$, or alternatively, a procedure that implements such an allocation, which is considered fair by the partners (in the sense that it respects the partners' subjective evaluations of claims in some appropriate sense see the discussion below) and which is also efficient (in the sense that $S - \sum_i s_i$ is minimized).

In contrast to our framework, Fischbacher et al. (2009) study the complementary problem of two players exerting real effort, consisting of answering a single multiple-choice question, after the division rule has been determined either by a neutral third party or by players themselves taking on the roles of proposer and responder in an ultimatum game. Despite the differences in subjects' bargaining power in these two treatments, the results are similar: The better-performing agent received a larger share of joint output, but this share does not reflect sub-

[3]By stand-alone value we mean the outside option, that is, the material payoff realized when no agreement is reached. While in the examples above the combined value of the inputs is worth much more than the sum of the stand-alone values, each input still has some value individually. We abstract from positive stand-alone values for the sake of simplicity.

[4]In contrast to the n privately known vectors of subjective claims described here, the more familiar *objective claims* (or bankruptcy) *problem* refers to one publicly known vector of objective claims, which is infeasible because the sum of the objective claims exceeds the available amount. See Moulin (2002) and Thomson (2003) for surveys, and Gächter and Riedl (2005, 2006) for experiments on the objective claims problem.

jects' marginal productivities. This complements the findings of Cappelen et al. (2007), who use monetary efforts and find most subjects to prefer an egalitarian distribution or one that is based on individual efforts.

As economists, we may consider various procedures that provide a solution to the subjective claims problem. One common way to resolve the problem would be through bargaining. For a scenario with multiple players a variety of structured bargaining protocols are available, for example, the unanimity bargaining procedures proposed by Torstensson (2009), Shaked (see Sutton 1986), and Krishna and Serrano (1996), or the majority procedures proposed by Baron and Ferejohn (1987 and 1989), and Morelli (1999). Besides the dynamic bargaining procedures, an alternative approach would be to apply a static mechanism that uses agents' reports as an input and yields a unique allocation as an output. In a large-scale experiment, we investigated representatives of both approaches. Specifically, we implemented three static mechanisms and three unanimity bargaining procedures. Due to the large amount of data we obtained, we decided to separate static and dynamic procedures and to report here only the comparison of the three static mechanisms.

The details of our experiment are as follows: First, we generated a subjective claims problem involving three partners in a lab experiment. This is done by having subjects perform real effort tasks within different cohorts and earn points depending on their relative performance within their cohort.[5] The points a subject earns are his contribution to the partnership, and all three partners of a partnership have earned their points from exerting real effort in a different cohort. The amount of effort and the performance in the real effort task are private information – subjects are only informed about partners' contributions in points. Furthermore, the production function translating individual contributions to the joint profit is non-linear. The joint profit is then distributed among partners with the help of three mechanisms. The mechanisms use reported subjective evaluations of claims as input and yield unique shares as output, summing up to no more than the value of the cake. The mechanisms differ in the amount and kind of information they

[5]We let subjects acquire their contributions to the cake in real effort tasks to induce strong entitlements. Numerous empirical studies confirm that earned rights induce stronger entitlements than those acquired by luck (see e.g. Hoffman and Spitzer 1985, Burrows and Loomes 1994, Schokkaert and Lagrou 1983).

process:

The **Modified Nash Demand Rule** (Mumy 1981) asks each partner for a report of the own entitlement or claim. If the sum of reported own claims does not exceed S, each partner receives the reported own claim. Otherwise each partner receives the own claim minus a penalty that is a multiple, greater than 1, of the difference between the sum of reported own claims and the amount available. This implies an inefficiency since less than the cake is divided in this case. Another potential source for inefficiencies under this mechanism are own claims that sum up to less than the cake.

The **Impartial Division Rule** (DeClippel, Moulin and Tideman 2008) asks each partner for a report on her assessment of the relative claims of the other partners. If there is a division of S that is consistent with the reported opinions about relative shares, then the mechanism assigns these shares. Otherwise the rule divides less than the total amount available, resulting in an inefficiency which increases in the degree of inconsistency of the reported relative shares.

The **Extended Divide-the-Dollar Rule** (Brams and Taylor 1994) asks each partner to report an evaluation of the own claim as well as one of the partners' claims. If the three own claims sum up to S or less, each partner receives the reported evaluation of the own claim. Otherwise, own claims are paid out sequentially up to the point where S is depleted, giving priority in the order of greed levels, starting with the lowest one. To calculate a partner's greed level, the mechanism subtracts the average of the claims assigned to this partner by the other two members of the partnership from the reported own claim. If this difference is positive, the partner is greedy and the difference is his greed level. The only source for inefficiencies under this mechanism are own claims that sum up to less than the value of the cake.

None of these mechanisms has been tested in lab experiments before. The mechanisms shall be compared in terms of fairness and efficiency. The efficiency comparison refers to the fraction of S that is finally paid out to the partners. Efficiency is not a criterion that can stand alone in a subjective claims problem, but it must be seen in relation with the fairness of the implemented allocation.[6] As to

[6] While one may reject a mechanism for the sole reason that it performs badly in terms of efficiency, one cannot recommend a mechanism in this context only because it is highly efficient. For instance, a mechanism that allocates the entire cake to one partner is maximally efficient,

the fairness comparison, we are interested in the partial view of the stakeholders, since, after all, it is the involved parties who have to live with the final outcome, and since they have more information than anyone else on partners' contributions. Specifically, we combine three different pieces of information to evaluate the fairness of a procedure: First, before introducing the mechanisms, subjects are asked in a hypothetical fairness question what they consider a fair way to divide the cake. The answers to this fairness question shall serve as a first yardstick for the evaluation of the allocations produced by the three mechanisms. A second yardstick is subjects' evaluation of the mechanisms' procedural fairness, which is solicited after subjects have been exposed to the three division procedures but before they receive feedback about their payoffs. The third yardstick we use in our comparison is subjects' evaluation of the mechanisms' allocative fairness, which requires them to compare the actual outcomes of the three mechanisms.

Our results clearly show that the two mechanisms which explicitly force agents to assess the partners' claims, that is, the Impartial Division Rule and the Extended Divide-the-Dollar Rule, yield outcomes that are closer to subjects' fairness evaluations. This is in line with the finding from our companion paper on bargaining with subjective claims (Gantner, Horn and Kerschbamer 2013), where a bargaining rule that explicitly asks the proposing player to make a complete division proposal and the other players to accept or reject this proposal performs best in terms of fairness, while a rule that asks players to make only a proposal regarding the own share leads to inefficiencies and distortions caused by a self-serving bias.[7] Being forced by the procedure to quantify the partners' claims induces agents to make proposals that are closer to their own fairness assessments. The Modified Nash-Demand Rule that uses only agents' own claims as input is not only worse in terms of fairness, but due to its rule of imposing a fine when the sum of claims exceeds the available cake size, it is also far less efficient. A comparison of the Extended Divide-the-Dollar Rule and the Impartial Division Rule shows that the evaluation depends on the relative position of a partner. In the Extended Divide-the-Dollar

but it is probably considered as rather unfair by most of the partners.

[7]See also Karagözoglu and Riedl (2010) for a study on free-form bilateral bargaining in a similar context. Subjective entitlements in their scenario arise because there is no or only ordinal information on the partner's contribution, or because of noise in the production process, and they find that information about relative performance is needed to move away from the equal split.

Rule, low contributors are systematically advantaged and high contributors disadvantaged, and thus it is not surprising that low contributors typically prefer the Extended Divide-the-Dollar Rule, while high contributors tend to prefer the Impartial Division Rule.

2 Three Mechanisms to Resolve the Subjective Claims Problem

The subjective claims problem we consider features the three partners A, B, and C, who have jointly produced the cake S, which now has to be divided amongst them. We denote the subjective evaluation of claims by partner i by $c^i = (c^i_A, c^i_B, c^i_C)$, where c^i_j stands for the amount agent j should receive from partner i's perspective. Throughout we assume that $c^i_A + c^i_B + c^i_C = S$ for $i = A, B, C$. Each of the three mechanisms we consider yields an allocation $s = (s_A, s_B, s_C)$, where $\sum_i s_i \leq S$, which we will evaluate in terms of efficiency (in the sense that $S - \sum_i s_i$ is minimized) and in terms of how well it represents the subjective evaluations of claims of the three partners.[8] The three mechanisms we compare differ in the amount and kind of information they process. Suppose the partners are asked to report their subjective evaluations of claims and denote the report of agent i by $m^i = (m^i_A, m^i_B, m^i_C)$, where m is mnemonic for message. (Note that if agent i reports truthfully then $m^i = c^i$). Then, from partner i's report, only i's 'own claim' m^i_i is used as input in the first mechanism considered below; only 'others' claims' m^i_j and m^i_k, with $\{i, j, k\} = \{A, B, C\}$, are used in the second mechanism; and the whole vector of reported claims, m^i, is used in the third mechanism.

2.1 The Modified Nash-Demand Rule

Mumy (1981) proposed a mechanism where each agent i is asked to report only the own claim m^i_i and where the amount agent i receives is given by

[8] Our analysis will not provide a simple quantifiable measure for the comparison of the three mechanisms' performance, since we do not specify a social utility function to be maximized. Rather, we will consider various benchmarks for the evaluation of a mechanism's performance, which are all based on subjects' evaluations.

$$s_i = m_i^i - \max\{a(\sum_j m_j^j - S), 0\},$$

where $a > 1$. In words, if the sum of reported own claims does not exceed S, then each agent receives exactly the reported own claim. Otherwise each agent receives the reported own claim minus a 'fine' that is proportional to the difference between the sum of reported own claims and the available amount. Note that there are two sources of inefficiencies under this mechanism. One arises when the sum of reported own claims exceeds S, while the other arises when this sum falls short of S. Both kinds of inefficiencies increase linearly in the difference between the sum of reported own claims and the sum of money available. As for the theoretical prediction, it is far from trivial to see what it would be for the case where agents are not only interested in their own material payoff but also in the fairness of an allocation as we assume in this paper, since it depends on the exact shape of agents' preferences and their information about the partners' preferences in that case. In order to keep things tractable, we refer here and in the following only to the theoretical benchmark for the case where it is common knowledge that all agents are exclusively interested in their own monetary payoff.[9] Under this assumption, any combination of reports such that $\sum_j m_j^j = S$ is an equilibrium under this mechanism.

2.2 The Impartial Division Rule

In the second mechanism proposed by DeClippel, Moulin and Tideman (2008), each agent is asked to report an evaluation of the relative shares that the other two agents deserve. That is, for $\{i, j, k\} = \{A, B, C\}$ each agent i is asked only how much partner j should get compared to partner k. Denoting partner i's report for the ratio of j's share to k's share by r_{jk}^i (so $r_{jk}^i = m_j^i / m_k^i$ in the notation used before), the Impartial Division Rule yields the following payoffs for the three partners:

[9] The selfish benchmark is mentioned for completeness only. Our focus is not on whether subjects' behavior in the experiment is consistent with this benchmark or not, but rather on which outcomes occur and how far they are from subjects' views of a fair division.

$$s_A = \frac{S}{1 + r^B_{CA} + r^C_{BA}};\ s_B = \frac{S}{1 + r^A_{CB} + r^C_{AB}};\ s_C = \frac{S}{1 + r^A_{BC} + r^B_{AC}}.$$

If there is a division of S that is consistent with the reported opinions about relative shares, then the mechanism assigns these shares and the whole cake is distributed. Otherwise the rule distributes strictly less than S in a way which guarantees that the report of a partner has no effect on the share she gets.[10] In the latter case the difference between the size of the cake and the sum of the assigned shares counts as an efficiency loss and this loss increases in the degree of inconsistency of the reported relative shares – see Tideman and Plassmann (2008) for a discussion.[11] As for the theoretical prediction, the assumption of common knowledge about agents' exclusive interest in own monetary payoffs implies that any combination of reports constitutes an equilibrium for this rule, as no agent can affect his own share by his report.

2.3 The Extended Divide-the-Dollar Rule

The third mechanism we include in our comparison was proposed by Brams and Taylor (1994). The authors start with a setting with publicly known, player-specific entitlements $e = (e_A, e_B, e_C)$ that sum to the available amount S. The mechanism proposed for this 'objective entitlements' setup asks each agent i to report only the own claim m^i_i. If the sum of reported own claims does not exceed S, then each agent receives exactly the reported own claim; otherwise, the claims are given priority in order of their 'greed level' defined by $m^i_i - e_i$, starting with the lowest greed level.[12] Relevant for our subjective claims environment is an extended version of this mechanism, where players' entitlements are endogenously determined. For this purpose, each player i is now asked to report a complete

[10] DeClippel et al. (2008) show that for three players this is the only division rule that is impartial and consensual. Impartiality requires (i) that the rule depends only on the partners' reports of the relative shares other partners deserve and (ii) that the share of any agent is determined exclusively by the reports of other agents. Consensuality requires that if there is a way to divide the cake that is consistent with the relative shares reported by the agents, then this should be the outcome.

[11] For more than three agents, DeClippel et al. (2008) characterize a family of adjusted rules that always distribute exactly the available amount.

[12] Ties result in an allocation where the shares of those involved in the tie are proportional to their entitlements.

vector $m^i = (m^i_A, m^i_B, m^i_C)$ describing i's subjective evaluation of her own claim and of the claims of i's two partners. If own claims are feasible – in the sense that they sum up to S or less – they are implemented, just as under the Modified Nash-Demand Rule. If own claims sum up to more than S, then this mechanism assigns to each player i a greed level defined as the difference between the reported own claim, m^i_i, and i's entitlement, which is the average of the claims assigned to i by her two partners, $(m^j_i + m^k_i)/2$. Own claims are then paid out sequentially in ascending order of greed levels up to the point where S is depleted. The only source for inefficiencies under this mechanism are own claims that sum up to less than the cake size S. While Brams and Taylor derive formal results for several other variants of the Divide-the-Dollar game, the discussion of this variant remains informal. Own investigations (under the assumption that agents care only about their own monetary payoff) reveal that the game induced by this rule has –in general– no pure-strategy equilibrium and that the existence and properties of mixed strategy equilibria heavily depend on the size of S and on the grid on S.

3 Experimental Design

In the experiment, each subject is part of a group consisting of three partners. The experiment consists of four parts. In Part I each group acquires an amount of money S by the partners' individual performances in a real effort task. In Part II subjects are asked about their opinion on what would be a fair division of S among the three partners (the "fairness question"). In Part III, the sum of money is actually divided according to the three procedures. In Part IV, subjects are asked to evaluate the procedures as well as the resulting divisions with respect to fairness. The experiment was programmed and conducted with the software z-Tree (Fischbacher 2007).

Part I: Real Effort Task and Emerging Cake Size The real effort task consists of a general knowledge quiz. Prior to the quiz, subjects are randomly assigned to one of three same-size cohorts and they are informed that (i) each subject in a cohort will be exposed to the same set of questions; (ii) each subject in a cohort will receive points depending on her relative performance (in terms of correctly answered quiz questions within a given time period) within her cohort;

(iii) after the quiz each subject will be assigned to a group of three partners, each coming from a different cohort; (iv) the points a subject acquires in the quiz will be her contribution to the joint profit of the group; and (v) the joint profit of the group will later be distributed amongst group members with the help of several procedures, which are all fully paid out. Each cohort consists of 6 subjects, and the points assigned to subjects are as follows: The two high performers within a cohort (i.e., ranks 1 and 2) are assigned 4 points, the two medium performers (ranks 3 and 4) receive 3 points, and the two low performers (ranks 5 and 6) get 2 points. After the real effort task, subjects are informed about their own rank within their cohort and the points they achieved. Then they are assigned to a group consisting of three partners (labelled A, B, C), one from each of the three cohorts. Upon assignment to a group, subjects are informed about their two partners' contributions in points, but not about their partners' precise performance or rank within their respective cohort. The points subjects bring into the group enter a non-linear production function, which determines the size of the cake S to be distributed:[13]

$$S = 12 + \textit{(points A)}\cdot\textit{(points B)}\cdot\textit{(points C)}$$

By using a relative performance measure *within* cohorts, but selecting the three partners from *different* cohorts, we intend to induce conflicting subjective evaluations of claims. A division according to the number of contributed points might be considered unfair because contributions in points are only a noisy signal for the actual performances in the quiz and because subjects have no possibility to directly compare their own quiz performance to that of the other two group members. The non-linear production function further aggravates the difficulty of finding a fair division.[14]

Groups in the experiment are composed such that we have groups with a small cake size of $S = 24$, groups with a medium cake size of $S = 36$ and groups with a large cake size of $S = 60$; see Table 1 for details. Note that with this choice of group compositions we have groups where two partners contribute the same low number of points (albeit they may have performed differently in terms of correctly

[13] All of this information was contained in the instructions, which are available upon request.

[14] Similarly, in our oil production example, one can directly compare the size of land parcels each party contributes. However, due to physical characteristics of oil and gas deposits, a division according to the size of land is unlikely to be considered fair by the partners.

Cake Size	Points			# Observations	
S	A	B	C	Groups	Subjects
S=24	2	2	3	90	270
S=36	2	3	4	90	270
S=60	3	4	4	90	270

Table 1: Group Compositions in Experiment

answered questions), groups in which all partners have different contributions in points and groups where two partners have the same high contribution.

Part II: The Fairness Question After being informed about their partners' contributions in points and the resulting cake size, subjects are privately asked what they consider a fair division of the jointly produced cake. That is, each subject i is asked to report a vector of his subjective evaluation of claims, $m^i = (m^i_A, m^i_B, m^i_C)$, where the entries have to sum up to S, knowing that the answer to this question is irrelevant for her earnings in the experiment. The answers to the fairness question shall serve as one of the benchmarks for our comparison of the three cake-division mechanisms in terms of allocative fairness.

Part III: Actual Division of the Cake In each experimental session, each group is successively exposed to each of the three mechanisms in random order.[15] Group composition and cake size remain constant across mechanisms. In each case, the mechanism is first described and subjects have to answer some control questions to make sure that they understand how the mechanism works. Then subjects are asked for the necessary input for the mechanism, after which they receive the description of the next mechanism. Feedback about the allocations implemented by the three mechanisms is given only at the end of the experiment (see Part IV below) in order to avoid that the outcome of a mechanism affects subjects' behavior for other mechanisms. All three mechanisms are paid off, and for each point earned in the experiment subjects were paid 25 cents.

Part IV: Procedural and Allocative Fairness Before subjects receive feedback about their payoffs under the three cake-division mechanisms, they are asked to rank the mechanisms in terms of procedural fairness. After receiving informa-

[15] As already mentioned, we also tested various bargaining procedures in different sessions. Only one procedure was tested per session, and it was presented after the three mechanisms described here.

tion about their payoffs, subjects are asked to rank the allocations implemented by the three mechanisms in terms of allocative fairness.

4 Experimental Results

4.1 Fairness Question

Our formulation of the subjective claims problem assumes that the partners are interested not only in their own material payoff, or share of S, but also in the fairness of the allocation. While in experimental studies without entitlements equality is a generally accepted fairness norm, the survey by Karagözoglu (2012) shows how norms of equity and desert influence bargaining behavior when agents jointly produce the cake that is to be divided. Regarding the self-serving bias in fairness judgements, which is shown in numerous studies when stakes are involved (for field experiments see Babcock, Wang and Loewenstein 1996, for lab experiments see Kagel et al. 1996, Konow 2000), Konow (2003) notes that although biases sometimes widen the range of predicted outcomes, behavior still is constrained by fairness. This is consistent with our presumption that subjects in a subjective claims problem have some kind of social preference, i.e. they have a subjective evaluation of claims describing the allocation of the cake they consider fair, and which they want to see implemented.

One might argue that known fairness standards (apart from the egalitarian norm) should not play a role for fairness evaluations in a subjective claims problem, since efforts are unknown and contributions are difficult to compare. Yet, due to the absence of other (better) benchmarks, it is likely that some known fairness standards serve as anchors for subjective evaluations of claims. Thus, subjects' responses to the fairness question are expected to lie – at least to some extent – around allocations implied by some known fairness norms, with some adjustment, possibly to a subject's own advantage.[16] The upper part of Table 2 displays the allocations implied by three well-known division standards. We refer to the *egalitarian standard* when S is distributed equally among the partners, to the *propor-*

[16]Furthermore, the literature on moral wiggle room (e.g. Dana et al. 2005) shows that if multiple fairness standards can plausibly be applied, subjects tend to appeal to the one that yields the highest payoff for them.

tional standard when shares of S are assigned proportionally to the points each partner has contributed, and to the *liberal standard* when each partner receives an equal share of the fixed part of the production function and the remainder of S is divided proportionally to the points contributed.[17] Below we will report the relative frequencies of answers to the fairness question that are consistent with a given standard. When counting observations as consistent with a given standard, we allow for intervals that typically round numbers to the next half unit in case the standard does not yield integers.[18] These intervals are given in the *'adjusted'* row. Note that an impartial arbitrator, who is assumed to know only the size of the cake, could only appeal to the egalitarian standard. On the other hand, subjects in the experiment are informed about their partners' contributions and can thus appeal to any of the three standards in forming their subjective evaluations of claims.

The middle part of Table 2 displays for each contribution type and cake size how much subjects, on average, report as being fair for themselves (*'own fair share'*), what their partners report as being fair for them (*'fair share from others'*), and what the average fair share for each position is when all three partners' reported fairness evaluations are included in the calculation of the average (*'average fair share'*). For the small and the large cake size we pool the data for the two partners with the same contribution in points (i.e. for partners A and B in $S = 24$ and for partners B and C in $S = 36$), since there is no sound basis to differentiate between them (except for the label), and since they are indeed not significantly different according to the two-sample Kolmogorov-Smirnov test for equality of distributions. A first lesson we learn from the middle part of the table is that there is a significant self-serving bias in the answers to the fairness question: The own fair share is larger than the fair share from others in all relevant comparisons – the paired t-test shows significant differences for all contribution types and cake sizes at $p < 0.01$. Own fair

[17] The term "liberal" is used differently in part of the literature – see Cappelen et al. (2007), for instance.

[18] Sometimes the interval is larger. For instance, for $S = 24$ the proportional standard predicts 6.86 for the two low contributors and 10.28 for the medium contributor. If we count an observation of 6.5 for the low contributors as consistent with this standard, this would leave 11 for the medium contributor, therefore we consider observations in the interval $[10, 11]$ as consistent with the proportional standard for this type. This is certainly an ad-hoc criterion, however, we tried various other criteria, such as allowing for a fixed deviation in both directions of a point prediction, and results are rather stable.

cake size	S=24		S=36			S=60	
partner	A/B	C	A	B	C	A	B/C
contribution	2	3	2	3	4	3	4
fairness standard							
egalitarian	8	8	12	12	12	20	20
proportional	6.86	10.28	8	12	16	16.36	21.82
adjusted	[6.5, 7]	[10, 11]	8	12	16	[16, 17)	(21.5, 22]
liberal	7.43	9.14	9.33	12	14.67	17.10	21.45
adjusted	(7, 7.5]	[9, 9.5]	[9, 9.5]	12	[14.5, 15]	[17, 18]	[21, 21.5]
observed fair shares							
own fair share	8.04	9.90	10.22	12.64	16.36	18.91	22.65
fair share from others	7.14	8.72	8.61	11.48	14.30	16.00	20.95
average fair share	7.44	9.10	9.15	11.87	14.99	16.97	21.51
observations consistent with fairness standard (in %)							
egalitarian	42.7	18.8	28.8	15.5	8.8	41.1	25.0
proportional	38.3	62.2	26.6	26.6	31.1	15.5	27.2
liberal	5.5	2.2	4.4	5.5	8.8	27.2	9.4
equal shares to equal contributions	90.5	96.6		–		98.8	89.4
reflects contribution ordering	47.2	38.8	62.2	75.5	80.0	53.3	64.4
low contributor gets more than proportional	50.5	23.3	66.6	53.3	41.1	73.3	38.3

Table 2: Fairness Standards and Observed Assignments of Fair Shares

shares also exceed the shares the proportional standard would predict for almost all types and cake sizes for common significance levels. There are two exceptions: Those subjects who bring the most points to the partnership report own fair shares below the proportional share in $S = 24$ (t-test: $p < 0.02$), and own fair shares that are not significantly different from what the proportional standard would predict in $S = 60$ (t-test: $p = 0.19$). This observation is consistent with a taste for a more 'compressed' distribution than the one implied by proportionality, i.e., one that implies smaller payoff differences across subjects. This taste for a more compressed distribution can be seen more clearly in the assignments from others, where the self-serving bias does not obliterate part of the effect: Fair shares from others are significantly different from the proportional standard in most comparisons (t-test: $p < 0.01$ for all types and cake sizes except for the medium contributor in $S = 60$, where $p < 0.09$), and while high and medium contributors are consistently assigned less than what proportionality would predict, low contributors are assigned more.

The lower part of Table 2 displays for each contribution type and cake size the observed relative frequency for each of the three division standards, as well as that of some intuitive criteria such as assigning equal shares to equal contributors, or shares that reflect the contribution ordering.[19] In accordance with our goal of implementing a subjective claims problem characterized by heterogeneous (and conflicting) assessments regarding the fair division of the cake, an overall consistency with common fairness standards is not observed. In particular, for the cake size of $S = 36$ – where all contributions are different – only about half of the observed assignments of fair shares are consistent with one of the three prominent standards discussed above. This confirms our presumption that evaluations of claims are subjective in our scenario, i.e. it is not clear which standard is of relevance here. Looking at the data in more detail confirms the insights from the average data (discussed in the previous paragraph): The self-serving bias here manifests itself in the choice of fairness standards – while low contributors tend to report own fair shares that are (roughly) consistent with the egalitarian standard, high contributors tend to report fair shares consistent with the proportional standard. The self-serving bias can also be seen in the last row of the table: While low contributors frequently report fair shares that assign more to low contributors than what proportionality predicts, higher contributors do so to a lesser extent. It is important to note, however, that a considerable fraction of *all* contribution types in all cake sizes assign a larger share than the one predicted by the proportional standard to the lowest contributor within their group. This confirms our earlier finding of a taste for a more compressed distribution than the one implied by proportionality.

4.2 Results for the Three Mechanisms

Since the randomized order in which the procedures where presented to subject showed no effect on the results for common significance levels, we will report pooled data of all sessions for a given procedure.

[19] Here we refer to the strict ordering, i.e. the egalitarian standard would not reflect contribution ordering in our definition, as higher contributions are not rewarded with higher shares.

4.2.1 Modified Nash-Demand Rule

The results for the mechanism proposed by Mumy (1981) are displayed in Table 3. Recall that only each partner i's own claim, m_i^i, is elicited by this mechanism.

Mechanism Input: Again, we use pooled data for partners with the same contribution in points, as the Kolmogorov-Smirnov test shows no difference in the two distributions. Reported own claims respect the ordering of contributions in points (median test: $p < 0.001$ for all comparisons of claims with different contributions), but they do not correspond to the proportional standard: While low contributors ask systematically more (WSR and t-test: $p < 0.001$ for all cake sizes), high contributors ask less (WSR: $p < 0.03$ for all cake sizes, only the t-test for $S = 36$ gives $p = 0.5$) than proportionality would predict. This finding corresponds to what was observed in the fairness question. But there are also some notable differences: Low contributors' reported own claims under this mechanism tend to be higher than own fair shares in the fairness question (WSR: $p < 0.03$ and paired t-test: $p < 0.001$ for $S = 24$; t-test: $p < 0.08$ while WSR: $p < 0.28$ for $S = 36$). Differences between own claims under the mechanism and own fair shares in the fairness question are also present for medium contributors. However, here the sign of the difference depends on the cake size, probably because the relative position of the medium contributor varies in cake size: For $S = 24$ medium contributors claim less than what they stated as own fair share (WSR: $p < 0.001$ and t-test: $p < 0.03$), for $S = 36$ they claim more (WSR: $p < 0.03$ and t-test: $p < 0.01$) and for $S = 60$ there is no significant difference between reported own claim and stated own fair share (WSR: $p = 0.91$ and t-test: $p = 0.35$). Interestingly, there is no difference between reported own claim and stated own fair share for high contributors independent of the cake size.

Mechanism Output: Since subjects (especially low contributors) tend to ask for more under the mechanism than what they reported as own fair share in the fairness question, and since we found a self-serving bias already in the fairness question, it seems clear that this mechanism will suffer from inefficiencies, in particular for small and medium cake sizes, where low contributors claim far more than they reported as own fair share. And this is indeed what we find: Mean payoffs are significantly lower than mean claims, as can be seen in Table 3, and this is true also for pairwise comparison on the individual level (MWU: $p < 0.01$ for all

	cake size	S=24		S=36			S=60	
	partner	A/B	C	A	B	C	A	B/C
	contribution	2	3	2	3	4	3	4
claims	mean m_i^i	9.06	9.50	11.02	14.21	16.00	19.18	22.54
	median m_i^i	8	9	10	12	15	18.75	21
payoffs	mean s_i	5.47	6.12	6.40	8.75	10.75	14.24	16.98
	mean s_i/m_i^i	0.70	0.63	0.66	0.70	0.68	0.78	0.77
fine		4.43		6.53			5.8	
efficiency		0.71		0.72			0.80	

Table 3: Modified Nash-Demand Rule: Claims, Fines and Payoffs

types and cake sizes). Payoffs are thus also lower than what subjects considered their own fair share in the fairness question, and furthermore, payoffs from this mechanism are also lower than fair shares from others as stated in the fairness question (WSR: $p < 0.001$ for all types and cake sizes). The efficiency of this mechanism, i.e. the share of S that is paid out, is rather low – it reaches values between 71% for the small cake size and 80% for the large cake size. While there are two sources of inefficiency, claiming too little and too much, the latter turns out to be the main culprit for the low payoffs. The sum of claims is lower than the available amount in less than 25% of cases for all cake sizes, while it is higher in more than 50%.[20] Overall, our results suggest that the self-serving bias in fairness evaluations already present in the fairness question is exacerbated by this mechanism: Subjects tend to report own claims that are higher than own fair shares in the fairness question, and this results in realized payoffs that are not only lower than reported own claims, but also lower than the stated own fair shares and even the fair shares from others in the fairness question.

4.2.2 Impartial Division Rule

The main characteristic of the mechanism proposed by DeClippel, Moulin and Tideman (2008) is its impartiality, which relies on two properties. First, each agent is asked to report only an evaluation of the other two partners' relative shares; that is, for $\{i, j, k\} = \{A, B, C\}$, each agent i is asked only for an $r_{jk}^i = m_j^i/m_k^i$. Secondly, the report of agent i has no effect on the payoff of agent i. With

[20]Efficiency is, on average, over 90% when the sum of claims is smaller than the cake size, while it is only between 53% and 72% when the sum of claims exceeds the cake size.

three partners, however, the rule distributes the entire cake only if the reported relative shares are consistent; otherwise it distributes strictly less.[21] It is therefore interesting to see how well this mechanism performs in practice.

Mechanism Input: Since only a report about the relative share of the two partners is required as input under this mechanism, we compare elicited relative shares to the respective ratios implied by subjects' answers to the fairness question (*observed fair ratio*) in Table 4. For the large cake size, mechanism inputs do not differ from observed fair ratios. They are also consistent with proportionality. Together these results seem to be in conflict with our earlier finding that the answers to the fairness question are to a large extent inconsistent with proportionality. However, this is not the case, since here we refer only to a *relative* comparison of the partners' shares and thereby ignore the self-serving bias in the fairness evaluation and its consequence of leaving less for others when claiming more for oneself. The proportional standard then seems to predict the report of others' relative shares quite well, which is the input required by this mechanism. For the medium cake size, note that the median report r^i_{jk} for each of the three roles corresponds precisely to the proportional standard. Looking at the whole distribution, we find no significant differences between the observed reports and the ratios derived from the proportional standard for subjects in the roles of partners B (medium contributor) and C (high contributor). Only the low contributor assigns less to the high contributor than what his relative payoff according to the proportional standard would be (i.e. $r^A_{BC} > 0.75$, WSR: $p < 0.01$ and t-test: $p < 0.05$). This is also true for the small cake size: The reported ratios of the low contributors A and B differ from the proportional standard ($r^A_{BC} = 2/3$ and $r^B_{AC} = 2/3$ are rejected, WSR and Sign test: $p < 0.01$), they assign a smaller relative payoff to C compared to his relative contribution in points.[22] The deviation from the contribution ratios is again at the expense of the relatively higher contributor.

Overall, subjects' evaluations of others' claims show that there is consensus on equal shares for equal contributions (87% in $S = 60$ and 94% in $S = 24$ assign ratio $1 : 1$ when partners have the same contribution), while with different con-

[21] The reports of the three partners are consistent iff $r^C_{AB} = r^B_{AC}/r^A_{BC}$.

[22] Two outliers of r^B_{AC}=50 are excluded from reported descriptive statistics for $S = 24$ in Table 4. Including them would yield a mean reported ratio of 1.4 for the low contributors and give a distorted picture of the results.

tributions, the proportional standard seems to guide most subjects' evaluations. Only low contributors deviate systematically from this rule, reporting ratios to the disadvantage of higher contributors. A similar result is found when comparing the relative evaluations of the partners' claims for this mechanism to those reported in the fairness question: they differ significantly only when low contributors are involved in the comparison. Deviations of the mechanism reports from the stated fair ratios are systematically favoring the low contributor. Not only do low contributors themselves deviate from their stated fair ratios, but they are also assigned more from other types (e.g. from partners B and C in $S = 36$) compared to the observed fair ratio in the fairness question.

	cake size	**S=24**		**S=36**			**S=60**	
	partner	A/B	C	A	B	C	A	B/C
	contribution	2	3	2	3	4	3	4
observed r^i_{jk}:	mean	0.86	0.99	0.80	0.48	0.66	1.09	0.75
	median	0.67	1	0.75	0.5	0.67	1	0.75
observed fair ratio		0.84	1.00	0.83	0.60	0.71	1.0	0.76
proportional		0.67	1	0.75	0.5	0.67	1	0.75
realized s^i_{jk}:		0.84	1.00	0.80	0.51	0.62	1.06	0.75
efficiency		0.98		0.99			0.99	

i's report r^i_{jk} is represented as r^A_{BC}, r^B_{AC}, r^C_{AB}, and similarly, s^i_{jk} as s^A_{BC},s^B_{AC}, s^C_{AB}

Table 4: Impartial Division Rule: Reports and Payoffs Compared to Fairness Standards

Mechanism Output: As Table 4 shows, realized payoff ratios under this mechanism are rather close to reported input ratios (the ratio of j's to k's payoff share, s_j/s_k, is displayed in partner i's cell of the respective cake size): The WSR cannot reject that the mechanism implements results that are not significantly different from subjects' reports for all cake sizes. As a result, the comparisons above between mechanism inputs and proportional standard also hold for realized payoffs. Regarding the comparison between realized payoff ratios and observed fair ratios in the fairness question, the mechanism yields even better results: There are no significant differences for the large and small cake size; for the medium cake size, low contributors are disadvantaged compared to the observed fair ratios in the fairness question. The mechanism thus seems to implement payoff ratios that are consistent with subjects' fairness ideas for most types. Only high contributors receive larger relative shares than in the fairness question, but the spread of the distribution is still smaller than what the proportional standard would suggest.

Figure 1 compares the absolute size of realized payoffs to the respective assignments from the fairness question. This mechanism implements absolute payoffs that are between the own fair share and the fair share from others in the fairness question. This seems justified, considering the self-serving bias in the answers to the fairness question. A mechanism that performs well in a subjective claims problem needs to take such a bias into account. This mechanism gives good incentives to report true evaluations of others – assuming that those elicited in the fairness question are true. This and the property that it avoids self-evaluations seem to be the main strengths of the mechanism. A possible weakness is the fairly intransparent payoff rule which does not easily reveal the consequences of changing the own input report on assigned shares.

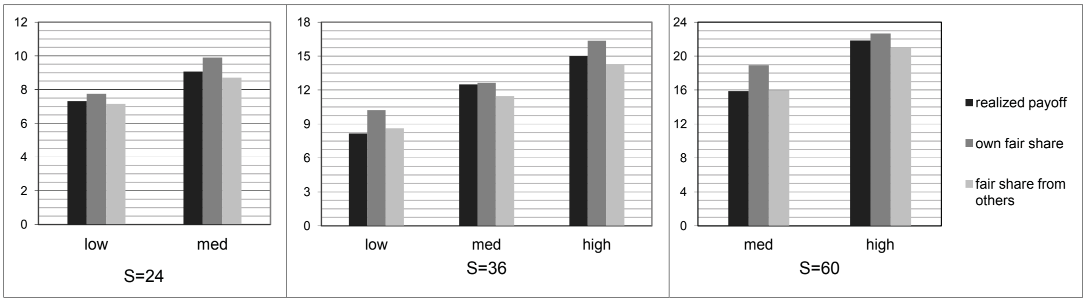

Figure 1: Fairness and Impartial Division Mechanism

4.2.3 Extended Divide-the-Dollar Rule

In the mechanism proposed by Brams and Taylor, each partner i is asked to report an $m^i = (m^i_A, m^i_B, m^i_C)$. Since all partners are paid out their claims in ascending order of their greed levels, the mechanism incorporates an incentive to include the partners' point of view in deciding which own claim to report in order to keep the own greed level low. Table 5 displays own claims, entitlements, greed levels and payoffs for this mechanism, as well as the corresponding reports from the fairness question as comparison.[23] Note that this mechanism is particularly well-suited for a comparison with the results from the fairness questions, as they both ask for precisely the same information.

cake size	**S=24**		**S=36**			**S=60**	
partner	A /B	C	A	B	C	A	B/C
contribution	2	3	2	3	4	3	4
own claim	8.49	9.8	11.06	12.77	15.22	19.30	22.15
entitlement	7.08	8.4	9.09	11.83	13.55	17.21	20.49
greed level	1.42	1.41	1.98	0.95	1.67	2.08	1.66
own fair share	8.04	9.90	10.22	12.64	16.36	18.91	22.65
fair share from others	7.14	8.72	8.61	11.48	14.30	16.00	20.95
payoff	7.31	8.95	9.25	12.07	13.84	16.93	21.05
efficiency	0.98			0.97		0.98	

Table 5: Extended Divide-the-Dollar Rule: Reports and Payoffs Compared to Fairness Question Results

Mechanism Input: Subjects' own claims differ significantly from their entitlements (WSR: $p < 0.01$ for each comparison), which implies a greed level different from zero for all types and cake sizes. Overall, greed levels do not depend on subjects' contributions towards the cake (WSR: $p = 0.49$ for the comparison between low vs. medium contributor in $S = 24$; $p = 0.7$ for low vs. high contributor in $S = 36$; and $p = 0.8$ for medium vs. high contributor in $S = 60$). Only exception is the medium contributor in $S = 36$: his greed level is significantly lower than the low contributor's (WSR: $p < 0.02$ and t-test: $p < 0.03$) and also lower than the high contributor's (WSR: $p = 0.12$ and t-test: $p < 0.09$). This is probably due to the fact that only for this type all fairness standards discussed above point towards the same allocation of 12. In the experiment, 2/3 of all subjects in this role claimed precisely this amount, and also 2/3 of all subjects in this role were assigned an entitlement of 12 or more from their partners, which explains the low greed level.

Recall that two factors affect greed levels and therefore also realized payoffs: own claims and entitlements. Observing subjects report own claims different from own fair shares in the fairness question, or report assignments to others different

[23] Remember that the 'entitlement' of partner i is the average of the two partners' evaluations of i's claim.

from those reported as others' fair shares in the fairness question, could potentially be interpreted as strategic behavior. For instance, by reporting a low claim for a partner, his entitlement decreases, which increases his greed – holding everything else constant. This, in turn, makes it more likely that the subject's own greed is comparatively lower, and since partners with lower greed levels get paid off first, the subject's own claim is more likely to be fully paid off. If, on the other hand, a subject reduces his own claim, this increases the chance of being fully paid off (since the own greed level decreases), however, at the cost of receiving a lower amount when the subject gets paid off (since payoffs correspond at most to own claims).

We first compare reported own claims under this mechanism to reported own fair shares in the fairness question. They do not differ for low contributors in the small cake size and for medium contributors in all cake sizes. This suggests that low and medium contributors tend to submit truthful reports of their evaluation of own claims. High contributors, on the other hand, systematically claim less than their stated own fair share (WSR: $p < 0.05$ and t-test: $p < 0.03$ for $S = 36$; WSR: $p < 0.01$ and t-test: $p < 0.01$ for $S = 60$), which can be interpreted as an attempt to lower their greed levels. But, we also find that low contributors in the medium cake size claim more than they stated as own fair share (WSR: $p < 0.05$ and t-test: $p < 0.03$), which would – ceteris paribus – imply higher greed levels. Knowing that there was a significant self-serving bias in the fairness question, one might expect that reporting even higher own claims would further increase these subjects' greed. However, payoffs finally depend on relative greed levels. We thus need to check whether low contributors systematically assign smaller entitlements to a certain type compared to what they assigned to them as fair share. This would mean that they try to compensate high reports of their own claim by actively increasing some other partner's greed level. In fact, low contributors (partner A) in the medium cake size do precisely that – they assign slightly more to partner B, and considerably less to C (t-test: $p < 0.001$) compared to what they stated as fair for each of these partners. Note that the disadvantaged partner C is the high contributor, who would take away a large share of the cake in case he gets paid off first. In comparison, the high contributor C has some extra amount to allocate between his partners compared to the fairness question, since his own claim under this mechanism is lower than his stated own fair share in the fairness question.

C allocates this extra amount by reporting, on average, 0.4 more for the medium contributor compared to what he assigned to B in the fairness question, and 0.8 more to the low contributor. This might be interpreted as a strategic move, too, as C would prefer B to be the last one being paid off, since B takes away more than A from the cake. Consistent with this strategic reasoning is also what we find for $S = 60$: Both high contributors report, on average, 0.7 more for the low contributor than for their fellow high-contribution partner (t-test: $p < 0.03$ for B, and $p < 0.1$ for C).

Overall, we find that for each cake size, the partner with the highest contribution in a given partnership is assigned a smaller entitlement compared to the fair share from others in the fairness question (WSR: $p < 0.01$, t-test: $p < 0.01$ for $S = 36$ for C; WSR: $p < 0.07$ and t-test: $p < 0.03$ for $S = 24$ for C; and WSR: $p < 0.03$ and t-test: $p < 0.01$ for pooled types B and C in $S = 60$). The high contributor thus gets systematically disadvantaged by a lower entitlement. Lower contributors, on the other hand, receive entitlements at least as high as the fair shares from others in the fairness question, which is consistent with the strategic incentive of lowering high contributors' entitlements: Since each subject has to report shares that add up to the full amount of the cake, and most subjects preferred not to report a higher own claim, this leaves some extra amount (compared to the fairness allocation) to be allocated. Assigning this to low contributors does increase the chances that those advance in the order partners are paid off, but they take away a smaller share of the cake than anyone else would. Therefore, reducing the entitlements of higher contributors and increasing those of lower contributors serves the same purpose.

Mechanism Output: As can be seen in Table 5, final payoffs in this mechanism are always smaller than own claims (WSR: $p < 0.01$ for all types and cake sizes). This is due to the observed self-serving bias in own claims and its consequence that not all claims can be paid off when the sum of own claims exceeds the total amount available. Only for subjects in the role of partner B in the medium cake size this difference is small, as 82 out of 90 subjects receive the full own claim (recall that they had lower greed levels than other types). The high own claims also imply that most of the time the full cake size is allocated, which can be seen in the high efficiency of this mechanism: for all cake sizes, 97% or more of the available amount is paid off.

Regarding the mechanism's performance, we are interested in a mechanism providing an allocation that is not only efficient but also considered fair by the partners. The discussion of mechanism inputs above revealed that neither own claims nor entitlements differ significantly for partners with same contribution levels within a partnership, thus we would expect that the mechanism implements similar payoffs for these partners. This is indeed true for low contributors in $S = 24$ (MWU: $p > 0.6$), as well as for high contributors in $S = 60$ (MWU: $p > 0.3$). Since we found that high contributors in each partnership are systematically disadvantaged by partners' reports, we check whether this is reflected in the payoffs. To do so, we compare – in Figure 2 – by how much realized payoffs differ from own fair shares as well as from fair shares from others as reported in the fairness question.[24] Comparing each type's payoff to the own fair share in the fairness question, it is clear that the former must be lower than the latter for all types except the medium contributor in $S = 36$. This follows from our earlier finding that own fair shares are at least as high as own claims in the mechanism for all but one type, and we already saw above that payoffs were significantly lower than own claims for all types and cake sizes. Only the low contributor's claim in $S = 36$ was above his stated own fair share, thus only this type could have received a payoff that corresponds to his stated own fair share. However, even for this type, the difference between the realized payoff of 9.2 and the reported own fair share of 10.2 is significant (WSR: $p < 0.03$ and t-test: $p < 0.01$).

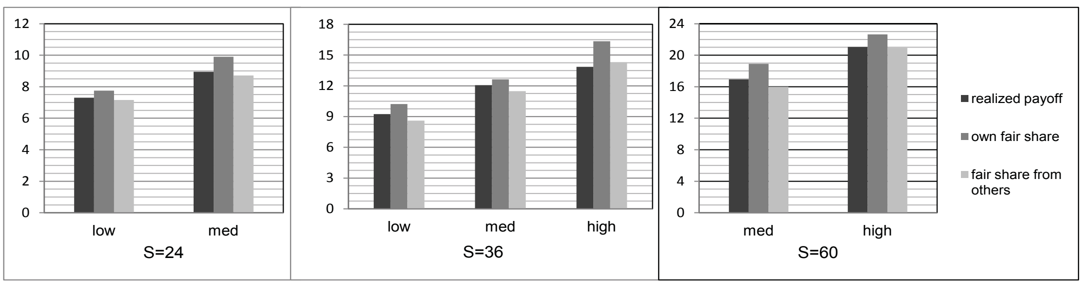

Figure 2: Fairness and Extended Divide-the-Dollar Mechanism

Comparing each type's payoff to the fair share from others in the fairness ques-

[24]One could also think of comparing realized payoffs to own claims as reported under this mechanism to evaluate the fairness of the implemented allocation. However, since high contributors lowered their claims compared to what they stated in the fairness question, this comparison would not reflect subjects' ideas of fairness.

tion, we find that all types in the small cake size get a slightly higher average payoff than what others think is fair, but the difference is not significant. For $S = 36$, the low and medium contributor get a significantly higher payoff than the fair share from others (t-test: $p < 0.01$); the high contributor's mean payoff is smaller, but the difference is not significant (WSR: $p = 0.17$ and t-test: $p = 0.11$). For $S = 60$, A gets significantly more (t-test: $p < 0.01$), while B and C receive about what others thought was fair for them. Overall, we see that high contributors' disadvantage due to lower entitlements and lower own claims carries over to their payoffs in this mechanism – they are the only ones who systematically receive an amount comparable to what others think is fair for them, while others receive more. Recall that the self-serving bias in the answers to the fairness question implies that summing up the fair shares from others would not add up to the full cake size, since high own fair shares leave less for others. Therefore, receiving a payoff which does not differ from the fair share from others means that another partner must have received more than his fair share from others, which then distorts overall fairness.

Finally, with regard to the discussed fairness standards, all types in all cake sizes receive a payoff that is significantly different from what the egalitarian and proportional standard would imply. Unique exception is again the medium contributor in $S = 36$ whose payoff is not significantly different from 12 (which, as already discussed, also corresponds to the payoff implied by other distribution standards).

4.3 Comparison across Mechanisms

In this subsection we compare the three mechanisms in terms of subjects' ex-post evaluation of the procedural and allocative fairness. Before knowing their payoffs, subjects were asked to rank the mechanisms in terms of procedural fairness. Figure 3 shows the mean rank that each contribution type in a given cake size assigns to each mechanism for its procedural fairness: The Modified Nash-Demand Rule is ranked worse than the others in most cases, while there is mostly no significant difference in the mean rank between the Impartial Division Rule and the Extended Divide-the-Dollar Rule. Indicating the significance level of 5% for a Wilcoxon signed-rank test by ** and the level of 1% by ***, the following comparisons hold: Extended Divide-the-Dollar Rule = Impartial Division Rule $\succ^{***}$ Modified Nash-Demand Rule for all contribution types and all cake sizes except for the medium contributor in the large cake size and the low contributor in the medium cake size, for whom we have Extended Divide-the-Dollar Rule $\succ^{**}$ Impartial Division Rule = Modified Nash-Demand Rule. Thus, the Extended Divide-the-Dollar Rule overall is seen as best with respect to procedural fairness by subjects when they do not know which division allocations the rules finally implement.

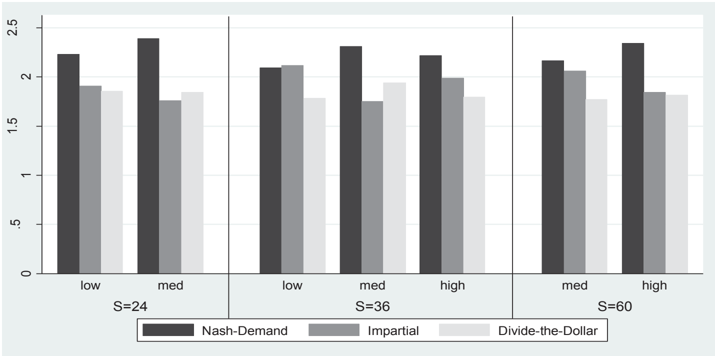

Figure 3: Mean Ranks of Mechanisms' Procedural Fairness by Cake Size

After being informed about their payoff from each mechanism, subjects were asked to rank the mechanisms according to the fairness of outcomes. Figure 4 displays the results of subjects' view on the allocative fairness of the three mechanisms. Here we see that the Modified Nash-Demand Rule again is ranked last for all comparisons while the preference over the Impartial Division Rule versus the Extended Divide-the-Dollar Rule depends on subjects' contribution type. The following comparisons hold: Extended Divide-the-Dollar Rule = Impartial Division Rule $\succ^{***}$ Modified Nash-Demand Rule for all types in S=24, for the high and medium contributor in S=36 and for the medium contributor in S=60. For the high contributor in the large cake size we find Impartial Division Rule $\succ^{***}$ Extended Divide-the-Dollar Rule $\succ^{***}$ Modified Nash-Demand Rule, while for the low contributor in S=36 we find Extended Divide-the-Dollar Rule $\succ^{***}$ Impartial Division Rule = Modified Nash-Demand Rule. The Impartial Division Rule might have looked slightly less appealing when judged only in terms of procedural fairness as one cannot state own claims, and only relative evaluations of others are possible. However, after seeing the allocation this mechanism produces, it moves upwards in the ranking of agents in the large cake size, while for all other types its ranking does not change. The Extended Divide-the-Dollar Rule probably has a more intuitive procedure, but its potential for strategic behavior, which harms high contributors in our experiment, makes its allocations less fair for these partners in the end.

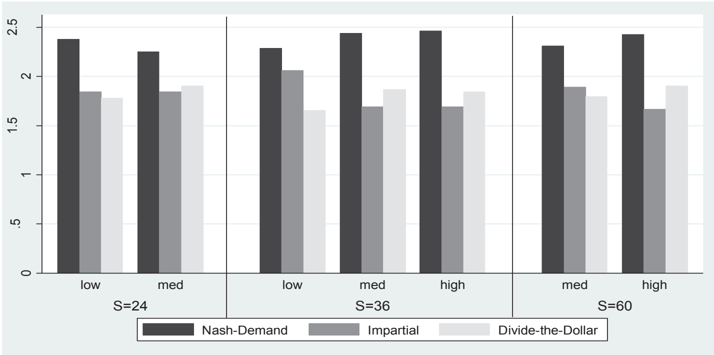

Figure 4: Mean Ranks of Mechanisms' Allocative Fairness by Cake Size

5 Conclusion

Three cake-division mechanisms have been compared experimentally in terms of fairness and efficiency in a subjective claims problem. All mechanisms use reported subjective evaluations of claims as input and yield unique shares summing up to no more than the value of the cake as output. The mechanisms differ in the amount of information they process: The Modified Nash-Demand Rule (Mumy 1981) only requires each partner's own claim as input, the Impartial Division Rule (DeClippel, Moulin and Tideman 2008) asks for each agent's assessment of her partners' relative claims, and the Extended Divide-the-Dollar Rule (Brams and Taylor 1994) requires each partner's assessment of her own as well as the partners' claims.

We found that the Modified Nash-Demand Rule performs rather badly, since the considerable self-serving bias, i.e. the difference between what subjects consider fair for themselves and what others think is their fair share, implies that not the entire amount available is paid out. This is mainly due to the mechanism's built-in fine that is intended to deter agents from reporting too high own claims. The other two mechanisms perform significantly better in terms of efficiency, as at most 2-3% of the cake is not allocated. They also rank high in terms of perceived procedural and allocative fairness. The relative ranking of these two mechanisms depends on the relative position of the evaluating partner in terms of contribution to the cake. While low contributors typically prefer the Extended Divide-the-Dollar Rule, high contributors tend to prefer the Impartial Division Rule. This is consistent with our finding that low contributors are systematically advantaged and high contributors disadvantaged by the Extended Divide-the-Dollar Rule.

Overall, we think that the Impartial Division Rule in particular deserves high attention in this type of division problem for two reasons: First, because no partner is systematically disadvantaged by this mechanism, and second, because its property of impartiality (own shares are only determined by others' reports, and one can only report evaluations of others) and the relative (rather than absolute) evaluation it requires make it less susceptible to the self-serving bias and its consequence of large differences between what one thinks one deserves and what one finally receives.

The ubiquitous self-serving bias also explains the findings of our companion study on the performance of three unanimity bargaining procedures to resolve the same subjective claims problem: A procedure that only requires agents to state their own demand has a disadvantage for the last mover, as little is left for him after the other two players make their demands. Bargaining procedures which force agents to consider the entire vector of claims turn out to be closer to subjects' own fairness assessments, and also to assessments of impartial spectators expressed in a vignette.

While bargaining may seem a natural way to resolve the subjective claims problem, our companion paper shows that all bargaining protocols considered lead to inefficiencies. By contrast, as the present paper has shown, a carefully chosen static mechanism implements a fair allocation at almost no efficiency loss. The fact that a well designed static mechanism –such as the Impartial Devision Rule– outperforms the best bargaining protocol in our comparison in terms of effciency is a remarkable result, especially in light of the fact that our experiments involve only three players. Indeed, we would expect the relative advantage of static over dynamic procedures to increase in the number of players.[25] Since the high-stake

[25]See e.g. Chatterjee and Sabourian (2000), and Cai (2000) for theoretical work, or Cadigan et al. (2009) for an experimental study on the so-called holdout problem, a situation in which the required agreement by multiple parties creates an incentive for individual strategic delay in

environments in the motivating examples typically involve at least three parties, and since in these examples each percentage point of difference in efficiency corresponds to a large dollar amount, investigating how the relative performance of static over dynamic procedures changes in the number of players seems a fruitful area for future research.

order to capture a larger share of the surplus.

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Fairness and efficiency in a subjective claims problem

Abstract
In a subjective claims problem several agents have contributed to the production of a cake which is to be divided among them. Since contributions are difficult to compare and the production function is nonlinear, agents'subjective evaluations of claims are likely to be conflicting. In a large-scale experimental study we compare the performance of three mechanisms which use agents' reports to resolve the subjective claims problem. The mechanisms differ with respect to the information they process, and they are compared in terms of efficiency and perceived allocative and procedural fairness.

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