Monopolistic Competition and Macroeconomic Dynamics

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Abstract

Modern macroeconomic models with a Keynesian flavor usually involve nominal rigidities in wages and commodity prices. A typical model is static and combines wage bargaining in the labor markets and monopolistic competition in the commodity markets. As central policy implication follows that deregulating labor and/or commodity markets increases equilibrium employment.

We reassess the consequences of deregulation in a dynamic model. It still increases employment at the fixed point, which corresponds to the static equilibrium solution. However, the fixed point loses stability through a Flip-bifurcation with stronger deregulation; numerical simulations illustrate that deregulation may even reduce average employment.

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1. Introduction

Modern macroeconomic models with a Keynesian flavor usually involve nominal rigidities in wages and/or commodity prices (see Gordon, 1990). A typical microfoundation for the latter recurs to the Dixit-Stiglitz model of monopolistic competition (Dixit and Stiglitz, 1977). Keynesian features studied in such models are the possibility of unemployment (e.g. d’Aspremont et al., 1990) and multiple equilibria (e.g. d’Aspremont et al., 1995, Linnemann, 2001) and that fiscal policy has income multiplier effects (Heijdra and Ligthart, 1997; Heijdra et al., 1998; see also Solow, 1998). Blanchard and Kiyotaki (1987) proposed a prototype macro-model combining monopolistic competition in the commodity markets with labor market imperfections that has now become a standard framework for analyzing unemployment problems. The widely used approach of specifying a wage setting and a price setting equation (see e.g. Nickel, 1990; Layard et al., 1991; or Blanchard, 2003; and for empirical applications various analyses by the European Commission, e.g. McMorrow and Roeger, 2000; Bains et al., 2002) ultimately rests upon such micro foundations. Blanchard and Giavazzi (2001) recently presented a structural model along these lines combing monopolistic competition with wage bargaining (similar also Jerger, 2001, in an open economy context). A characteristic result in those analyses is that deregulating the labor markets (i.e. reducing the bargaining power of workers and/or reducing the unemployment benefits) and/or deregulating the commodity markets (i.e. 

\[ \text{[Equation]} \]

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reducing the market power of commodity suppliers) increases equilibrium employment (see also Gersbach, 1999 and 2000).

However, those models are typically static models, which do not specify explicitly the economic process in time. In the following paper, we develop a dynamic macroeconomic model in which commodity markets are characterized by monopolistic competition and labor markets by wage bargaining.

In our analysis, the usual equilibrium solution is a fixed point of the dynamic model, which exhibits the usual comparative static properties (deregulating the labor and/or the commodity market increases employment). However, depending upon the parameters the fixed point may lose stability through a Flip-bifurcation giving rise to cyclical solutions and endogenous fluctuations. We show analytically that commodity and labor market deregulation may lead to instability; in numerical simulations, we even found cases in which deregulation leads to lower average employment. Both results, valid in a dynamic framework, contrast with the usual comparative static properties.

The paper is organized as follows: In the second section, we present our model; its basic structure closely follows Blanchard and Giavazzi (2001). Monopolistically competitive firms bargain with unions over employment and nominal wage rates on the basis of firms’ anticipated demand functions and workers’ nominal reservation wages. In the third section, we explicitly add two dynamic components. First, central to monopolistic competition is that firms neglect the reactions of other firms. They therefore base their decisions on a too high anticipated price elasticity; outside equilibrium, they are necessarily surprised by the market results. As a consequence, they will adjust their anticipated demand function over
time\textsuperscript{2}. Second, the nominal reservation wage also adjusts through time on the basis of the realized employment and the realized nominal wage rate. Whereas for the second process several plausible specifications do exist, the specification of the first one is directly implied by the external effect at the core of the monopolistic competition model itself. We show that the first process leads to diverging time paths: while it still holds that deregulation increases stationary employment, it leads to fundamental instability. In the fourth section, we study two different specifications for the reservation wage adjustment, each of which mirrors a different institutional set up. We show analytically that this second process may dampen the sharp instability result: deregulation may still destabilize the economy; however, the time path does not diverge but is attracted to a period-two cycle or eventually to a complex time path. Some simulations complete the dynamic explorations. The last section is left for concluding remarks.

2. The economic framework

The analysis, being framed in a dynamic setting, requires dealing explicitly with time. We assume that during the time unit, which for expository purposes we call ‘Week $t$’, all events occur according to a well-defined sequence. On Monday of Week $t$, firm $i$ and the associated union bargain over employment and the nominal wage rate on the basis of firms’ anticipated demand and workers’ nominal reservation wage. Production occurs during the

\textsuperscript{2} See Currie and Kubin (2002) for an exploration of the monopolistic competition dynamics. D’Aspremont et al. (1995) analyze endogenous fluctuations in what they call a Cournotian monopolistic competition model. In contrast to our approach, the dynamics in their model is driven by an overlapping generations structure and variable mark-ups are central for the occurrence of endogenous fluctuations.
Week from Tuesday to Friday. On Friday, commodities are delivered to the market. Market
equilibrium determines the realized price. Finally, on Saturday firms and unions update the
information relevant for decisions in the following period concerning the employment, the
nominal wage rate and the realized price.

2.1 Firms and Households

The economy comprises a fixed number \( m \) of firms, each firm producing one differentiated
good in a regime of monopolistic competition; and \( L \) households, which own the existing
firms, each household supplying one unit of labor. Each firm faces a union composed of
\( M_i = \frac{L}{m} \) members.

All firms share the same production technology, which involves only one input, labor, used
in a fixed proportion. Assuming that during Week \( t \) firm \( i \) employs \( N_i' \) workers, the
production of good \( i \) is:

\[
x_i' = f(N_i') = N_i'
\]

for \( i = 1 \ldots m \).

Following Blanchard and Giavazzi (2001), household \( j \)'s preferences towards good \( i \) (for
\( i = 1 \ldots m \) and \( j = 1 \ldots L \)) are represented by a CES utility function:

\[
U_j' = \left( \frac{m^{-\frac{1}{\sigma}} \sum_{i=1}^{m} (c_i')^{\frac{\sigma-1}{\sigma}}}{m^{-\frac{1}{\sigma}}} \right)^{\frac{\sigma}{\sigma-1}}
\]

where \( 1 < \sigma < \infty \) is the elasticity of substitution between goods and \( c_i' \) is the consumption
level of good \( i \) for the period. Note that the inclusion of the factor \( m^{-\frac{1}{\sigma}} \) into the utility
function implies that the utility level depends only upon the overall consumption quantity
and not on the number of different consumption goods. This specification, therefore, does
not represent a preference for commodity variety and \( \sigma \) is a parameter related only to the market power of a single commodity supplier (see Blanchard and Kiyotaki, 1987; and also the discussion in Benassy, 1996). In Blanchard and Giavazzi (2001), it is the central parameter for studying the effects of commodity market deregulation.

Household \( j \)'s demand for good \( i \) is given by

\[
d_j' = \frac{Y_j}{mP_i}\left(\frac{p_{i}}{P_i}\right)^\sigma
\]

where \( Y_j \) is the household \( j \)'s nominal income and where

\[
P_t = \left( \frac{1}{m} \sum_{i=1}^{m} (p_i)^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \tag{3}
\]

is a price index corresponding to the price of one unit of utility. Summing through \( j \), we obtain the demand for commodity \( i \) during Week \( t \)

\[
d_t = \frac{Y}{mP_i}\left(\frac{p_{i}}{P_i}\right)^\sigma \tag{4}
\]

where \( Y \) is the overall nominal income of the economy. We neglect the Price Index Effect and the Ford Effect (see Yang and Heijdra, 1993; Dixit and Stiglitz, 1993; and d'Aspremont et al., 1990, 1996); the elasticity of demand with respect to the own price therefore reduces to \( -\sigma \). \(^3\)

\(^3\) Linnemeier (2001) showed that both effects are at the root of a possible multiplicity of equilibria. We prefer to demonstrate our results in the simplest possible framework with a unique equilibrium.
If the price of each good is the same, \( p'_i = p_i \), the price index becomes \( P'_i = p_i \) and the demand for an individual good simplifies to

\[
d_i = \frac{Y}{mp_i}
\]

(5)

In that case, the elasticity of demand with respect to the price is equal to \(-1\).

Moving on to the description of the events occurring during ‘Week \( t \)’, most of our discussion will be devoted to the crucial ones, that is, the bargaining process and the determination of the market equilibrium.

2.2 Bargaining

We employ the efficient bargaining model (McDonald and Solow, 1983; Layard and Nickell, 1990): During the Monday of Week \( t \) the parties (firm \( i \) and the associated union) bargain over employment and the nominal wage rate for the current period on the basis of a known set of data relating to workers’ reservation wage and firm \( i \)’s anticipated demand function. The solution maximizes the following Nash product

\[
\left( Z'_i - \tilde{Z}'_i \right)^{\beta} \left( \Pi'_i - \tilde{\Pi}'_i \right)^{1-\beta}
\]

(6)

where \( 0 < \beta < 1 \) represents the relative bargaining power of the unions, \( Z'_i \) denotes the value of the union’s objective function if an agreement is reached and \( \Pi'_i \) the corresponding value for the firm. \( \tilde{Z}'_i \) and \( \tilde{\Pi}'_i \) are the respective values, if no contract is concluded. We assume a utilitarian trade union. Observing that the CES indirect utility function is given by the nominal income deflated by an expected price index \( \tilde{P}'_i \), the value of the unions objective function is given by

\[
Z'_i = \frac{N'_i W'_i + \left( M'_i - N'_i \right) W_k'}{\tilde{P}'_i}
\]

(7)
where \( N'_t \) denotes employment, \( W'_t \) the nominal wage rate and \( W'_k \) the nominal reservation wage, which indicates the income opportunities expected to prevail outside the firm under consideration. With \( \hat{Z}'_t = \frac{M_k W'_k}{P_t} \), the union’s contribution to the Nash bargain is

\[
Z'_t - \hat{Z}'_t = \frac{N'_t}{P_t} (W'_t - W'_k)
\] (8)

Turning now to the firm’s contribution, we note that all profits are distributed to the owners’ households. Therefore, the firm’s objective is also defined in terms of the CES indirect utility function. With \( \Pi = 0 \) and using equation (1), it is thus given by

\[
\Pi'_t - \Pi'_t = \frac{p'_i x'_i - W'_t N'_t}{P_t} = \frac{N'_t}{P_t} (\hat{p}' - W'_t)
\] (9)

where \( \hat{p}'_i \) denotes the anticipated price for commodity \( i \). The framework of the Dixit-Stiglitz model of monopolistic competition suggests that the anticipated price is determined along the following lines. Typically, each firm assumes that other firms will not react to its own price decisions and that – when neglecting the Price Index effect and the Ford effect – the price elasticity of their demand function is constant and equal to \(-\sigma \). Therefore, a firm that knows it sold in the previous period a quantity \( \hat{d}_{t-1} \) at a price \( \hat{p}_{t-1} \) will anticipate its demand function for the current period to be given by:

\[
\tilde{d}_i = \hat{d}_{t-1} \left( \frac{\hat{p}_{t-1}}{\hat{p}'_i} \right)^\sigma = K'_i \left( \frac{\hat{p}'_i}{\hat{p}'_i} \right)^\sigma
\] (10)
where $K'_i$ is the position of the anticipated demand function.\(^4\)

Using equations (1) and (10) we obtain the anticipated price $\tilde{p}'_i$ as

$$\tilde{p}'_i = \left( \frac{K}{N'_i} \right)^{\frac{1}{\beta}}$$

(11)

We assume that this is common knowledge to both bargaining parties.

Efficient (Nash) bargaining between firm’s $i$ and the associated union corresponds to choosing $N'_i$ and $W'_i$ so as to maximize the following expression subject to (11):

$$\left( \frac{Z'_i - \tilde{Z}'_i}{\beta} \right) \left( \frac{\Pi'_i - \Pi\tilde{'}_i}{\beta} \right)^{-\beta} = \left( \frac{W'_i - \tilde{W}'_i}{\beta} \right) \left( \frac{\tilde{p}'_i - W'_i}{\beta} \right)^{-\beta} \frac{N'_i}{P_t}$$

(12)

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(12)

Observing that – consonant with the monopolistic competition set-up – the parties assume the expected price index $\tilde{P}_i$ to be independent of their decisions, the bargaining results in:

$$W'_i = \left( \frac{\sigma - 1 + \beta}{\sigma - 1} \right) W'_r$$

(13)

$$\tilde{MR} = \frac{\sigma - 1}{\sigma} \tilde{p}'_i = W'_r$$

(14)

$$N'_i = K_i \left[ \left( \frac{\sigma}{\sigma - 1} \right) W'_r \right]^{-\sigma}$$

(15)

Note the familiar result from the efficient bargaining problem, namely that a position is chosen off the profit maximum. In equation (14), the marginal revenue is equated to the

\(^4\) In other words: Firms assume that they can sell the same quantity at the same price as in the previous period and they assume that their quantity will react to a price change according to an elasticity of $-\sigma$. 9
nominal reservation wage and not to the marginal cost (corresponding to the nominal wage rate). It follows that the price is set as a mark-up over the reservation wage, \( \bar{p}_i = (1 + \mu)W^i_R \), where \( \mu \equiv 1/(\sigma - 1) \).

Following Blanchard and Giavazzi (2001), we interpret a decrease in the mark-up \( \mu \) (i.e. an increase in \( \sigma \)) as commodity market deregulation. It reduces the size of firm \( i \)'s anticipated surplus per unit of output, \( (\bar{p}_i - W^i_R) = \mu W^i_R \). Labor market deregulation is reflected in Blanchard and Giavazzi (2001) by a decrease in the relative bargaining power of the union, \( \beta \). As equation (13) shows, \( \beta \) determines the distribution of the anticipated surplus between wages and profits. Decreasing \( \beta \) reduces the workers’ share in the surplus, \( \beta \mu W^i_R \). In addition, our model, as developed in the following sections, allows for labor market deregulation in the form of changing the unemployment insurance scheme. This form of labor market deregulation may also impinge on the bargaining outcome (and on the size of the surplus) through its effects on the nominal reservation wage.

Note finally that equations (14) and (15) are not independent results, equation (15) can be obtained from equation (14) by taking constraint (11) into account.

During Week \( t \), at the end of contracting, firm \( i \) employs \( N^i_t \) workers, pays the nominal wage rate \( W^i_t \) and anticipates the price \( \bar{p}_t^i \). Figure 1 depicts the bargaining equilibrium. Given the reservation wage \( W^i_R \) and the position of the anticipated demand function \( K_t^i \), the bargain determines the employment level \( N^i_t \) and the wage rate \( W^i_t \) in firm \( i \). Each firm expects to sell its output at an anticipated price \( \bar{p}_t^i \).
2.3 Short term commodity markets equilibrium

The characteristics of the commodity markets equilibrium follow from the assumption that all firms behave identically – what Blanchard and Giavazzi (2001) call the symmetry assumption. Each firm pays the same wage $W'_i=W'_t$, hires the same number of workers, $N'_i=N_t$, and produce the same quantity $x'_i=x_t$, which is equal to their respective supplies $s'_i=s_t=x_t$. They envisage to sell these quantities at an identical anticipated price $\hat{p}'_i=\hat{p}'$. At the end of the production period, on Friday, each firm supplies $s_t$ to the (identical) true demand function $d_t$ (see Eq. (5)). In contrast to the anticipated demand functions, the true demand functions take into account the reactions of all other firms; each of them has an elasticity of $-1$ (instead of $-\sigma$). Therefore, producers cannot realize their price and quantity anticipations. In the following we assume that firms sell the entire quantity produced at the market-clearing price $\hat{p}_t$ (different from the anticipated price)\(^5\). Using (5), the realized price is determined as:

$$\hat{d}_i = s_i = x_i = N_i \quad \hat{p}_t = \frac{Y}{mx_i} = \frac{Y}{mN_i} \quad (16)$$

Figure 2 depicts the short run commodity markets equilibrium.

\(^5\) In principle, producers as monopolists can also decide to sell a smaller quantity than the produced one when entering the market. However, they do not have an incentive to do so: On the basis of an elasticity of demand greater (anticipated demand) or equal to 1 (true demand) in absolute terms, revenues do not increase with a quantity restriction. In addition, at that moment, production costs are already sunk and maximizing revenues also maximizes profits. Therefore, quantity restrictions cannot increase profits. Currie and Kubin (2002) also explore an alternative quantity rationing scheme.
At that point in the paper, it might be worthwhile to trace explicitly the money circuit corresponding to the market transactions described so far. The quantity of money, denoted by $Q$, is assumed to be given and money is only used as a means of transaction. On Monday morning, the households in their quality of firms’ owners hold the entire quantity of money. At the conclusion of the bargaining process, firms borrow part of it to pay the workers’ wages. If any tax is levied to finance an unemployment benefit, part of the quantity of money is redistributed to the unemployed. On Friday morning, the entire quantity of money is in the hand of the households in the form of wages, unemployment benefits and money holdings. On Friday, the households’ consumption demand is determined under a *cash in advance* constraint. Market prices are therefore determined as $\hat{p}_t = \frac{Q}{mX_t} = \frac{Q}{mN_t}$; i.e. by the pure quantity theory of money. Firms receive as revenue the entire quantity of money, which they redistribute totally to households as realized profits, given by the difference between revenues and wage payments, and as debt repayments corresponding to the wages paid in advance.$^6$ Therefore, total realized nominal income $Y$ as the sum of nominal wage payments and realized nominal profits is always equal to the quantity of money $Q$; the *cash in advance* constraint coincides with an income constraint in the consumer maximization problem. From now on, we take the quantity of money and thus the nominal income as numéraire.

Finally, on Saturday firms and unions update their information. The position of firm $i$’s anticipated demand curve changes on account of the realized price $\hat{p}_t$ and the realized demand $\hat{d}_t$. Inserting this information in equation (10) determines the position of the anticipated demand for the period $t+1$:

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$^6$ In order to keep our analysis simple, we assume no interest payments.
Similarly, workers’ reservation wage adjusts in the light of the realized nominal wage \( W_t \) and of the realized employment \( N_t \).

3. The dynamic system

3.1. Outline

The dynamic behavior of the model, therefore, involves two processes. First, since producers do not know the true elasticity of the demand function, the position of the anticipated demand function (and thus of the anticipated marginal revenue) shifts over time. For a given nominal reservation wage and marginal revenue curve, the efficient bargaining outcome, \( \widetilde{MR} = W'_R \), determines employment. Since all firms are identical, equation (15) can be written as:

\[
e_t = \frac{m}{L} K_t \left\{ \left( \frac{\sigma}{\sigma - 1} \right) W'_R \right\}^{-\sigma}
\]

(18)

where \( e_t = \frac{mN_t}{L} \) denotes the employment rate. Taking into account equations (16) and (17), it can also be written as

\[
e_t = \left( \frac{Y}{L} \right)^{\sigma} e_{t-1} \left( \frac{\sigma}{\sigma - 1} \right)^{-\sigma} \left( W'_R \right)^{-\sigma}
\]

(19)

Second, the nominal reservation wage, \( W'_R \), is also adjusted over time.

For expository purposes, in what follows we begin our study with the dynamics of the first process in isolation by assuming a nominal reservation wage invariant over time. After that
we specify the adjustment of the reservation wage explicitly and explore the dynamic properties of the full system.

3.2. Commodity market dynamics in isolation

If we assume a reservation wage fixed at an arbitrarily chosen level, \( W'_{R} = \bar{W}_{R} \), the implied dynamic process is

\[
e_t = \left( \frac{Y}{L} \right)^{\sigma} e_{t-1}^{-\sigma} \left( \frac{\sigma}{\sigma - 1} \right)^{\sigma} \left( \bar{W}_{R} \right)^{-\sigma}
\]

Equation (20) is a one-dimensional first-return map with the following fixed point and first derivative

\[
\bar{e} = \left( \frac{Y}{L} \right)^{\sigma} \left( \frac{\sigma}{\sigma - 1} \right)^{-1} \left( \bar{W}_{R} \right)^{-1}
\]

\[
\frac{\partial e_t}{\partial e_{t-1}} = \left( \frac{Y}{L} \right)^{\sigma} \left( \frac{\sigma}{\sigma - 1} \right)^{-\sigma} \left( \bar{W}_{R} \right)^{\sigma} \left( 1 - \sigma \right) e_{t-1}^{-\sigma} \frac{\partial e_{t-1}}{\partial e_{t-1}} = 1 - \sigma
\]

As long as \( \sigma > 1 \), the fixed point, given by equation (21), increases with \( \sigma \),

\[
\frac{\partial \bar{e}}{\partial \sigma} = \frac{\bar{e}}{\sigma (\sigma - 1)} > 0,
\]

but loses stability at \( \sigma = 2 \). Eq. (22) shows that the derivative of the first return map is negative for all values of the employment rate; therefore, the time path diverges for \( \sigma > 2 \). Deregulating the commodity market increases the stationary employment but may eventually lead to instability.

Figure 3 illustrates the period 2 cycle occurring precisely at \( \sigma = 2 \). The employment rate alternates between \( e_1 \) and \( e_2 \); the anticipated demand and anticipated marginal revenue functions shift accordingly. Starting with the anticipated demand function 1 and the corresponding anticipated marginal revenue function 1 the bargaining process results in the higher employment rate \( e_1 \). The realized market price is below the anticipated one. The
anticipated demand and marginal revenue curves shift downward to the position 2. The next bargaining results in the lower employment rate $e_2$. The market price is now above the anticipated one inducing an upward shift of the anticipated demand and marginal revenue back to their respective position 1.

3.3. The nominal reservation wage as a function of the employment rate

Next, we introduce a positive dependence on the employment rate of the nominal reservation wage, $W' = W_R(e_{t-1})$: A higher employment rate is considered to increase the expected employment probabilities outside the firm under consideration and thus to increase the reservation wage. As a consequence, the adjustment of the nominal reservation wage affects the commodity market dynamics. The resulting dynamic system is still one-dimensional:

$$e_t = \left( \frac{Y}{L} \right)^{\sigma} e_{t-1}^{\sigma} \left( \frac{\sigma}{\sigma - 1} \right)^{\sigma} (W_R(e_{t-1}))^{-\sigma} \tag{23}$$

The fixed point is

$$\bar{e} = \left( \frac{Y}{L} \right)^{\sigma} \left( \frac{\sigma}{\sigma - 1} \right)^{\bar{e}} (W_R(\bar{e}))^{-\bar{e}} \tag{24}$$

Note that rising $\sigma$ still increases the equilibrium employment rate

$$\frac{\partial \bar{e}}{\partial \sigma} = \frac{\bar{e}}{\sigma (\sigma - 1)} \left( 1 + \frac{\bar{e}}{W_R} \frac{\partial W_R}{\partial \bar{e}} \right) > 0$$

The first derivative of (23) is

$$\frac{\partial e_t}{\partial e_{t-1}} = \left( \frac{Y}{L} \right)^{\sigma} \left( \frac{\sigma}{\sigma - 1} \right)^{\sigma} (W_R)^{\sigma} (1-\sigma) e_{t-1}^{-\sigma} + \left( \frac{Y}{L} \right)^{\sigma} e_{t-1}^{\sigma} \left( \frac{\sigma}{\sigma - 1} \right)^{\sigma} (-\sigma) (W_R)^{\sigma-1} \frac{\partial W_R}{\partial e_{t-1}}$$

$$= 1 - \sigma - \sigma \frac{\bar{e}}{W_R(\bar{e})} \frac{\partial W_R}{\partial e_{t-1}} \tag{25}$$
The fixed point now loses stability at $\sigma < 2$; beyond that value the time path diverges. Therefore, a positive influence of the employment rate on the nominal reservation wage destabilizes the economy.\(^7\)

Figure 3 also illustrates this destabilizing effect. Start from the market result 1 and consider the determination of position 2. The anticipated demand function and the anticipated marginal revenue shift again towards their position 2; but now, in addition, the nominal reservation wage is increased because of the high realized employment rate $e_1$. Therefore, the new employment rate 2 will be below $e_2$ in Figure 3. The increase in the nominal reservation wage thus increases the fluctuations. The process is destabilized.

### 3.4. The nominal reservation wage as a function of the employment rate and of its lagged value

Finally we allow also for a positive influence of the lagged reservation wage upon its current value:

$$W'_R = W_R \left( e_{t-1}, W''_{R,t-1} \right)$$

(26)

For the moment\(^8\), we assume the function $W_R(\bullet)$ to be monotonically increasing in both arguments. The rationale for the dependence of the nominal reservation wage on the employment rate is as sketched above; for the second one it runs along the following lines:

\(^7\) Blanchard and Giavazzi (2001) consider the *real* reservation wage to be an implicit function of the employment rate. However, it is not possible to assess the stability properties of their model without specifying a relationship between the reservation wage and the employment rate. In what follows we put forward such a specification.

\(^8\) But see below the discussion in section 4.2.
a high reservation wage in \( t-1 \) will result in a high bargained nominal wage rate in \( t-1 \), which in turn is expected to raise the nominal reservation wage in \( t \).

The central dynamic system is now two-dimensional and given by equations (19) and (26).

In the fixed point the following holds

\[
\sigma = \left( \frac{Y}{L} \right) \left( \frac{\sigma}{\sigma - 1} \right)^{-1} \left( W_R \right)^{-1} \quad \text{(27)}
\]

\[
W_R = \theta \left( \sigma, W_R \right) \quad \text{(28)}
\]

The Jacobian evaluated at the fixed point is given by

\[
J_E = \begin{pmatrix}
1 - \sigma & -\sigma \frac{\bar{e}}{W_R} \frac{\partial W'_{R}}{\partial e_{t-1}} & -\sigma \frac{\bar{e}}{W_R} \frac{\partial W'_R}{\partial W'^{-1}_{R}} \\
\frac{\partial W'_{R}}{\partial e_{t-1}} & \frac{\partial W'_{R}}{\partial W'^{-1}_{R}} & \frac{\partial W'^{-1}_{R}}{\partial W'^{t-1}_{R}}
\end{pmatrix} \quad \text{(29)}
\]

The trace and the determinant are

\[
\text{tr} J_E = 1 - \sigma - \sigma \frac{\bar{e}}{W_R} \frac{\partial W'_{R}}{\partial e_{t-1}} + \frac{\partial W'_R}{\partial W'^{-1}_{R}} \quad \text{(30)}
\]

\[
\det J_E = (1 - \sigma) \frac{\partial W'_R}{\partial W'^{t-1}_{R}} \quad \text{(31)}
\]

Figure 3 can be used to illustrate that the additional effect introduced by equation (26) is potentially stabilizing. Start again with the market position realized in period 1 and consider the determination of the position 2. The anticipated demand function and the anticipated marginal revenue shift to their respective position 2. Two factors change the nominal reservation wage. As in the previous case, the high employment rate \( e_1 \) tends to increase it.

At the same time, the high employment rate \( e_1 \) implies that the bargained nominal wage in
period 1 was comparatively low. This would reduce the nominal reservation wage, thus introducing a stabilizing element.

Without specifying explicitly the dynamic adjustment process for the nominal reservation wage, it is difficult to assess the stability properties. We provide such a specification in the following section and explore the dynamics of the full model.

4. Fully specified dynamics and numerical simulations

4.1. The dynamic adjustment process for the nominal reservation wage

The reservation wage in the bargaining process represents the income expected by the trade union for members who do not find employment in the firm under consideration (see Layard and Nickel, 1990). It therefore depends on the expected probability of finding employment in other firms, on the wage rate expected to be paid by other firms and on the unemployment benefit \( B_t \). Consonant with the monopolistic competition set up, we assume

\[ (Z_t' - \tilde{Z}_t')^\beta (\Pi_t' - \bar{\Pi}_t')^{1-\beta} = \left[ (1 - \bar{\tau}_i) (W_t' - W_t') \right]^\beta \left( \tilde{p}_t' - W_t' \right)^{1-\beta} \frac{N_t'}{P_t}, \]

where \( \bar{\tau}_i \) denotes the anticipated tax rate, with \( \bar{\tau}_i = \tau_{t-1} \), and where \( Z_t' \) and \( \tilde{Z}_t' \) are now net of taxation. On Tuesday, the tax rate is adjusted to match the benefit payments required by the realized unemployment: \( \tau_i \sum_i W_i' N_i' = \tau W_e L = (1 - \tau_i) B_t (1 - e_i) L \) and money holdings are redistributed accordingly. The bargaining results and the following analysis are not changed by this extension.

9 We did not incorporate explicitly the financing of the unemployment benefit. However, we studied the case of financing it out of a general labor income tax that applies both to workers and unemployed (similar to Calmfors and Johansson, 2001): Equ (12) would be modified to
that the trade unions do not take into consideration reactions of other firms and the impact of their own decisions on the aggregate variables. The expected probability of finding employment in other firms and the expected wage rate outside the firm under consideration is therefore given by the respective values realized in the previous period.

\[ W'_k = e_{t-1} W_{t-1} + (1 - e_{t-1}) B_t \]  \hspace{1cm} (32)

or using equation (13)

\[ W'_k = e_{t-1} \left( \frac{\sigma + \beta - 1}{\sigma - 1} \right) W_{t-1}^{\gamma} + (1 - e_{t-1}) B_t \]  \hspace{1cm} (33)

For the unemployment benefit we consider two different specifications (see e.g. Layard et al., 1991, and Pissarides, 1998) mirroring two possible institutional set-ups. The first is close to a social assistance scheme according to which the compensation for the unemployed is fixed in real terms (corresponding to a certain CES utility level \( \omega \)) and the nominal payment is adjusted each period to the realized price, \( B_t = \omega \hat{p}_{t-1} \). In the second, the compensation for the unemployed worker corresponds to a fraction of the nominal wage rate she was receiving in the previous period, \( B_t = \phi W_{t-1} \), where \( 0 < \phi < 1 \) is the replacement ratio. The unemployment benefit systems in the OECD countries are found in between those two extreme cases (see Goerke, 2000).

Such specifications will allow us to clarify the role of changes in parameters, which relate to the unemployment benefit, as measures of (de)regulation in the labor market.

4.2 Unemployment benefit fixed in real terms

We consider first the institutional set up for the unemployment benefit close to a social assistance system. The real unemployment benefit corresponds to a CES utility level of \( \omega \), its nominal counterpart is given by \( B_t = \omega \hat{p}_{t-1} \). The dynamic system is
\[ e_t = \left( \frac{Y}{L} \right)^\sigma e_{t-1} \left( \frac{\sigma}{\sigma - 1} \right)^{-\sigma} W_R^t \]  

(34)

\[ W_R^t = e_{t-1} \left( \frac{\sigma - 1 + \beta}{\sigma - 1} \right) W_{R}^{t-1} + \left(1 - e_{t-1} \right) \frac{Y}{L e_{t-1}} \omega \]  

(35)

A crucial feature of this specification is that the realized market position impacts on the dynamics both through shifts of the demand function, implicit in equation (34), and through the determination of the nominal reservation wage, as shown in equation (35).

The fixed point, the partial derivatives, the trace and determinant are given by

\[ \bar{e} = \frac{\sigma - 1 - \sigma \omega}{\sigma - 1 + \beta - \sigma \omega} \quad \text{and} \quad \bar{W}_R = \frac{Y}{eL} \frac{\omega}{\sigma} \]  

(36)

\[ \frac{\partial W_{R}^t}{\partial e_{t-1}} = \frac{\sigma - 1 + \beta}{\sigma - 1} - \frac{Y}{L} \frac{1}{\bar{e}^2} \omega \quad \text{and} \quad \frac{\partial W_{R}^t}{\partial e_{t-1}} = \bar{e} \frac{\sigma - 1 + \beta}{\sigma - 1} > 0 \]  

(37)

\[ \text{tr } J_E = 1 - \sigma + \frac{\sigma - 1}{\sigma - 1} \sigma \omega - (\sigma - 1 + \beta) \bar{e} \quad \text{det } J_E = - (\sigma - 1 + \beta) \bar{e} \]  

(38)

Note that – in contrast to the assumption in sections 3.3 and 3.4 – the partial derivative need not be positive since with the assumed specification a high employment rate in period t - 1 not only means a high expected re-employment probability in period t but also – via a low realized price in t - 1 – a low nominal unemployment benefit in t. The latter indirect effect of the employment rate on the nominal reservation wage was neglected in the previous sections.

Appendix 1 investigates analytically the properties of the dynamic system (34)-(35). It is shown that the fixed point exhibits the usual comparative static results: Commodity and labor market deregulation – as reflected in a higher value of \( \sigma \) and in lower values of \( \beta \) and \( \omega \), respectively – engender a higher employment rate. The analysis further shows that the
fixed point is only stable for low values of $\sigma$ and for high values of $\beta$ and $\omega$. However, in contrast to the one-dimensional models studied previously, the stability loss now occurs through a Flip bifurcation giving rise to attracting cycles and eventually to chaotic fluctuations. Therefore, commodity and labor market deregulation – though expected to increase the stationary employment rate – may destabilize the economy, but does not lead to diverging time paths.

So far, we have explored analytically the dynamics properties of the system (34)-(35). We now turn to a small calibration exercise. Note that the fixed point solutions and the stability properties only depend upon three parameters: the parameter reflecting the degree of monopoly power in the commodity markets, $\sigma$ or $\mu$; and the two parameters related to the extent of labor market regulation, namely the bargaining power of trade unions $\beta$ and the parameter concerning the specification of the real unemployment benefit $\omega$. These parameters are subject to the following boundaries: $0 < \beta < 1$, $0 < \omega < \bar{w}_r$ and $1 < \sigma$, where $\bar{w}_r = \frac{\sigma - 1}{\sigma}$ represents the stationary real reservation wage. Choosing an upper limit for $\sigma$ therefore allows studying the entire parameter space numerically. In our exercise, we assumed $\sigma \leq 5$ (or $\mu \geq 0.25$).

We searched numerically for values of $\beta$, $\omega$ and $\sigma$ such that the fixed point approximates two macroeconomic stylized facts; namely that in 1990s Continental Europe the employment rate typically assumes values above 0.8 and that the wage share typically assumes values slightly below 0.6$^{10}$. In addition, we ask which dynamic properties the time 

$^{10}$ Taking the average over a significant group of European Countries, Giammarioli et al. (2002) estimate that in the 1990s the labor share was 0.586 in the Business sector, 0.58 in Industry and 0.536 in the Tradable Services.
path exhibits close to that fixed point. Our numerical exploration shows that parameter constellations exist which result in plausible fixed points, which are stable for highly regulated markets and which lose stability with commodity or labor market deregulation. The resulting fluctuations are relatively small, which contributes to the plausibility of the results. As an illustration, Figure 4 shows the time paths for the employment rate $e_{t+1}$ and the wage share $w_t = \frac{W_t e_t L}{Y}$ for $\omega = 0.506$ and $\sigma = 2.15$. With these parameter values, the bifurcation value relating to the bargaining power of workers is $\beta_{bif} \equiv 0.015$. For $\beta = 0.02$ fixed point is stable; for $\beta = 0.01$ it is unstable. The resulting period-two time paths exhibit fluctuations with comparatively small amplitudes, hitting no boundary condition. Figure 4 also shows that on a period two cycle both the upper and the lower values for the employment rate may lie below its equilibrium value; average employment rates over the cycle may be lower than its stationary counterparts. The indirect effect of the employment rate on the nominal reservation wage is in this case stronger than the direct one and causes a decline in average employment. This result is in stark contrast to the conventional comparative static analysis. \(^{11}\)

\(^{11}\) Further simulations with different values for $\sigma$ show that commodity market deregulation may also reduce average employment rates below their equilibrium values. In addition, in that case average employment rates may even decline with increased commodity market deregulation. A similar result emerges with deregulating labor market via reducing unemployment benefits.
4.3. The unemployment benefit as a fixed proportion of the nominal wage rate

We now consider another institutional set up according to which the unemployment benefit is a fixed replacement ratio $\phi$ of the nominal wage rate earned in the lost job, $B_t = \phi W_{t-1}$.

The system (19) and (33) can be written as

$$ e_t = \left( \frac{Y}{L} \right)^{\sigma} e_{t-1}^{\sigma} \left\{ \left( \frac{1}{\sigma - 1} \right) W_R \right\}^{-\sigma} $$

$$ W_R' = e_{t-1} \left( \frac{\sigma - 1 + \beta}{\sigma - 1} \right) W_R^{t-1} + (1 - e_{t-1}) \phi \left( \frac{\sigma - 1 + \beta}{\sigma - 1} \right) W_R^{t-1} $$

Note that the analytical structure of this specification is simpler than the previous one given that the realized market position only affects the dynamics through shifts of the demand function as shown in equation (39). The fixed point, the partial derivatives, the trace and the determinant are given by

$$ \bar{\sigma} = \frac{(\sigma - 1)(1 - \phi) - \phi \beta}{(\sigma - 1 + \beta)(1 - \phi)} \quad \text{and} \quad \bar{W}_R = \frac{Y}{\bar{\sigma} L} $$

$$ \frac{\partial W_R'}{\partial e_{t-1}} = \frac{\sigma - 1 + \beta}{\sigma - 1} \bar{W}_R (1 - \phi) > 0 \quad \text{and} \quad \frac{\partial W_R'}{\partial W_R} = 1 $$

$$ \text{tr} J_E = 2(1 - \sigma) + \frac{\sigma}{\sigma - 1} (\sigma - 1 + \beta) \phi \quad \text{det} J_E = (1 - \sigma) $$

The major conclusions from the previous case carry over: As is shown in Appendix 2, deregulation in the labor or the commodity market increases the stationary employment rate, but may eventually lead to a Flip bifurcation of the fixed point giving rise to attracting period two cycles. Therefore, the basic trade off remains present: Deregulation increases employment rates but may destabilize the economy.

Computer simulations (not presented here) show that in this case it is more difficult to find plausible parameter values for stable equilibrium points. We have been able to identify
stable two-cycles. The amplitude of the cyclical time path, however, is much higher than in the previous case, and the employment rate hits quite often its upper boundary of one. In the unstable region, further deregulation usually increases average employment rates even above their corresponding equilibrium values.

5. Conclusion

In the previous paper, we have analyzed the dynamics of a model following closely the prototype specification put forward by Blanchard and Giavazzi (2001) which combines monopolistic competition in the commodity market and efficient bargaining in the labor market. The dynamics results from two sources: First, inherent to monopolistic competition is that each single supplier overestimates the price-elasticity of his demand function; each single supplier is therefore necessarily surprised by the market outcome and will adapt his anticipated demand function accordingly. Second, efficient bargaining processes are based on a reservation wage indicating the expected income outside the firm under consideration. This anticipation is also adjusted in the light of realized market results.

While the second process may be specified in various forms, the first one is directly implied by the external effect at the core of the monopolistic competition model itself. It is difficult to imagine fundamentally different specifications without leaving the assumed market structure. We showed in the paper, that the first process destabilizes the economy: Deregulation does increase the stationary employment rate, but engenders diverging time paths. Introducing various plausible specifications for the adjustment process of the

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12 We found in computer simulations the instability problem to be less severe with adaptive expectations, which dampen the fluctuations of the anticipated demand function.
reservation wage dampens this sharp result: The stability loss occurs through a Flip bifurcation giving rise not to diverging time paths but to attracting period-two cycles and eventually to complex time paths. However, the basic trade-off remains: Deregulation increases the stationary employment but may lead to instability with time paths exhibiting in some cases even lower average employment rates.

References:


Appendix 1

In this appendix we study the dynamic system related to the case in which the employment benefit is fixed in real terms and its nominal counterpart is adjusted to the realized price, $B_t = \omega \hat{\rho}_{t-1}$. The dynamic system is

$$e_t = \left( \frac{Y}{L} \right)^{\frac{\sigma}{\sigma - 1}} \left( \frac{\sigma}{\sigma - 1} \right)^{\frac{\sigma}{\sigma - 1}} \left( W_{R}^{t-1} \right)^{\frac{1}{\sigma - 1}}$$

(A1.1)

$$W_{R}^{t} = \left( \frac{\sigma - 1 + \beta}{\sigma - 1} \right) e_{t-1} W_{R}^{t-1} + \frac{1 - e_{t-1}}{e_{t-1}} \frac{Y}{\omega}$$

(A1.2)

A1.1 Fixed point and comparative statics

The stationary real reservation wage and the stationary real wage are

$$\bar{w}_{R} = \frac{\sigma - 1}{\sigma} \quad \bar{w} = \frac{\sigma - 1 + \beta}{\sigma}$$

(A1.3)

It follows that

$$\bar{c} = \frac{\bar{w}_{R} - \omega}{\bar{w} - \omega} \quad \bar{W}_{R} = \frac{Y}{\bar{c}L} \frac{\sigma - 1}{\sigma} \quad \bar{P} = \frac{Y}{\bar{c}L}$$

(A1.4)

Note that if condition

$$\omega < \bar{w}_{R}$$

(A1.5)

holds and $\beta > 0$, then $\bar{w}_{R} < \bar{w}$ and $0 < \bar{c} < 1$.

Note that the stationary real reservation wage depends only on $\sigma$ whereas the stationary real wage rate depends on $\beta$ and $\sigma$ and the stationary employment rate depends on $\omega$, $\beta$ and $\sigma$.

We have
\[ \frac{\partial \bar{e}}{\partial \sigma} = \frac{\beta}{\sigma} \left[ \frac{\omega (1 - \beta) + \sigma (\bar{w} - \omega)}{\bar{w}} \right] > 0 \]  \hspace{1cm} (A1.6)

\[ \frac{\partial \bar{W}_k}{\partial \sigma} = \frac{Y}{L(\bar{e}\sigma)^2} \left( \bar{e} - \sigma (\sigma - 1) \frac{\partial \bar{e}}{\partial \sigma} \right) \]
\[ \frac{\partial \bar{w}}{\partial \sigma} = \frac{1 - \beta}{\sigma^2} > 0 \]
\[ \frac{\partial \bar{W}_k}{\partial \sigma} = \frac{1}{\sigma^2} > 0 \]  \hspace{1cm} (A1.7)

with \( \frac{\partial \bar{W}_k}{\partial \sigma} \geq (0) \) for \( \frac{\partial \bar{e}}{\partial \sigma} \leq (\frac{\bar{e}}{\sigma (\sigma - 1)}) \)

The impact of changes in \( \beta \) is given by

\[ \frac{\partial \bar{e}}{\partial \beta} = -\frac{\bar{w}_k - \omega}{\sigma (\bar{w} - \omega)^2} < 0 \]  \hspace{1cm} (A1.8)

\[ \frac{\partial \bar{W}_k}{\partial \beta} = -\left( \frac{\sigma - 1}{\sigma} \right) \frac{Y}{L\bar{e}^2} \frac{\partial \bar{e}}{\partial \beta} \quad \frac{\partial \bar{w}}{\partial \beta} = \frac{1}{\sigma} > 0 \]  \hspace{1cm} (A1.9)

The impact of changes in \( \omega \) is given by

\[ \frac{\partial \bar{e}}{\partial \omega} = -\frac{\beta}{\sigma} \frac{1}{(\bar{w} - \omega)^2} < 0 \]  \hspace{1cm} (A1.10)

\[ \frac{\partial \bar{W}_k}{\partial \omega} = -\left( \frac{\sigma - 1}{\sigma} \right) \frac{Y}{L\bar{e}^2} \frac{\partial \bar{e}}{\partial \omega} > 0 \]  \hspace{1cm} (A1.11)

A1.2 Bifurcation analysis

The Jacobian evaluated at the fixed point is

\[ J_E = \begin{bmatrix}
1 - \sigma - \sigma \left( \frac{\sigma + \beta - 1}{\sigma - 1} \bar{e} - \omega \frac{Y}{\bar{W}_k} \right) - \frac{\sigma \bar{e}^2}{\bar{W}_k} \left( \frac{\sigma + \beta - 1}{\sigma - 1} \right) \\
\left( \frac{\sigma + \beta - 1}{\sigma - 1} \right) \bar{W}_k - \omega \frac{Y}{L\bar{e}^2} \left( \frac{\sigma + \beta - 1}{\sigma - 1} \bar{e} \right)
\end{bmatrix} \]

Determinant and Trace are:

\[ \det J_E = -(\sigma + \beta - 1)\bar{e} \]  \hspace{1cm} (A1.12)
\[ \text{tr} \, J_E = 1 - \left[ (\sigma + \beta - 1)\bar{e} + \sigma \left( \frac{\bar{w}_R - \omega}{\bar{w}_R} \right) \right] \]  

(A1.13)

with \(-\infty < \text{det} \, J_E < 0\) and \(-\infty < \text{tr} \, J_E < 1\).

The violation of one of the following conditions would involve instability for the system (in brackets the type of bifurcation involved):

(i) \(1 - \text{det} \, J_E = 1 + (\sigma - 1 + \beta)\bar{e} > 0\)  
(Hopf bifurcation);

(ii) \(1 - \text{tr} \, J_E + \text{det} \, J_E = \left( \frac{\bar{w}_R - \omega}{\bar{w}_R} \right) \sigma > 0\)  
(Saddle node bifurcation);

(iii) \(1 + \text{tr} \, J_E + \text{det} \, J_E = 2 \left[ 1 - (\sigma - 1 + \beta)\bar{e} \right] - \left( \frac{\bar{w}_R - \omega}{\bar{w}_R} \right) > 0\)  
(Flip bifurcation).

Conditions (i) and (ii) always hold as long as \(\sigma > 1\) and \(\bar{w}_R > \omega\). The Flip bifurcation is the only possible. Condition (iii) corresponds to

\[ \frac{[ (3\sigma - 2)\omega + (4 - 3\sigma)\bar{w}_R ] \beta \bar{w}_R - (\sigma - 1)(\bar{w}_R - \omega) \left[ (3\sigma - 4)\bar{w}_R - \sigma\omega \right]}{[ (\sigma - 1)(\bar{w}_R - \omega) + \beta \bar{w}_R ] \bar{w}_R} > 0 \]  

(A1.14)

In this condition for stability, the denominator is always positive. Therefore, the system is stable for

\[ \left[ (3\sigma - 2)\omega + (4 - 3\sigma)\bar{w}_R \right] \beta \bar{w}_R > (\sigma - 1)(\bar{w}_R - \omega) \left[ (3\sigma - 4)\bar{w}_R - \sigma\omega \right] \]

We may distinguish three cases depending on \(\sigma\).

Case 1: \(\sigma < \frac{4}{3}\). The stability condition is satisfied for all other parameter values:

Case 2: \(\frac{4}{3} < \sigma < 2\). The stability of the system depends upon the parameter values. There are three sub-cases that we have to take into account depending on \(\omega\):
Case 2a: \( \omega < \frac{3\sigma - 4}{3\sigma - 2} \bar{w}_R \). It follows \((3\sigma - 4)\bar{w}_R > (3\sigma - 2)\omega > 2\sigma\omega\). In this case, the stability condition is never satisfied.

Case 2b: \( \frac{3\sigma - 4}{3\sigma - 2} \bar{w}_R < \omega < \frac{3\sigma - 4}{\sigma} \bar{w}_R \). The system is stable for

\[
\beta > \beta^{bf} = \frac{(\sigma - 1)(\bar{w}_R - \omega)\[(3\sigma - 4)\bar{w}_R - \sigma\omega\]}{[(3\sigma - 2)\omega + (4 - 3\sigma)\bar{w}_R]\bar{w}_R} > 0
\]  

(A1.15)

If \( \beta \) falls below this value, the system loses stability through a Flip bifurcation.

Case 2c: \( \frac{3\sigma - 4}{3\sigma - 2} \bar{w}_R < \frac{3\sigma - 4}{\sigma} \bar{w}_R < \omega < \bar{w}_R \). The system is stable for

\[
\beta > 0 > \beta^{bf} = \frac{(\sigma - 1)(\bar{w}_R - \omega)\[(3\sigma - 4)\bar{w}_R - \sigma\omega\]}{[(3\sigma - 2)\omega + (4 - 3\sigma)\bar{w}_R]\bar{w}_R}
\]

That is, the system is always stable.

Case 3: \( 2 < \sigma \). As for Case 2, the stability of the system depends upon the parameter values. Depending on \( \omega \) we may identify two sub-cases:

Case 3a: \( \omega < \frac{3\sigma - 4}{3\sigma - 2} \bar{w}_R \). As in Case 2a, the stability condition is never satisfied.

Case 3b: \( \frac{3\sigma - 4}{3\sigma - 2} \bar{w}_R < \omega < \bar{w}_R < \frac{3\sigma - 4}{\sigma} \bar{w}_R \). As in Case 2b, the system is stable for

\[
\beta > \beta^{bf} = \frac{(\sigma - 1)(\bar{w}_R - \omega)\[(3\sigma - 4)\bar{w}_R - \sigma\omega\]}{[(3\sigma - 2)\omega + (4 - 3\sigma)\bar{w}_R]\bar{w}_R} > 0
\]

(A1.16)

If \( \beta \) falls below this value, the system loses stability through a Flip-bifurcation.
Appendix 2

In this appendix we examine some of the properties of the dynamic system related to the case in which the employment benefit is a proportion of the nominal wage rate earned in the lost job, $B_t = \phi W_{t-1}$. The dynamic system is

$$e_t = \left( \frac{Y}{L} \right) \epsilon_{t-1}^{1-\sigma} \left( \frac{\sigma}{\sigma - 1} W'_R \right)^{-\sigma} \tag{A2.1}$$

$$W'_R = \left( \frac{\sigma - 1 + \beta}{\sigma - 1} \right) \left[ 1 - (1 - \phi)(1 - e_{t-1}) \right] W'^{-1}_R \tag{A2.2}$$

A2.1 Fixed point and comparative statics

From (40) and (39), we solve for the stationary employment rate and reservation wage

$$\bar{e} = \frac{(\sigma - 1)(1 - \phi) - \phi \beta}{(\sigma - 1 + \beta)(1 - \phi)} \tag{A2.3}$$

$$\bar{W}_R = \frac{\sigma - 1}{\sigma} \frac{Y}{\bar{e}L} \tag{A2.4}$$

Note that if condition

$$(\sigma - 1)(1 - \phi) - \phi \beta > 0 \tag{A2.5}$$

holds and $\beta > 0$, then $0 < \bar{e} < 1$.

The stationary price is $\bar{p} = \frac{Y}{\bar{e}L}$ and the stationary nominal wage is $\bar{W} = \frac{\sigma - 1 + \beta}{\sigma - 1} \bar{W}_R$. It follows that the stationary real wage and the stationary real reservation wage are identical to the previous case:

$$\bar{w} = \frac{\sigma - 1 + \beta}{\sigma} \qquad \bar{w}_R = \frac{\sigma - 1}{\sigma} \tag{A2.6}$$
From (A2.3), the stationary employment rate depends on $\phi$, $\beta$ and $\sigma$. The comparative statics of the stationary state involves:

$$\frac{d\bar{e}}{d\sigma} = \frac{\beta}{(\sigma - 1 + \beta)(1 - \phi)} > 0$$  \hspace{1cm} (A2.7)

$$\frac{d\bar{W}_R}{d\sigma} = \frac{1}{(\sigma \bar{e})^2} \left( e - \sigma (\sigma - 1) \frac{d\bar{e}}{d\sigma} \right)$$

$$\frac{d\bar{w}}{d\sigma} = \frac{1 - \beta}{\sigma^2} > 0$$

$$\frac{d\bar{W}_R}{d\sigma} = \frac{1}{\sigma^2} > 0$$  \hspace{1cm} (A2.8)

with $\frac{d\bar{W}_R}{d\sigma} \geq (\leq) 0$ for $\frac{d\bar{e}}{d\sigma} \leq (\geq) \frac{\bar{e}}{\sigma (\sigma - 1)}$

The impact of changes of $\beta$ is given by

$$\frac{d\bar{e}}{d\beta} = -\frac{\sigma - 1}{(\sigma - 1 + \beta)(1 - \phi)} < 0$$  \hspace{1cm} (A2.9)

$$\frac{d\bar{W}_R}{d\beta} = -\left(\frac{\sigma - 1}{\sigma}\right) \frac{Y}{L^2} \frac{d\bar{e}}{d\beta} > 0$$

$$\frac{d\bar{w}}{d\beta} = \frac{1}{\sigma} > 0$$  \hspace{1cm} (A2.10)

The impact of changes of $\phi$ is given by

$$\frac{d\bar{e}}{d\phi} = -\frac{\beta}{(\sigma - 1 + \beta)(1 - \phi)^2} < 0$$  \hspace{1cm} (A2.11)

$$\frac{d\bar{W}_R}{d\phi} = -\left(\frac{\sigma - 1}{\sigma}\right) \frac{Y}{L^2} \frac{d\bar{e}}{d\phi} > 0$$  \hspace{1cm} (A2.12)

### A2.2 Bifurcation analysis

The Jacobian evaluated at the fixed point is

$$J_E = \begin{bmatrix}
1 - \sigma - \sigma \left(\frac{\sigma - 1 + \beta}{\sigma - 1}\right) (1 - \phi \bar{e}) & -\sigma \frac{\bar{e}}{\bar{W}_R} \\
\sigma - 1 + \beta & \frac{\sigma - 1 + \beta}{\sigma - 1} (1 - \phi \bar{W}_R) & 1
\end{bmatrix}$$
Determinant and Trace are:

\begin{align*}
\det J_E &= 1 - \sigma \\
\text{tr } J_E &= 2(1 - \sigma) + \frac{\sigma}{\sigma - 1} (\sigma - 1 + \beta) \phi \\
\end{align*}

(A2.13) \quad \text{with } -\infty < \det J_E < 0.

The violation of one of the following conditions would involve instability for the system (in brackets the type of bifurcation involved):

(i) \quad 1 - \det J_E = \sigma > 0 \quad \text{(Hopf bifurcation);} \\

(ii) \quad 1 - \text{tr } J_E + \det J_E = \frac{\sigma}{\sigma - 1} \left[ (\sigma - 1)(1 - \phi) - \phi \beta \right] > 0 \quad \text{(Saddle node bifurcation);} \\

(iii) \quad 1 + \text{tr } J_E + \det J_E = 1 + 3(1 - \sigma) + \frac{\sigma}{\sigma - 1} (\sigma - 1 + \beta) \phi > 0 \quad \text{(Flip bifurcation).}

Conditions (i) and (ii) always hold as long as condition (A2.5) holds. The Flip bifurcation is the only possible. Condition (iii) corresponds to

\begin{align*}
(3 - \phi)\sigma^2 - \left[ 7 - (1 - \beta) \phi \right] \sigma + 4 < 0 \\
\end{align*}

(A2.15)

If condition (A2.15) is not satisfied, the system loses stability through a Flip-bifurcation.

Define that value of \( \sigma \) corresponding to the highest root that satisfies condition (A2.15) with an equality sign as

\begin{align*}
\zeta(\phi, \beta) = \frac{7 - (1 - \beta) \phi + \sqrt{(1 + (1 - \beta)^2 \phi^2 + 2(1 + 7 \beta) \phi)}}{2(3 - \phi)} \\
\end{align*}

(A2.16)

The system is stable for

\begin{align*}
\phi > \phi_{bf} \equiv \frac{3\sigma - 4}{\sigma - 1 + \beta} \left( \frac{\sigma - 1}{\sigma} \right) \\
\end{align*}
\[ \beta > \beta_{bf} = \frac{(3 - \phi)\sigma - 4(\sigma - 1)}{\phi} \]

and that \[ \sigma < \sigma_{bf} \]

where \( \sigma_{bf} = z(\phi, \beta) \) for \( 0 < \beta < 1 \) and \( 0 < \phi < 1 \).

Note finally that \( 0 < \phi_{bf} < 1 \), iff \( z(0, \beta) < \sigma < z(1, \beta) \) and \( 0 < \beta_{bf} < 1 \) iff \( z(\phi, 0) < \sigma < z(\phi, 1) \);

and that \( z(0, \beta) = \frac{4}{3} < \sigma_{bf} < \frac{7}{4} + \frac{1}{4\sqrt{17}} = z(1,1) \).
Figure 1: Bargaining equilibrium Week t

\[ \hat{p}_{t-1} = \frac{\sigma}{\sigma - 1} W_R' \]

\[ W_i' = \frac{\sigma - 1 + \beta}{\sigma - 1} W_R' \]

\[ \hat{d}_t = \frac{\sigma - 1}{\sigma} \hat{p}_i' \]

\[ \tilde{d}_i \]

\[ N_i' \]

\[ N_i, x_i, d_i \]
Figure 2: Short run commodity market equilibrium Week t
Figure 3: Period-two Cycle
(a) employment rate

$\beta = 0.02$

$\bar{v} \approx 0.756$

0 

Beta 

100

(b) wage share

$\beta = 0.02$

$\bar{w} \approx 0.544$

0 

Time 

100

$\beta = 0.01$

$\bar{w} \approx 0.54$

0 

Time 

100

Figure 4: Time paths