# Mergers under Asymmetric Information—Is there a Lemons Problem?

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ABSTRACT: We analyze a Bayesian merger game under two-sided asymmetric information about firm types. We show that merger returns, defined as the difference between pre- and post-merger profits, are not necessarily higher for low-type firms. This has two implications. First, under very general conditions, equilibria exist where mergers occur, and there is no presumption that there is inefficiently low trade. Second, in these equilibria it is typically not the case that only low-type firms enter an agreement. Thus, the standard prediction of the lemons market model—if any, only low-type firms are traded—does not hold in general for the market for firms.

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# 1 Introduction

Since the nineteen sixties, there have been several extended periods of large-scale merger activity. Nevertheless, some authors (e.g. Ravenscraft and Scherer 1987, 1989; Healy et al. 1992; Scherer 2002) have questioned the overall profitability of mergers. Also, numerous case studies suggest that at least one of the parties involved in a merger is likely to consider the deal a failure with the benefit of hindsight.[1] Thus, the market for corporate control is simultaneously characterized by high trade volumes and asymmetric information. This sits uneasily with the standard lemons rationale that asymmetric information and inefficiently low trade levels, or even market collapse, should go hand in hand (Akerlof 1970). Is it therefore misleading to think of the market for corporate control as a market for lemons, as previous literature suggests?[2]

In the present paper, we want to argue that the lemons market model is indeed not the ideal way to think of mergers, in spite of the considerable uncertainty surrounding them. In particular, we show that if both buyers and sellers have private information about their own type, low (i.e., relative unprofitable or inefficient) types are not necessarily more inclined to trade. Instead, high types may be more willing to trade than low types, so that the standard prediction of the lemons market model—if any, only low types are traded—is inappropriate for describing merger decisions.

As a starting point, consider the following standard lemons story. Suppose the potential seller of an object (a firm, for instance), knows the true value $v$ of the object. The potential buyer, in turn, only knows the distribution $F(v)$ of the object's value. Crucially, for any price $p$ at which the object might

[1] For example, in the merger with *Hypobank*, *Bayerische Vereinsbank* discovered "the full horror of its partner's balance sheet" only after two years (*The Economist*, July 29, 2000). Further well-known examples of "disastrous" mergers include *Daimler/Chrysler* and *Time Warner/AOL* (*The Economist*, February 3, 2005).

[2] See, e.g., Besanko et al. (2002, p. 317), Gioia and Thomsen (2002), Arend (2004), Rhodes-Kropf and Viswanathan (2004).

be sold, the net gain $p - v$ from the transaction is decreasing in the object's value. Thus, owners of an object with low value have a strong incentive to sell. As a result, in a Bayesian equilibrium of a game where buyers and sellers take the price as given and trade involves only part of the sellers, only objects with a value below some *cut-off level* will be traded. In many cases, however, there is *no trade* at all: For instance, if the buyers' valuations are not systematically higher than sellers' valuations, potential buyers anticipate that the value of an object that a seller is willing to trade must be lower than its price, and trade does therefore not occur.

Can we expect such no-trade and cut-off results to hold in merger markets in general? To some extent, the answer depends on the mechanisms determining whether a merger takes place. To stay as closely as possible to the standard lemons model, suppose first that there is some arbitrary price for which each party must decide whether to consent to a merger. In this simple setting, the *buyer's merger returns* are given by

$$\{\text{post-merger profit}\} - \{\text{price}\} - \{\text{pre-merger profit}\},$$

whereas the *seller's merger returns* are

$$\{\text{price}\} - \{\text{pre-merger profit}\}.$$

Assuming that high types correspond to low marginal costs,[3] the profits of the buyer and the seller depend on the types of both firms in an oligopolistic setting. Therefore, unlike in the standard lemons model, the returns from trade depend on the types of both firms. When firms compete in the same industry, pre-merger profits are usually increasing in own type, but decreasing in the competitor's type. Post-merger profits should typically be increasing in both types, at least if the costs of the new entity reflect the costs of the constituent parts to some extent.

In such a setting, high-type sellers will have lower merger returns than low-type sellers, as in the standard case. For buyers, however, it is less clear

[3] Alternatively, the type of a firm could correspond to its fixed costs, or to a demand parameter.

that low types have higher merger returns: The positive effect of a high type on post-merger profits may or may not dominate over the positive effect on pre-merger profits. Which of the two effects dominates depends critically on the environment in which the merger takes place, and, in particular, on the technology of the merged firm.

To illustrate this, we analyze the merger returns of buyers and sellers in a simple linear Cournot model with symmetric information.[4] First, consider the case where the buyer imposes its technology on the seller, so that the merged entity produces with the buyer's marginal costs (*buyer-dominated mergers*). In this case, the marginal effect of a higher type on post-merger profits may well be higher than the marginal effect on pre-merger profits, so that the buyer's merger returns are *increasing* (rather than decreasing) in own type.[5] Second, suppose—somewhat less plausibly—that the merged entity produces with the seller's marginal costs (*seller-dominated mergers*). Then the buyer's merger returns will be decreasing in own type. Third, assume that the merged entity produces with the marginal costs of the more efficient firm (*rationalization mergers*). Here, the buyer's merger returns may again be increasing rather than decreasing in own type. Compared to the standard lemons problem, we must therefore take into account that

(i) a firm's incentive to merge depends on the types of both players, and

(ii) low-type firms are not necessarily more inclined to merge.

These basic insights from a setting with symmetric information are useful for analyzing merger decisions in the presence of *two-sided asymmetric*

[4] In spite of the well-known result that, in such a setting, two-firm mergers do not increase joint profits unless they lead to monopoly (Salant et al. 1983), Barros (1998) has shown that joint profits may rise due to rationalization effects when there is cost-heterogeneity. In principle, therefore, there is scope for efficiency-enhancing mergers.

[5] The seller's merger returns are decreasing in own type, as in all other cases considered below.

*information*.[6] We consider a reduced-form merger game where two firms are matched whose types $z_i, i = 1, 2$, are drawn from distributions that are common knowledge.[7] These distributions may be interpreted to reflect the residual uncertainty about the other firm's type after merger negotiations between the two firms have ended. That is, we assume that firms are unable to fully reveal their own type. Given the evidence of unpleasant surprises in merger decisions (such as the discovery of the "full horror" of the partner's balance sheet), this appears quite natural. Knowing their own type, both firms then state whether they consent to the merger. If both firms consent, the merger takes place. If at least one firm declines, there is no merger. Following the merger game, an oligopoly game is played. If no merger occurs, both firms earn their pre-merger profits. If a merger occurs, the joint profit is shared according to some predetermined rule. In the simplest case, one firm buys the other one at a constant price $p$, as sketched in the Cournot example. However, our analysis is also consistent with other ways of profit sharing.

Our main results are the following. First, if merger returns under symmetric information are monotone decreasing in own type for both firms, the standard results from the adverse selection literature carry over: In the *two-sided lemons equilibrium*, no trade is often the only equilibrium and, if trade takes place, only low-type firms consent to the merger. Second, if only the seller's returns are decreasing in own type, whereas the buyer's returns are increasing, the merger pattern looks dramatically different: Equilibria with trade exist quite generally, and in these *lemons-and-peaches-equilibria*, low-

[6]The approach of deriving insights for games with uncertain payoffs from information about the players' objective functions under certainty has been developed by Athey (2001). We adopt a similar approach towards analyzing merger decisions under asymmetric information.

[7]Hviid and Prendergast (1993) provide an analysis of merger games with one-sided asymmetric information. Assuming that the target firm has private information about its profitability, they show that an unsuccessful bid may increase the profitability of the target but reduce the profitability of the bidding firm (relative to the profitability before the merger offer) due to learning from rejection.

type sellers merge with *high*-type buyers. Further, there will typically be non-degenerate type sets for which buyers regret the merger ex post, and non-degenerate type sets for which sellers regret the merger ex post. The last statement is true even if the seller receives a reimbursement that is independent of the competitor's type (e.g. a cash payment): As the buyer turns out to be worse than expected, the seller realizes that its stand-alone profits would have exceeded the takeover price. Finally, independent of the properties of merger returns, there is always a *no-merger equilibrium* where firms merge with probability zero: If both firms believe that the other firm will not consent to the merger, irrespective of its type, it is a (weakly) best response not to consent, and beliefs are correct in equilibrium.

The remainder of the paper is organized as follows. In Section 2, we introduce the general analytical framework. Section 3 analyzes the monotonicity properties of the merger return functions in the linear Cournot model with cash payment under symmetric information. Section 4 shows how various conceivable properties for these functions lead to different implications for the Bayesian equilibria of our merger game that are consistent with the examples from the Cournot case. Section 5 concludes.

# 2 Analytical Framework

We consider an oligopoly with an exogenous number of firms $n \geq 2$. Two of these firms, denoted as $i = 1, 2$, are matched to play a merger game. Each of the two firms is characterized by a type $z_i \in \mathbb{R}$. We allow the stand-alone and post-merger profits of firm $i$ to be affected by both $z_i$ and $z_j, i, j = 1, 2, j \neq i$. There is asymmetric information about the value of $z_i$, i.e. firms know their own $z_i$, but not their competitor's $z_j$. The ex ante probability of $z_i$ is described by a probability distribution $F_i$ with density $f_i$ and compact support $[\underline{z}_i, \overline{z}_i] \subset \mathbb{R}$. $F_i$ is common knowledge. We allow for ex ante heterogeneity between firms, i.e. firms' types $z_i$ may be drawn from different distributions.[8]

Firms simultaneously announce whether they are willing to merge. The decision of firm $i$ is summarized in a variable $s_i$ such that $s_i = 1$ if it consents to an agreement and $s_i = 0$ if it rejects it. A merger occurs if and only if $s_i = 1$ for $i = 1, 2$. If no merger occurs, each firm earns its stand-alone oligopoly profit $\pi_i (z_i, z_j)$. If a merger occurs, the merged entity earns total profit $\pi^M (z_i, z_j)$. These functions are defined on some set $\mathcal{Z} = \mathcal{Z}_1 \times \mathcal{Z}_2$, where $[\underline{z}_i, \overline{z}_i] \subseteq \mathcal{Z}_i$. The properties of $\pi_i$ and $\pi^M$ reflect more primitive assumptions on the nature of product market interaction (e.g. Bertrand vs. Cournot) and the interpretation of the type variable. Note that, to simplify exposition, $\pi_i$ and $\pi^M$ do not explicitly depend on state variables of outsiders to the merger. However, one might imagine that there are state variables $z_3, ..., z_n$ for the remaining firms about which both merger parties are symmetrically uninformed. $\pi_i (z_i, z_j)$ and $\pi^M (z_i, z_j)$ should then be interpreted as expected profits when $z_1$ and $z_2$ are known, but $z_3, ..., z_n$ are not. Throughout the paper, we shall require the following assumption on the firms' stand-alone profits to be satisfied.

**Assumption 1** *$\pi_i$ is non-decreasing in $z_i$; $\pi^M$ is non-decreasing in $z_1$ and $z_2$.*

Thus, by definition, higher types are more profitable: The higher a firm's type, the higher its stand-alone profits and the higher the profits of the merged entity that it becomes a part of. We do not specify the way in which post-merger profits are split between the owners of the two firms. Rather, we use a fairly general approach and assume that the owners of the firms obtain post-merger profits $\pi_i^M$ that may (but need not) depend on $z_i$ and $z_j$. For instance, when mergers are cash-financed, the buyer, say firm 1, obtains

[8]This is of particular importance for vertical or conglomerate mergers where firms produce entirely different goods. Even the interpretation of the firms' types might differ: For vertical mergers, for instance, the types might correspond to the costs of input production for the upstream firm and marketing ability for the downstream firm.

$\pi_1^M(z_1,z_2) = \pi^M(z_1,z_2) - p$, where $p \geq 0$ is the transaction price, whereas the seller obtains $\pi_2^M(z_1,z_2) = p$, which is independent of types. Alternatively, our approach applies to stock-financed mergers where $\pi_i^M(z_1,z_2) = \alpha_i \pi^M$, with $\alpha_i \geq 0$ denoting the profit share of firm $i$'s owners in the new entity.[9] Our next assumption requires budget balance.

**Assumption 2** *The profits of the merging firms' owners add up to the joint profits of the merged firm, i.e.* $\pi_1^M(z_1,z_2) + \pi_2^M(z_2,z_1) = \pi^M(z_1,z_2)$.

Next, we require the following assumption.

**Assumption 3** $\partial\pi_i/\partial z_j \leq \partial\pi_i^M/\partial z_j$ *for* $i = 1,2, j \neq i$.[10]

This assumption is particularly appealing for horizontal mergers. When firms compete in the same industry, stand-alone profits typically fall if the competitor becomes more efficient. Thus, the left-hand side of the inequality is negative. In addition, because of Assumption 1, total post-merger profits (weakly) increase if firm $j$ becomes better. If the owners of both firms benefit from these higher profits, then $\pi_i^M$ is non-decreasing in $z_j$ and the right-hand side of the inequality is non-negative.[11]

Finally, we introduce the following definition:

**Definition 1 (merger returns)** *(i) Firm i's **individual** merger returns are given by*

$$g_i(z_i,z_j) \equiv \pi_i^M(z_i,z_j) - \pi_i(z_i,z_j).$$

*(ii) The merging firms' **joint** merger returns are given by*

$$g(z_1,z_2) \equiv \pi^M(z_1,z_2) - \pi_1(z_1,z_2) - \pi_2(z_1,z_2).$$

[9] See Eckbo et al. (1990), Fishman (1989) and Hansen (1987) for an analysis of the choice of the "medium of exchange" in takeovers.

[10] We are implicitly assuming that the profit functions are differentiable here for notational convenience only.

[11] In vertical relationships, however, both upstream and downstream firms usually benefit from the other party being high-type. Therefore, the left-hand side might be positive and the relation does not necessarily hold.

The functions $g_i$ and $g$ correspond to merger returns under certainty or expected returns under symmetric uncertainty about the types of players $3, ..., n$.[12] To obtain expressions for expected returns for $i$ under *asymmetric* information about player $j \neq i$, suppose, in our merger game, for $i = 1, 2$, firm $i$ plays a strategy $s_i(z_i)$. Let $B_i \equiv B_i(s_i) \equiv \{z_i \,|s_i(z_i) = 1\}$ denote the set of types $z_i$ for which firm $i$ consents to a merger. Then

$$\begin{aligned} G_i(z_i; B_j, f_j) \equiv\ & \mathbb{P}[B_j]\, \mathbb{E}_{z_j}\left[\pi_i^M(z_i, z_j)\,|z_j \in B_j(s_j)\right] + \\ & (1 - \mathbb{P}[B_j])\, \mathbb{E}_{z_j}[\pi_i(z_i, z_j)\,|z_j \notin B_j(s_j)] - \mathbb{E}_{z_j}[\pi_i(z_i, z_j)] \\ =\ & \int_{B_j} g_i(z_i, z_j)\, f_j(z_j)\, dz_j \end{aligned} \tag{1}$$

denotes the *expected returns* from consenting to a merger for firm $i$ with type $z_i$, when players $j$ are distributed as $f_j$, and only players in $B_j$ consent to a merger. Note that the expectation in (1) is taken only over $B_j$ (i.e., the set of types for which firm $j$ consents to a merger): For types outside $B_j$, player $i$ obtains the stand-alone profit, irrespective of its own decision.

# 3 Properties of Merger Returns

Most of the results for our merger game with two-sided *asymmetric* information about firm types depend crucially on the properties of the merger return function, $g_i = \pi_i^M - \pi_i$, under *symmetric* information. To set the stage, we therefore analyze the properties of $g_i$ in a linear Cournot setting with symmetric information and cash payment in this section. We shall discuss the equilibria of the linear Cournot model under asymmetric information in Section 4, applying our results from the analysis of the more general reduced-form merger game.

[12] In the latter case, we suppress the types of firms $3, ..., n$ to avoid excessive notation.

## 3.1 A Linear Cournot Setting

Suppose that, initially, there are 3 firms with marginal costs $c_i, i = 1, ..., 3$, and inverse demand is given by $P(Q) = a - Q$, where $Q = \sum_i^n q_i$ is aggregate output and $a > 0$. We consider a merger game between firms 1 and 2. Define the type of firm $i = 1, 2$ as $z_i \equiv -c_i$, i.e. the negative of marginal costs. Suppose that $z_1$ and $z_2$ are uniformly and independently distributed with compact support $\mathcal{Z} = \mathcal{Z}_1 = \mathcal{Z}_2 = [\underline{z}, \overline{z}]$. For notational simplicity, we assume that the marginal costs of the outside firm 3 are known to be $c_3$.[13] The marginal costs of the merged entity are given by $c_M$, and we write $z_M \equiv -c_M$. We suppose that $z_M(z_1, z_2)$ is a non-decreasing function, so that a merger between high-cost firms tends to lead to a high-cost merged firm.

To examine the properties of the merger return function under *symmetric* information, we suppose for the moment that all types are common knowledge. Thus, firms play a three-firm linear Cournot game with known marginal costs $(c_1, c_2, c_3)$ when there is no merger and a two-firm game with marginal costs $(c_M, c_3)$ when a merger has occurred. In this setting, firm $i$'s stand-alone profits are given by

$$\pi_i(z_i, z_j) = \frac{(a + 3z_i - z_j + c_3)^2}{16},$$

whereas the new firm's post-merger profits are

$$\pi^M(z_i, z_j) = \frac{(a + 2z_M(z_i, z_j) + c_3)^2}{9}.$$

Figure 1 illustrates the model for $a = 2$ and $c_3 = 1$. The triangle $ABC$ delineates the admissible set of those $(c_1, c_2)$ for which all firms produce positive outputs.

<Figure 1 around here>

[13] With added notational complexity, all results generalize to the case where the parties to the merger are symmetrically uninformed about $c_3$.

We now proceed to show that merger returns are not necessarily decreasing in own type—so that simple analogies from Akerlof's (1970) lemons model are not appropriate to characterize the potential equilibria of the merger game. In particular, we demonstrate that the properties of the merger return functions $g_i, i = 1, 2$, depend crucially on the merged firm's technology $z_M(z_i, z_j)$.

## 3.2 Case A: Buyer-Dominated Mergers

First, consider the case where the merged firm adopts the buyer's technology ($z_M \equiv z_1$).[14] Figure 1 indicates that, for buyer-dominated mergers, the set of marginal costs for which the merger is efficient (i.e., where joint merger returns are positive), is bounded by the triangle *ABE*. That is, only if the buyer has a substantial efficiency advantage over the seller can a buyer-dominated merger be efficient. For cost combinations within the triangle *BDE*, Salant et al. (1983) show that mergers are inefficient.

For buyer-dominated mergers with cash payment, the buyer's merger returns are given by

$$\begin{aligned} g_1(z_1, z_2) &\equiv \pi^M(z_1, z_2) - \pi_1(z_1, z_2) - p \\ &= \frac{(a + 2z_1 + c_3)^2}{9} - \frac{(a + 3z_1 - z_2 + c_3)^2}{16} - p, \end{aligned} \tag{2}$$

whereas the seller's merger returns are

$$g_2(z_1, z_2) \equiv p - \pi_2(z_2, z_1) = p - \frac{(a + 3z_2 - z_1 + c_3)^2}{16}. \tag{3}$$

Clearly, Assumption 3 is satisfied, so that $\partial g_i / \partial z_j > 0, i, j = 1, 2, i \neq j$, because higher competitor types negatively affect own stand-alone profits but

[14] This makes sense when the productivity of a firm is mainly determined by its managers' *tacit knowledge*. In this case, it is not clear whether this knowledge will find its way into the new firm. For instance, when firm 1 buys firm 2 at some given price, there may be no incentive for the managers of firm 2 to inform the new firm's management about useful business knowledge they might have, so that the new firm will have to adopt the buyer's technology.

leave post-merger profits unaffected.[15] Also, for the seller, merger returns are unambiguously decreasing in own type ($\partial g_2/\partial z_2 < 0$): High-type sellers earn high stand-alone profits, whereas the takeover price is independent of types.

For the buyer, however, it is generally ambiguous whether merger returns are increasing or decreasing in own type: Not only do high-type buyers earn higher stand-alone profits, they also earn higher profits in the merged entity. It is straightforward to show that the latter effect dominates over the former if the seller is "not too inefficient" relative to the buyer. Figure 1 illustrates this: For marginal costs $(c_1, c_2)$ within the triangle $BCF$, merger returns for the buyer are *increasing* in own type ($\partial g_1/\partial z_1 > 0$). For costs $(c_1, c_2)$ within the triangle $ABF$, however, merger returns are decreasing in own type ($\partial g_1/\partial z_1 < 0$). This indicates that the monotonicity properties of the buyer's merger return function crucially depend on the relevant type set.

### 3.3 Case B: Seller-Dominated Mergers

Next, consider the other polar case where the merged firm adopts the technology of the seller ($z_M \equiv z_2$). Figure 1 indicates that the set of types for which the merger is efficient is bounded by $BCD$. That is, only if the seller has a substantial efficiency advantage over the buyer can a seller-dominated merger be efficient.

In this setting, the seller's merger returns are given by (3) and thus remain decreasing in own type ($\partial g_2/\partial z_2 < 0$). The buyer's merger returns,

$$g_1(z_1, z_2) = \frac{(a + 2z_2 + c_3)^2}{9} - \frac{(a + 3z_1 - z_2 + c_3)^2}{16} - p,$$

are also decreasing in own type ($\partial g_1/\partial z_1 < 0$), as stand-alone profits are increasing in $z_1$, whereas post-merger profits are independent of $z_1$ by definition.

[15] The buyer's post-merger profit, $\pi^M$, does not depend on $z_2$, and the seller's post-merger profit is simply $p$.

## 3.4 Case C: Rationalization Mergers

Finally, consider the case where the merged firm adopts the technology of the more efficient firm ($z_M = \max(z_1, z_2)$), which is the most common assumption in the literature. Again, joint merger returns can only be positive for sufficiently heterogeneous firms. However, as either technology can be implemented, mergers are efficient within both triangles $ABE$ and $BCD$ in Figure 1.

The merger returns for the seller are given by (3); they are thus decreasing in own type ($\partial g_2/\partial z_2 < 0$). For the buyer, merger returns are

$$g_1(z_1, z_2) = \frac{(a + 2\max(z_1, z_2) + c_3)^2}{9} - \frac{(a + 3z_1 - z_2 + c_3)^2}{16} - p.$$

The monotonicity properties of the buyer's merger returns again depend crucially on the firms' types: (i) If the seller is more efficient than the buyer ($z_1 < z_2$), the rationalization merger corresponds to the seller-dominated merger, so that the buyer's returns are unambiguously decreasing in own type ($\partial g_1/\partial z_1 < 0$). (ii) If the buyer is more efficient than the seller ($z_1 \geq z_2$), the rationalization merger corresponds to the buyer-dominated merger, so that buyer's merger returns are either *increasing* or decreasing in own type, depending on the relevant type set.

## 3.5 Summary

The above discussion illustrates that low-type firms have not necessarily more to gain from mergers than high-type firms: Even for the standard linear Cournot oligopoly model and the simplest assumption on the division of post-merger profits—cash compensation for the seller, post-merger profits of the new firm for the buyer—different types of mergers are likely to give rise to very different merger returns functions, as summarized in the following statement.

**Observation 1 (merger return properties)** *In the linear Cournot model with symmetric information and cash compensation for the seller, the* ***seller's***

***merger returns*** *are always* ***decreasing*** *in own type. The* ***buyer's merger returns****, in contrast, are*

*(i)* ***increasing or decreasing*** *in own type for buyer-dominated mergers, depending on the relevant type set;*

*(ii)* ***decreasing*** *in own type for seller-dominated mergers;*

*(iii)* ***increasing or decreasing*** *in own type for rationalization mergers, depending both on the relevant type set and whether the buyer is more efficient than the seller.*

With these properties of merger returns under symmetric information, it is difficult to see how the conventional wisdom from the standard lemons market model carries over to mergers under asymmetric information. To better understand to what extent the lemons market rationale is useful for understanding mergers under two-sided asymmetric information, we now provide an analysis of the Bayesian equilibria of our reduced-form merger game under various assumptions on the merger return functions. Before doing so, we remark that Assumptions 1 to 3 hold in our linear Cournot example with cash compensation, so that results relying on these assumptions will be applicable.

# 4 Analyzing Merger Equilibria Under Asymmetric Information

We now return to the reduced-form merger game with two-sided asymmetric information introduced in Section 2. We characterize its Bayesian equilibria in general terms, depending on the properties of the merger return functions.

## 4.1 Two-Sided Lemons Equilibria

Though we believe the other cases are more relevant and more interesting, we start with conditions under which low types are more likely to merge in equilibrium, that is, there is a cut-off equilibrium where only low types consent to a merger. We call this a *two-sided lemons equilibrium.*[16]

We shall use the following terminology.

**Definition 2** *The function $G_i : [\underline{z}_i, \overline{z}_i] \rightarrow \mathbb{R}$ satisfies* ***strong downward single crossing (SSC$^-$)*** *if, for all $z_i', z_i'' \in [\underline{z}_i, \overline{z}_i]$ such that $z_i' > z_i'', G_i(z_i') \geq 0$ implies $G_i(z_i'') \geq 0$ and $G_i(z_i') > 0$ implies $G_i(z_i'') > 0$.*

This definition is closely related to the familiar single-crossing property of incremental returns (Milgrom and Shannon 1994).[17] We first give a cut-off condition in terms of expected merger returns, and then consider more primitive conditions on merger returns without asymmetric information.[18]

**Lemma 1 (cut-off property)** *Suppose $G_i(z_i; B_j, f_j)$ satisfies $SSC^-$ in $z_i$ for all $B_j \subset \mathcal{Z}_j$, $i = 1, 2$, $j \neq i$ and all $f_j$. Then every Bayesian Equilibrium $(s_1^*, s_2^*)$ in pure strategies with $\mathbb{P}[B_i(s_i^*)] > 0$ for $i = 1, 2$ satisfies the cut-off-property, that is, there are cut-off values $z_i^* \in \mathcal{Z}_i$ such that*

$$s_i^*(z_i) = \begin{cases} 1, & \text{if } z_i \leq z_i^*; \\ 0, & \text{if } z_i > z_i^*; \end{cases} \qquad i = 1, 2.$$

[16] Equilibria with a monotone relation between types and strategies are common in Bayesian games: Examples include first-price auctions where the type is the bidder's valuation and the strategy is the bid, double auctions where the types of buyers and sellers are valuations and costs, and the strategies are bids and asks (Chatterjee and Samuelson 1983), wars of attrition where the type is the valuation for the prize and the strategy is the quitting period, and games of public good provision where types correspond to the costs of providing a public good and actions correspond to the provision decision (see Fudenberg and Tirole (1991) for a discussion of these games). Athey (2001) analyzes more generally under which conditions such monotonicity results arise.

[17] Let $\Pi_i(s_i, z_i; B_j, f_j)$ define the expected payoff from strategy $s_i$ for a firm with type $z_i$, facing a competitor characterized by $B_j$ and $f_j$. Then $\Pi_i(s_i, z_i; B_j, f_j)$ satisfies the Milgrom-Shannon Single-Crossing Property in $(-s_i, z_i)$ if and only if $G_i$ satisfies $SSC^-$.

[18] Using the equivalence between $SSC^-$ and the Milgrom-Shannon condition, Lemma 1 is a special case of Theorem 1 in Athey (2001).

**Proof.** See Appendix. ■

The intuition for Lemma 1 is as follows: $SSC^{-}$ states that, for any distribution of $z_j$, if some type $z_i$ consents to a merger, so will any lower type $z_i' < z_i$, no matter what the distribution of $z_j$ is. The result applies this property to the distribution of $z_j$ corresponding to the equilibrium behavior of $z_j$.

Lemma 1 immediately implies the following result.

**Proposition 1 (two-sided lemons)** *If $g_i(z_i, z_j)$ is monotone decreasing in $z_i, i, j = 1, 2, i \neq j$, then every Bayesian Equilibrium satisfies the cut-off property.*

The intuition for this result is simple: If higher types face lower merger returns for arbitrary realizations of types, then clearly they must gain less in expectation. In the linear Cournot setting discussed above, the condition that merger returns are decreasing in own type for both firms corresponds to a seller-dominated merger (see section 3.3). In this particular case, only low types (if any) consent to a merger under two-sided asymmetric information.

As we demonstrated in Borek et al. (2003), Proposition 1 generalizes to a wide class of functions $g_i$ that we call "essentially monotone decreasing". This class, which is closely related, but not identical to the class of functions satisfying downward single crossing, includes many functions that are non-monotone decreasing in $z_i$ (e.g. many functions for which $g_i$ is single-peaked in $z_i$). Even so, the cut-off property does not hold for some of the merger return functions discussed in Section 3, as we shall show now.

## 4.2 Lemons-and-Peaches Equilibria

Consider the case where merger returns are increasing in own type for one firm, say firm 1, and decreasing in own type for the other, corresponding to a buyer-dominated merger where the buyer is not too inefficient in the linear Cournot setting (see Section 3.2).

Under these circumstances, Proposition 1 has the following straightforward implication:

**Proposition 2** *Suppose $g_1$ is monotone increasing in $z_1$, and $g_2$ is monotone decreasing in $z_2$. Then, in any Bayesian Equilibrium in pure strategies with $\mathbb{P}[B_i(s_i^*)] > 0$ for $i = 1, 2$, there exist $z_1^H$ and $z_2^L$ such that $s_1(z_1) = 1$ if and only if $z_1 \geq z_1^H$ and $s_2(z_2) = 1$ if and only if $z_2 \leq z_2^L$.*

**Proof.** Redefine the firms' types as $y_1 = -z_1$ and $y_2 = z_2$ and apply Proposition 1. ■

Proposition 2 reveals that the lemons rationale may be highly misleading in the context of mergers: As argued in Section 3, even though a high-type firm 1 foregoes higher stand-alone profits than a low-type firm 1 when entering a merger, it gains more from the merger, as it also performs better in the merged entity. In this case, only high types of firm 1 ("peaches") will consent to the merger in Bayesian equilibrium. For firm 2, the lemons rationale remains correct: In equilibrium, only low types will consent to the merger. As a result, we obtain an equilibrium where low types of firm 2 merge with high types of firm 1. We call this a *lemons-and-peaches equilibrium.* As the linear Cournot model with buyer-dominated mergers satisfies the conditions of Proposition 2 when firms are not too asymmetric, we expect a lemons-and-peaches equilibrium to emerge under these circumstances.

## 4.3 Rationalization Mergers

We finally examine the case where the merger returns of one firm, say firm 1, are non-monotone in own type. In the linear Cournot example, for instance, firm 1's merger returns were decreasing in own type for $z_1 < z_2$ and increasing for $z_1 \geq z_2$ (for firm 2 not too inefficient). In such a setting, we obtain the following partial characterization of the reduced-form merger game.

**Proposition 3** *Suppose $g_1$ is monotone increasing in $z_1$ for $z_1 \geq z_2$, and $g_2$ is monotone decreasing in $z_2$. Then* *(i) there exists a* $\widetilde{z}_2 \in \mathcal{Z}_2$ *such that* $s_2(z_2) = 1$ *if and only if* $z_2 \leq \widetilde{z}_2$.

*(ii) if, for some* $z_1 > \widetilde{z}_2, s_1(z_1) = 1$, *then* $s_1(z_1') = 1$ *for all* $z_1' > \widetilde{z}_2$.

**Proof.** (i) Follows immediately as $g_2$ is decreasing in $z_2$ by assumption. (ii) Suppose $z_1 > \widetilde{z}_2$. If $s_1(z_1) = 1$, we have $\int_{\underline{z}}^{\widetilde{z}_2} g_1(z_1, z_2) f_2(z_2) dz_2 \geq 0$. Since $g_1$ is increasing in $z_1$ for $z_1 \geq z_2$, and $z_1' > z_1 > \widetilde{z}_2$, we obtain $\int_{\underline{z}}^{\widetilde{z}_2} g_1(z_1', z_2) f_2(z_2) dz_2 \geq 0$. ■

Proposition 3 excludes equilibria such as those sketched in Figure 2, where bold lines correspond to sets of types consenting to a merger. Thus, there are no equilibria where $B_1 = \left[z_1^{\min}, z_1^{\max}\right]$ and

$$\widetilde{z}_2 \leq z_1^{\min} < z_1^{\max} < \overline{z} \quad \text{or} \quad z_1^{\min} \leq \widetilde{z}_2 < z_1^{\max} < \overline{z}.$$

<Figure 2 around here>

## 4.4 Necessary and Sufficient Conditions for Mergers

Our analysis in the above section strongly restricts the possible set of equilibria. However, it does not exclude the existence of equilibria without mergers. In this section, we therefore analyze more carefully whether trade actually occurs in equilibrium.[19]

### 4.4.1 The No-Merger Equilibrium

The first result shows that, for arbitrary distributions of types, there is always a degenerate cut-off equilibrium where no type merges.

**Proposition 4 (no-merger)** *Each strategy pair* $(s_1, s_2)$ *with*

$$\mathbb{P}[B_i(s_i)] \equiv \int_{B_i} f_i(z_i) dz_i = 0, \quad i = 1, 2,$$

*is a Bayesian Equilibrium of the merger game.*

[19] The analysis is closely related to the analysis of no-trade equilibria in the standard adverse selection literature.

**Proof.** See Appendix. ■

The result is very intuitive: If both firms believe that the other firm will not consent to a merger—no matter what its type is—it is a (weakly) best response not to consent, and beliefs are correct in equilibrium. Thus, there always is an equilibrium where firms merge with probability zero. Note, however, that the no-merger equilibrium is Pareto-dominated in terms of expected profits whenever a cut-off equilibrium exists where firms consent to a merger with strictly positive probability.

### 4.4.2 Decreasing Buyer Returns

The next proposition deals with the case that merger returns are decreasing in own type for both the buyer and the seller. It gives a sufficient condition under which there is no other than the no-merger equilibrium.

**Proposition 5** *Suppose* $\mathcal{Z}_i = \mathcal{Z}_j = \mathcal{Z}$. *Further assume that, for* $i, j = 1, 2, j \neq i$,

*(i)* $g_i(z_i, z_j)$ *is monotone decreasing in* $z_i$, *and*

*(ii)* $g_i(z_i, z_j) \leq 0$ *for* $z_i = z_j$.

*Then there is no Bayesian Equilibrium where a positive measure of firms consents to a merger.*

**Proof.** See Appendix. ■

Proposition 5 states that if mergers between identical types are not profitable under certainty for either firm and low types have stronger incentives to merge, then no trade is the only equilibrium.[20]

[20] The result generalizes to the case where $\mathcal{Z}_1 \neq \mathcal{Z}_2$ by demanding more generally that $g_i(z_i, z_j) \leq 0$ on the diagonal of $\mathcal{Z}$ rather than for $z_i = z_j$. Also, even this condition can be generalized further. However, though the present formulation of the proposition is not the most general, it is the easiest one to apply.

The result is useful in applications such as the linear Cournot model where, for homogeneous firms, joint merger returns are negative. Then, individual merger returns must also be negative for at least one of the firms for any budget-balancing split of profits. Suppose further that the transaction is stock-financed rather than cash-financed, as in the examples of Section 3. Specifically, assume that firms agree to predetermined shares $\alpha_i \in [0,1], \alpha_1 + \alpha_2 = 1$, of the new firm's joint profits rather than cash payment, so that $\pi_i^M(z_i, z_j) = \alpha_i \pi^M(z_i, z_j)$. Clearly, if $\alpha_i$ is sufficiently close to $1/2$, both firms must have negative merger returns for identical types. Further, it is straightforward to show that if the firms' profit shares are not extremely asymmetric, merger returns are decreasing in own type for both firms. Therefore, Proposition 5 applies and there is no equilibrium where firms merge with strictly positive probability.

#### 4.4.3 Increasing Buyer Returns

So far, we have established that, for merger returns that are decreasing in own type, no trade is the likely outcome, at least in a setting corresponding to the Cournot example. We have already seen, however, that this particular monotonicity requirement is surprisingly restrictive, even though it can be justified, for example, for seller-dominated mergers in the linear Cournot model. We therefore consider the case where $g_1$ is monotone increasing in $z_1$, and $g_2$ is monotone decreasing in $z_2$, as for many buyer-dominated mergers in the linear Cournot model. Thus, in equilibrium, high-type buyers acquire low-type sellers.

Let
$$G_1^L(z_1, z_2) \equiv \int_{\underline{z}_2}^{z_2} g_1(z_1, \widetilde{z}_2) f_2(\widetilde{z}_2) d\widetilde{z}_2$$
denote the expected returns of firm 1, anticipating that only low types of firm 2 (below $z_2$) will consent to a merger. Further, let

$$G_2^H(z_1, z_2) \equiv \int_{z_1}^{\overline{z}_1} g_2(\widetilde{z}_1, z_2) f_1(\widetilde{z}_1) d\widetilde{z}_1$$

denote the expected returns of firm 2, anticipating that only high types of firm 1 (above $z_1$) will consent to the merger. Clearly, an equilibrium with cut-off types $\left(z_1^H, z_2^L\right)$ requires $G_1^L\left(z_1^H, z_2^L\right) = 0 = G_2^H\left(z_1^H, z_2^L\right)$. We use the notation

$$G_1(z_1) \equiv G_1^L(z_1, \overline{z}_2); \quad G_2(z_2) \equiv G_2^H(\underline{z}_1, z_2)$$

to describe the expected payoffs in the boundary case where player $i$ expects all types $z_j \in \mathcal{Z}_j, j \neq i$, to consent. Then we obtain the following result.

**Proposition 6** *Assume that $g_1$ and $g_2$ are continuous functions, with $g_1$ monotone increasing in $z_1$ and $g_2$ monotone decreasing in $z_2$.*

*(i) Suppose*

$$G_1(\overline{z}_1) < 0 \vee g_2(\overline{z}_1, \underline{z}_2) < 0, \tag{4}$$

*or*

$$\exists z_2 \in \mathcal{Z}_2 \text{ such that } \max\left(G_1^L(\overline{z}_1, z_2), g_2(\overline{z}_1, z_2)\right) < 0, \tag{5}$$

*or*

$$\exists z_1 \in \mathcal{Z}_1 \text{ such that } g_1(z_1, \underline{z}_2) > 0 > G_2^H(z_1, \underline{z}_2). \tag{6}$$

*Then there is no equilibrium with mergers for a positive measure of types.*

*(ii) Suppose neither of conditions (4)-(6) holds. Then there is an equilibrium with mergers for a positive measure of types.*

**Proof.** See Appendix. ■

The conditions under which the no-merger equilibrium is unique are fairly restrictive. Condition (4) requires that merger returns are negative even in very favorable cases: Even the best type of player 1 does not want to merge with an average player 2, or the worst type of player 2 does not want to merge with the best type of player 1. Condition (5) requires the existence of a $z_2$ that is low enough that even the best type 1 does not want to merge if the cut-off value is $z_2^L = z_2$, but on the other hand high enough that $z_2$

himself does not want to merge with the best type of $z_1$. Finally, condition (6) requires the existence of a type 1 player that is good enough that he wants to merge with the lowest type of player 2, but bad enough that player 2 does not want to merge if he expects type 1 players who consent to be from $[z_1', \overline{z}_1]$.

Figure 3 illustrates the result. The downward sloping line $G_1^L(z_1, z_2) = 0$ is the 'zero-returns curve' for player 1: If a player of type $z_1$ expects types $z_2$ and below to consent to a merger, the expected merger returns are zero if and only if $(z_1, z_2)$ is on $G_1^L(z_1, z_2) = 0$. The line is downward sloping because $g_1(z_1, z_2)$ and therefore $G_1^L(z_1, z_2)$ are increasing in both $z_1$ and $z_2$, so that a higher type of player 1 is willing to merge with lower type 2 players. Similarly, consider $G_2^H(z_1, z_2) = 0$. If a player of type $z_2$ expects types $z_1$ and above to consent to a merger, the expected merger returns are zero if and only if $(z_1, z_2)$ is on $G_2^H(z_1, z_2) = 0$. By analogous reasoning as before, as $g_2(z_1, z_2)$ and thus $G_2^H(z_1, z_2)$ are increasing in $z_1$ and decreasing in $z_2$, $G_2^H(z_1, z_2) = 0$ is upward sloping, so that a lower type of player 2 is willing to merge with lower type 1 players.

If condition (5) holds, $G_1^L(z_1, z_2) = 0$ intersects the right boundary of the type space above $G_2^H(z_1, z_2) = 0$. Intuitively, both types of players do not expect to gain enough from a merger. If condition (6) holds, $G_1^L(z_1, z_2) = 0$ intersects the lower boundary of the type space to the left of $G_2^H(z_1, z_2) = 0$. Intuitively, type 1 players are too keen on merging relative to type 2 players. Thus, if a type 1 player consents, this is not necessarily very positive information for type 2 players: Though the type 1 players that consent to a merger tend to be better than those that do not, they are still not good enough on average, even for the worst possible type of player.

<Figure around 3 here>

## 4.5 Ex-Post Efficiency of Lemons and Peaches Equilibria

Finally, we briefly discuss the ex-post efficiency properties of the lemons-and-peaches equilibrium. As in the example in Section 3, we refer to player 1 as buyer and player 2 as seller.

**Proposition 7 (ex-post efficiency)** *Suppose $g_1$ is monotone increasing in $z_1$ and $g_2$ is monotone decreasing in $z_2$. Then, whenever an interior equilibrium $\left(z_1^H, z_2^L\right)$ exists, the following four cases coexist for non-degenerate sets of types.*

*(i) Mergers with **buyer regret (BR)**: In particular, low-type buyers regret merging with low-type sellers.*

*(ii) Mergers with **seller regret (SR)**: In particular, high-type sellers regret merging with low-type buyers.*

*(iii) **Pareto-improving mergers (PIM)**: In particular, both high-type buyers and high-type sellers benefit from the merger ex post.*

*(iv) **Inefficient Non-Mergers (INM)**: In particular, some efficient mergers of high-type sellers and high-type buyers do not occur.*

**Proof.** See Appendix ■

The intuition of Proposition 7 is straightforward. As cut-off types break even on average, they must regret a merger with the lowest type they expect to consent to a merger. Therefore, a buyer of type $z_1^H$ regrets a merger with a seller of type $\underline{z}_2$. Similarly, a seller of type $z_2^L$ regrets a merger with a buyer of type $z_1^H$. By continuity, there are regions like *BR* and *SR* in Figure 3 where buyers and sellers, respectively, regret the merger ex post (results (i) and (ii) in the proposition). The most interesting aspect of this is that sellers may regret the transaction even if the price is independent of their type: The reason is that, when the buyer's type turns out to be lower than expected, they realize that their stand-alone profits would have been higher than the transaction price.

Similarly, because cut-off types break even on average, they must have positive merger returns if they merge with the best type of the other player. In particular, a merger of a buyer with type $\overline{z}_1$ and a seller of $z_2^L$ will benefit both firms. By continuity, there is a region like *PIM* where both high-type buyers and high-type sellers benefit ex post from the merger (result (iii) in the proposition). Finally, there is a region like *INM* where mergers do not occur even though they would be ex post efficient. This is because, in the cut-off equilibrium, some high-type sellers slightly above $z_2^L$ do not consent to the merger for fear of selling out too cheaply.

# 5 The Standard Lemons Market Benchmark

As outlined in the introduction, our analysis of the cut-off and no-trade equilibria in Section 4 is related to the standard literature on one-sided asymmetric information emanating from Akerlof (1970). This literature focuses on the circumstances under which an object of given quality—which is only known to the seller—will be traded at some price $p$, so that the seller's post-merger profits are independent of types. Also, stand-alone profits are independent of competitor types. In this section, we sketch how cut-off and no trade results for this standard setting can be regarded as special cases of our framework. Thus, translated to our setting the following conditions hold:

**Condition 1** *The buyer's distribution function $F_1(z_1)$ is degenerate.*

**Condition 2** *Both-firms stand-alone profits, $\pi_i$, are independent of $z_j, j \neq i$.*

**Condition 3** *The seller's post-merger profit, $\pi_2^M$, is independent of $z_2$.*

These conditions make sense when a well-known firm diversifies into some other market by buying an unknown firm. We then interpret $\pi^M$ as the sum of the buyer's stand-alone profit and the profit he obtains in the new activity. A cut-off result for this setting is a special case of our Proposition 1.

**Corollary 1** *Suppose conditions 1-3 hold. Then every Bayesian equilibrium satisfies the cut-off property with respect to* $z_2$.

To understand the result, note that it suffices to consider the behavior of the seller, as the buyer's distribution function $F_1$ is degenerate by condition 1. Furthermore, the seller's merger returns $g_2 = p - \pi_2(z_2)$ are monotone decreasing in own type, since its post-merger profit does not depend on own type by condition 3; Proposition 1 thus immediately implies the result.

The next result gives a condition for no trade in the case of one-sided asymmetric information.

**Proposition 8** *Suppose conditions 1-3 hold. Further assume that*

$$\int_{\underline{z}_2}^{z_2} \left(\pi^M \left(z_1, \widetilde{z}_2\right) - \pi_1 \left(z_1\right)\right) f_2 \left(\widetilde{z}_2\right) d\widetilde{z}_2 < \pi_2 \left(z_1, z_2\right) \text{ for all } z_1, z_2. \qquad (7)$$

*Then there is no equilibrium where a positive measure of players merges.*

**Proof.** By condition 2, $\pi_2^M$ must be a fixed constant $p$. Thus player 2 consents to a merger if and only if $p \geq \pi_2 \left(z_1, z_2\right)$. By corollary 1, any equilibrium satisfies the cut-off property. Denote the cut-off type of player 2 as $z_2^*$. By (7), we have

$$\int_{\underline{z}_2}^{z_2^*} \left(\pi^M \left(z_1, \widetilde{z}_2\right) - \pi_1 \left(z_1\right)\right) f_2 \left(\widetilde{z}_2\right) d\widetilde{z}_2 < \pi_2 \left(z_1, z_2^*\right).$$

Thus, if $p \geq \pi_2 \left(z_1, z_2^*\right)$, so that the cut-off type of player 2 consents to merger, the expected merger returns for player 1 are negative for every potential cut-off type $z_2^* > \underline{z}_2$. ■

Condition (7) states that, for any cut-off type $z_2$, the buyer's expected merger returns, gross of the takeover price, are bounded above by the seller's stand-alone profits. If this condition holds, the buyer's merger returns are not sufficiently high to compensate the seller for foregone stand-alone profits, and the merger will thus not occur. This outcome is clearly (ex-post) inefficient whenever there exist some combinations of types for which the merger increases joint profits.

Even though conditions 1-3 are appropriate in some very specific settings, this paper demonstrated that they are misleading for a more general theory of mergers under asymmetric information.

# 6 Conclusions

This paper has shown that mergers under asymmetric information differ from standard lemons problems in two ways. First, the net gains from trade for the buyer and the seller typically depend on the types of both firms. Second, low types are not necessarily more inclined to trade than high types. As a result, mergers should be analyzed in a setting with two-sided asymmetric information about firm types, where the prediction of the standard lemons model—if any, only low type firms will be willing to merge—usually does not hold.

Using a simple linear Cournot model with cash payment, we illustrate that the buyer's merger returns under symmetric information may well be increasing (rather than decreasing) in own type when the buyer has a strong impact on the technology adopted by the merged firm. The merger pattern under two-sided asymmetric information crucially depends on the properties of the buyer's merger return function under symmetric information: For mergers with sufficient seller influence on the merged firm's technology, a two-sided lemons cut-off equilibrium with low-type sellers and low-type buyers emerges. However, for buyer-dominated mergers, a lemons and peaches equilibrium may emerge where low-type sellers merge with high-type buyers.

The standard Akerlof (1970) lemons model further overemphasizes the no-trade problem in the context of mergers. Though it is still true that asymmetric information may prevent some efficient mergers, there is no general presumption that trade is inefficiently low. In fact, equilibria with trade arise quite generally for the case that seller returns are decreasing in types, but buyer returns are increasing. The market for firms is thus unlikely to collapse.

This does not imply, however, that the market for firms is efficient in the sense that mergers occur if and only if they increase joint profits. This suggests a natural extension of the paper: It would be interesting to find out whether there are general incentive compatible and individually rational mechanisms with balanced budgets that avoid ex-post regret on both sides of the market. The extension is non-trivial because it involves solving a mechanism design problem with interdependent valuations. However, our allocation problem has a relatively simple structure: The only task is to induce firms to merge if and only if the merger is efficient (i.e., increases joint profits). Therefore, it does not seem hopeless to design mechanisms that solve the problems discussed in this paper. Nevertheless, at least if ex-post implementability is required, it is generally hard to achieve efficiency (Gaertner and Schmutzler 2005).

# Appendix

## Proof of Lemma 1

Firm $i$'s expected merger return, facing firm $j$ with strategy $s_j$, is $G_i(z_i; B_j, f_j)$. If $G_i(z_i; B_j, f_j)$ is positive, firm $i$ will consent to the merger, otherwise it will reject the merger. By assumption, $G_i(z_i; B_j, f_j)$ satisfies $SSC^-$ in $z_i$. Thus, if a $z_i \in \mathcal{Z}_i$ exists such that $G_i(z_i; B_j, f_j) > 0$, then there exists a $z_i^\circ(s_j)$ such that $G_i(z_i; B_j, f_j) \geq 0$ if and only if $z_i \leq z_i^\circ(s_j)$. Now define

$$\tilde{R}_i(s_j) = \begin{cases} z_i^\circ(s_j), & \text{if } z_i^\circ(s_j) \leq \overline{z_i}; \\ \overline{z_i}, & \text{if } z_i^\circ(s_j) \geq \overline{z_i} \text{ or if } z_i^\circ(s_j) \text{ does not exist.} \end{cases}$$

Then firm $i$'s optimal reaction is

$$R_i(z_i, s_j) = \begin{cases} 1, & \text{if } z_i \leq \tilde{R}_i(s_j); \\ 0, & \text{if } z_i > \tilde{R}_i(s_j). \end{cases}$$

In particular, for an equilibrium strategy $s_j$, the best reply has the required cut-off structure.

## Proof of Proposition 4

Suppose firm $i$ plays strategy $s_i(z_i)$ with $\mathbb{P}[B_i(s_i)] = 0$. Then the probability that a merger takes place is zero and therefore firm $j \neq i$ is indifferent between any strategies it can play; in particular, every strategy $s_j(z_j)$ with $\mathbb{P}[B_j(s_j)] = 0$ is a best response.

## Proof of Proposition 5

**Proof.** Assume that, for all $i \in \{1, 2\}$, there is no $\widehat{z}_i$ such that $g_i(\widehat{z}_i, \widehat{z}_i) \geq 0$. Suppose w.l.o.g. that there is a non-trivial cut-off equilibrium $(z_1^*, z_2^*)$ with $z_1^* \geq z_2^*$. As $g_1(z_1^*, z_1^*) < 0$ and $g_1$ is non-decreasing in $z_2$, $g_1(z_1^*, z_2) < 0$ for all $z_2 \leq z_2^*$. Therefore, expected equilibrium profits for firm 1 are $\int_{\underline{z}_2}^{z_2^*} g_1(z_1^*, z_2) f_2(z_2) dz_2 < 0$, contradicting the condition that

$$\int_{\underline{z}_2}^{z_2^*} g_1(z_1^*, z_2) f_2(z_2) dz_2 = 0$$

for the cut-off values $(z_1^*, z_2^*)$. ■

## 6.1 Proof of Proposition 6

**Proof.** (i) Suppose (4) holds. Then, merger returns are negative for at least one of the firms even under the most favorable assumption about the other player's cut-off value, so that a merger will never occur.

Suppose now that (4) does not hold, but (5) does. If $G_1^L(\overline{z}_1, z_2) < 0$ or $g_2(\overline{z}_1, z_2) = G_2^H(\overline{z}_1, z_2) < 0$ for all $z_2 \in \mathcal{Z}_2$, there clearly is no equilibrium with mergers. Thus, suppose $G_1^L(\overline{z}_1, z_2) > 0$ for some $z_2 \in \mathcal{Z}_2$ and $G_2^H(\overline{z}_1, z_2) > 0$ for some $z_2 \in \mathcal{Z}_2$. By (5), $G_1^L(\overline{z}_1, \underline{z}_2) < 0$. Because (4) does not hold, $G_1(\overline{z}_1) = G_1^L(\overline{z}_1, \overline{z}_2) > 0$, and $G_1^L(z_1, z_2) = 0$ intersects the right boundary of the type space in some point $(\overline{z}_1, z_2')$. By (5), $G_2^H(z_1, z_2) = 0$ intersects the right boundary in some $(\overline{z}_1, z_2'')$, where $z_2'' < z_2'$ (see Figure 4, Panel (a)). Therefore, $G_2^H(\overline{z}_1, z_2) < 0$ for all $z_2 > z_2''$. Thus, for player 2 to consent to a merger if $z_1^H = \overline{z}_1$, we require $z_2^L \leq z_2''$. However,

$G_1^L(z_1, z_2'') < G_1^L(z_1, z_1') = 0$, so that player 1 will not consent to a merger if $z_2^L \leq z_2''$. As a result, a no-merger equilibrium comes about.

Now suppose (6) holds. $G_1^L(z_1, z_2) = 0$ then intersects the lower boundary of the type space in $(z_1', \underline{z}_2)$, whereas $G_2^H(z_1, z_2) = 0$ intersects with the lower boundary in $(z_1'', \underline{z}_2)$, with $z_1' < z_1''$ (see Figure 4, Panel (b)). Therefore, each player 1 with type $z_1 \geq z_1'$ will consent to a merger even with the worst type of player 2. Thus, an equilibrium would require $z_1^H \leq z_1'$. As $z_1' < z_1''$, however, $G_2^H\left(z_1^H, \underline{z}_2\right) < 0$, $\forall z_1^H \leq z_1'$. That is, again a no-merger equilibrium comes about.

<Figure 4 around here>

(ii) Since condition (4) does not hold, we have $G_1(\overline{z}_1) = G_1^L(\overline{z}_1, \overline{z}_2) > 0$ and $G_2^H(\overline{z}_1, \underline{z}_2) = g_2(\overline{z}_1, \underline{z}_2) > 0$. Table 1 summarizes all equilibrium constellations corresponding to these signs, for all combinations of signs for $G_1^L(\underline{z}_1, \underline{z}_2)$ and $G_2^H(\underline{z}_1, \overline{z}_2)$.

First suppose $G_1^L(\underline{z}_1, \underline{z}_2) < 0$ and $G_2^H(\underline{z}_1, \overline{z}_2) < 0$.

a) If $g_2(\overline{z}_1, z_2) < 0$, (5) holds, and a no-merger equilibrium occurs.

If $g_2(\overline{z}_1, \overline{z}_2) > 0$, then one of the following three cases must apply.

b) There is an interior intersection of $G_1^L(z_1, z_2) = 0$ and $G_2^H(z_1, z_2) = 0$ and therefore an interior equilibrium as in Figure 3.

c) $G_1^L(z_1, z_2) = 0$ intersects the upper boundary of the type space to the right of $G_2^H(z_1, z_2) = 0$ in $\left(z_1^b, \overline{z}_2\right)$ as in Figure 5, Panel (a). Then there is an equilibrium such that the types of player 1 above $z_1^b$ and all types of player 2 consent.

d) $G_2^H(z_1, z_2) = 0$ intersects the left boundary of the type space in $(\underline{z}_1, z_2^s)$ above the intersection point of $G_1^L(z_1, z_2) = 0$. Then there is an equilibrium where all types of player 1 and all types of player 2 below $z_2^s$ consent to a merger.

Next suppose $G_1^L(\underline{z}_1, \underline{z}_2) < 0$ and $G_2^H(\underline{z}_1, \overline{z}_2) > 0$. By $G_2^H(\underline{z}_1, \overline{z}_2) > 0$, all types of player 2 consent to the merger, no matter what their expectations about player 1 are.

e) Suppose $G_1^L(\underline{z}_1, \overline{z}_2) > 0$. For the cut-off value $\overline{z}_2$ the expected merger returns are positive even for the lowest possible type of player 1. There is thus an equilibrium where all types on both sides of the market consent to the merger.

f) If $G_1^L(\underline{z}_1, \overline{z}_2) < 0$, the line $G_1^L(z_1, z_2) = 0$ intersects the upper boundary in $\left(z_1^b, \overline{z}_2\right)$ as in Figure 5, Panel (a). Then there is an equilibrium where all types of player 1 above $z_1^b$ and all type 2 players consent to a merger.

Next, suppose $G_1^L(\underline{z}_1, \underline{z}_2) > 0$ and $G_2^H(\underline{z}_1, \overline{z}_2) < 0$.

g) The case $G_2^H(\underline{z}_1, \underline{z}_2) < 0$ corresponds to (6). Thus, there is no merger equilibrium by part (i).

h) Now suppose $G_2^H(\underline{z}_1, \underline{z}_2) > 0$. By $G_1^L(\underline{z}_1, \underline{z}_2) > 0$, all types of player 1 consent, no matter what the cut-off value of player 2 is. $G_2^H(z_1, z_2) = 0$ intersects the left boundary in $(\underline{z}_1, z_2^s)$, with $z_2^s > \underline{z}_2$ (as in Figure 5, Panel (b)), and we thus have an equilibrium where all types of player 1 and all types of player 2 below $z_2^s$ consent to a merger.

Finally, suppose $G_1^L(\underline{z}_1, \underline{z}_2) > 0$ and $G_2^H(\underline{z}_1, \overline{z}_2) > 0$.

i) Then, all types of player 1 and 2 face positive expected merger returns even in the worst possible scenario, so that in equilibrium, all types on both sides of the market consent to a merger.

<Figure 5 around here>

■

Table 1: Summary of equilibria for $G_1^L(\overline{z}_1, \overline{z}_2) > 0$ or $G_2^H(\overline{z}_1, \underline{z}_2) > 0$

| | $G_1^L(\underline{z}_1, \underline{z}_2) < 0$ | $G_1^L(\underline{z}_1, \underline{z}_2) > 0$ |
|---|---|---|
| $G_2^H(\underline{z}_1, \overline{z}_2) < 0$ | a) no merger equilibrium<br>b) interior equilibrium<br>c) type 1 players above $z_1^b$ and all type 2 players merge<br>d) all type 1 players and type 2 players below $z_2^s$ merge | g) $G_2^H(\underline{z}_1, \underline{z}_2) < 0$: no merger equilibrium<br>h) $G_2^H(\underline{z}_1, \underline{z}_2) > 0$: all type 1 players and type 2 players below $z_2^s$ merge |
| $G_2^H(\underline{z}_1, \overline{z}_2) > 0$ | e) $G_1^L(\underline{z}_1, \overline{z}_2) > 0$: all types merge<br>f) $G_1^L(\underline{z}_1, \overline{z}_2) < 0$: type 1 players above $z_1^b$ and all type 2 players merge | i) all types merge |

## 6.2 Proof of Proposition 7

**Proof.** (i) Because $G_1^L(z_1^*, z_2^*) = 0$ and $g_1$ is increasing in $z_1$, we have $g_1\left(z_1^*, \underline{z_2}\right) < 0$. Thus, by continuity, $g_1(z_1, z_2) < 0$ for $(z_1, z_2)$ that are sufficiently close to $(z_1^*, \underline{z_2})$.

(ii) is analogous: Seller regret occurs for $(z_1, z_2)$ close to $(z_1^*, z_2^*)$.

(iii), (iv) Because $G_1^L(z_1^*, z_2^*) = G_2^H(z_1^*, z_2^*) = 0$ and both $g_1$ and $g_2$ are increasing in $z_1$, we have $g_1(\overline{z}_1, z_2^*) > 0$ and $g_2(\overline{z}_1, z_2^*) > 0$. By continuity, there is thus an $\varepsilon$-neighborhood of $(\overline{z}_1, z_2^*)$ such that $g_1(z_1, z_2) > 0$ and $g_2(z_1, z_2) > 0$ for all $(z_1, z_2)$ in this neighborhood. In this neighborhood, all the points with $z_2 < z_2^*$ satisfy (iii); the points above $z_2^*$ satisfy (iv). ■

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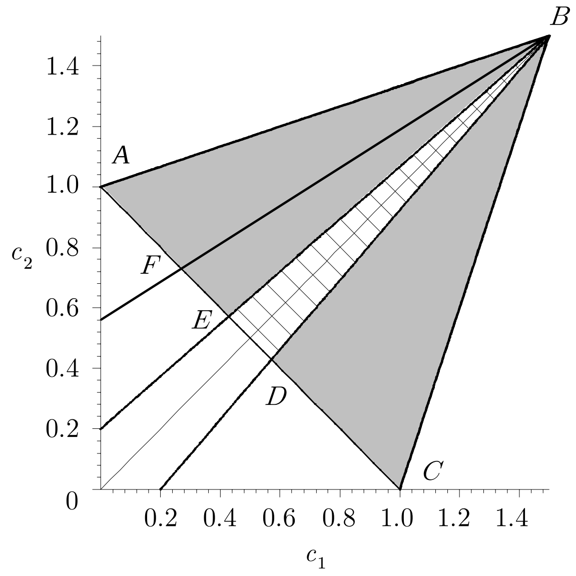


Figure 1: Admissible range of marginal costs ($a = 2, c_3 = 1$).

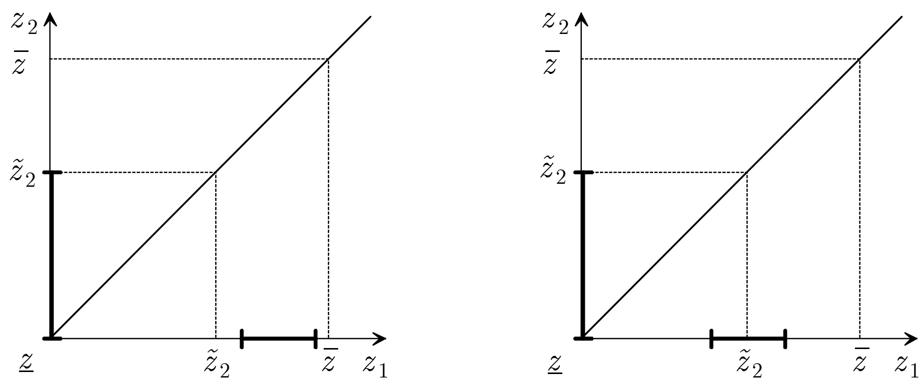


Figure 2: Equilibria excluded by Proposition 2

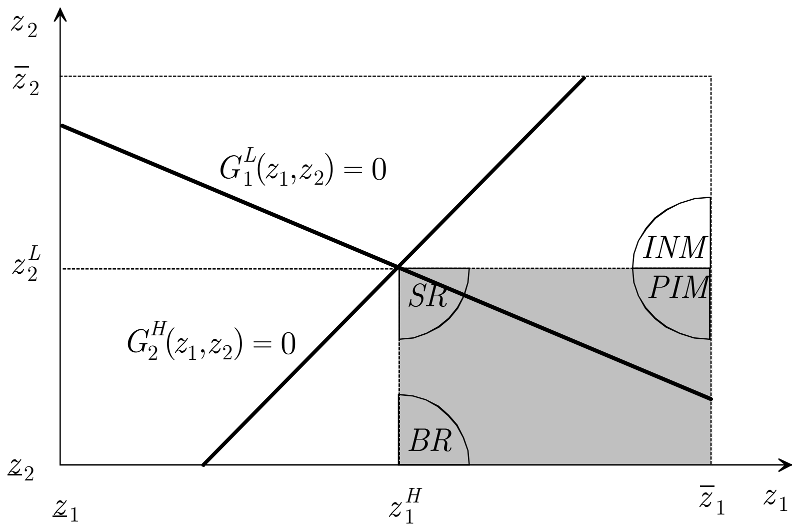


Figure 3: Characteristics of interior lemons and peaches equilibria

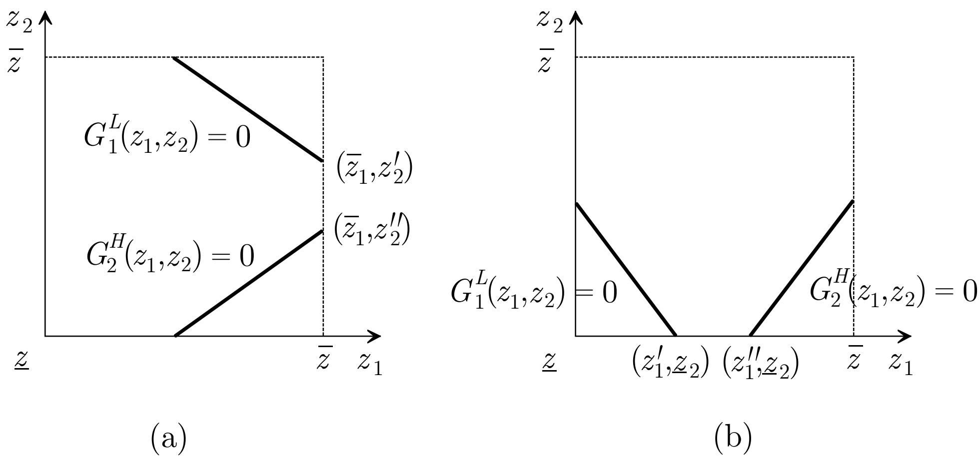


Figure 4: No-merger equilibria implied by (5) [Panel (a)] and (6) [Panel (b)]

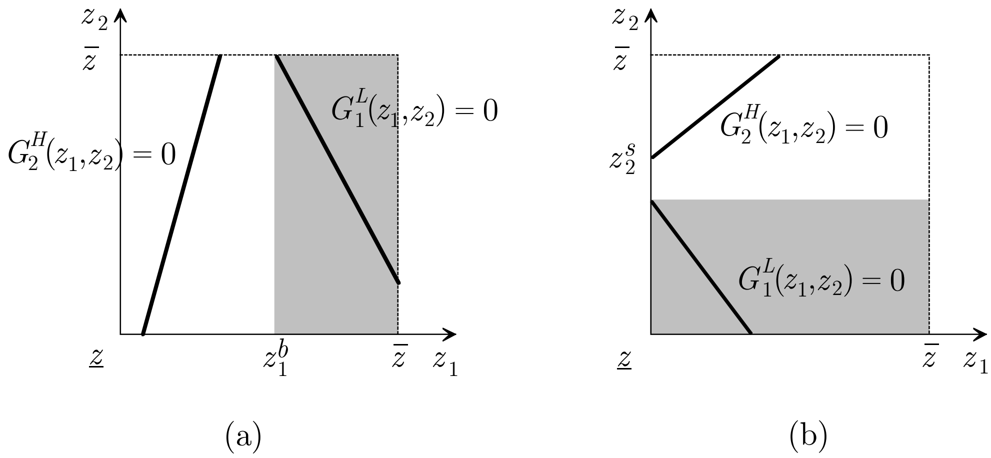


Figure 5: Lemons and peaches equilibria where all types on one side of the market consent