Platform Competition and Welfare: Media Markets Reconsidered*

Attila Ambrus† Markus Reisinger‡

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Preliminary Version

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†Department of Economics, Harvard University, Cambridge, MA 02138, e-mail: ambrus@fas.harvard.edu

‡Department of Economics, University of Munich, 80539 Munich, Germany, e-mail: markus.reisinger@lrz.uni-muenchen.de

Abstract

This paper presents a model of broadcast competition where viewers dislike advertising while producers want to reach many viewers with their commercials. In contrast to the existing literature, in our model viewers can watch different channels. We show that this leads to clear cut welfare results. In example, there is always over-provision of advertising because stations do not take into account viewers' utility loss from advertising. Moreover, separate ownership of channels is better than joint ownership. Although there is no direct competition for viewers, because they can multi-home, stations have a strong incentive to be the sole provider of their viewers to advertisers which leads to indirect competition. Because of this effect stations' profits are non-monotonic in the differentiation. We show that the results of our model fit well with recent developments in the German broadcasting market.

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1 Introduction

Advertising expenditures for television commercials have grown largely in recent years in almost all industrialized countries. In Germany expenditures for television advertising amounted to 7.74 Mrd. Euro in 2004 which is four times higher than the expenditures in the year 1990.[1] Similar numbers can be found in almost all western countries. At the same time there has always been a discussion if regulation concerning the amount and the content of commercials is necessary.[2] In the European Union there exists nowadays an advertising ceiling of 12 minutes per hour but the media commission of the European Union is currently discussing whether to abolish this law. By contrast, in USA an advertising cap was abolished already in 1981 by the Federal Communication Commission. So the question arises if there is too much advertising in an unregulated broadcasting market which would render a regulation necessary or if such a regulation would be harmful.

In recent years this question has gained a lot of attention in economic research after the seminal paper by Anderson & Coate (2005). This paper and many of its successors (see e.g. Kind, Nilssen & Sorgard (2003) or Choi (2004)) give an ambiguous answer to this question. There can either be too much or too little advertising in equilibrium dependent on parameter values. A prevalent assumption in these papers is that while producers can advertise on many channels, viewers are confined to watch only one channel. Of course, viewers can only watch one programme at a time but they often switch between channels and usually watch more the one programme during the whole time they spend watching TV. In this paper we deviate from this artificial assumption and allow viewers to endogenously watch more than one channel. Making the model more realistic on this aspect we are now able to get clear cut results in many respects. We show that there is always too much advertising in equilibrium, either if TV stations are controlled by the same firm or if there is competition between stations. The

[1]It is also 44% of all advertising expenditures in 2004 while it accounted only for 25% in 1990 (http://www.agf.de/daten/werbemarkt/werbespendings/).

[2]See Motta & Polo (1997) for a history and the problems concerning advertising regulation on TV.

reason is that stations do not take into account the direct utility loss of viewers from additional advertising. They are only concerned with the commercial price reduction due to less viewers but not with the utility loss of the inframarginal viewers. Additionally, we show that competition between the stations is always preferable to monopoly from a welfare point of view because in duopoly the firms display less commercials in equilibrium. We also receive a clear cut result concerning viewer prices, namely that (positive) viewer prices are always welfare enhancing. Again, the intuition is simple. If the viewer price is positive in equilibrium each station is broadcasting less commercials which increases welfare.

There exists also an interesting effect concerning viewers who watch both channels. The viewers who are most valuable for a station are the ones who solely watch this station and do not multi-home.[3] The reason is that the station is then a monopolist in providing these viewer to advertisers and can demand a high price for them. Instead, viewers who watch more channels are less valuable. So if the content of the stations' programme becomes more attractive more viewers are watching but also more viewers are multi-homing. We show that this second effect can dominate and stations' profits decrease.

We also give an example from the German broadcasting market and argue that some recent developments in this market fit well with the results of our model. In Germany public stations are not allowed to advertise after 8:00 p.m. There is currently a discussion if this law should be abolished and private stations are strongly opposing this abolishment. This reaction can not be explained by previous models because as a consequence of this abolishment private stations would only get more viewers. Instead, we argue that this abolishment would set an end to the monopoly position of the private stations and would therefore decrease their profits.

The rest of the paper is organized as follows. The next section presents the existing literature. The model is set out in section 3. In section 4 we derive the equilibrium for

[3]The term multi-homing was coined by Armstrong (2005). It means that viewers subscribe to both channels. Subscribing to only one is called single-homing.

joint and separate ownership of the stations. Section 5 gives the welfare maximizing result and compares it with the market outcomes. Recent developments in the German television industry and how they relate to our model are explained in section 6. The consequences of a viewer charge is analyzed in section 7. Section 8 presents a comparison to the single-homing case and section 9 concludes.

2 Related Literature

The recent literature on media markets started with the seminal paper by Anderson & Coate (2005). They were the first to analyze platform competition in the now widely used framework that stations must compete for the two sides of the market, namely viewers and advertisers.[4] The conclusions of Anderson & Coate are that the market outcome usually is not efficient, but the direction of the inefficiency is ambiguous. They show that there can be too little advertising, if the nuisance costs from advertising are low or competition between stations is high, and too much, if the opposite is true. As a consequence viewer pricing might increase or decrease welfare. They also show that a monopoly can do better or worse than duopoly dependent on parameter values.[5]

Choi (2004) extends the model of Anderson & Coate by allowing free entry of stations, modelling competition along the lines of Salop (1979).[6] He is concerned with the question how regulating the number of stations or the amount of advertising affects welfare. His result is that regulating only one of these variables might lead to an even stricter distortion of the second one (as in the theory of the second best) and it is

[4] In many examples of the two-sided markets literature, the two sides draw positive externalities on each other, like for example in the credit card industry. See Rochet & Tirole (2003, 2004) or Wright (2003).

[5] In section 7 of their paper Anderson & Coate briefly analyze the case of switching viewers in a two period model. They show that the problem of under-advertising is mitigated in this case. Their analysis already points in the direction of the results of our model.

[6] For a similar kind of model see Gal-Or & Dukes (2004).

therefore hard to give a policy advice.[7]

Kind, Nilsson & Sorgard (2003) also develop the result that there can be over- or under-provision of advertising in a media market. In additions they analyze the consequences of vertical mergers in a platform competition framework, which means a merger between a platform and an advertiser. They show that such a vertical merger might not always be profitable in this kind of frameworks. This is the case if competition is very intense because the amount of advertising would be too low in equilibrium.[8]

Peitz & Valletti (2004) explore the question whether advertising levels and welfare are higher under competition between either pay-tv or free-to-air stations. They show that if competition between platforms is sufficiently intense, moving from free-to-air to pay-tv is socially desirable. The intuition is that, in this case, free-to-air platforms advertise very little because reducing commercials is the only instrument to gain viewers, while pay-tv platforms can also vary their viewer charge. They also analyze location choice of platforms and find that there is maximal differentiation in the pay-tv case, while this does not necessarily hold with free-to-air stations.[9]

Armstrong (2005) analyzes many different kind of frameworks for platform competition. In section 5 of his paper he explores a model of advertising where platforms can either charge on a per-consumer basis or on a lump-sum basis. He shows that charging on a per-consumer basis leads to higher profits of the platforms because it relaxes competition between platforms. In his model, in contrast to the previous ones, viewers do not dislike advertising, but are neutral instead.

Summing up, the literature so far usually assumes that viewers can only watch one channel. As a consequence if competition for viewers is fierce this leads to an

[7]Crampes, Haritchabalet & Jullien (2005) also analyze a model with free entry and compare price and quantity competition. They find that price competition is more profitable than quantity competition.

[8]See also Barros, Kind & Sorgard (2003) for a similar kind of result.

[9]Dukes & Gal-Or (2003) analyze a similar question and find that, if viewers do not dislike advertising, there is minimal differentiation under free-to-air competition.

inefficiently low amount of advertising in equilibrium. The possibility of multi-homing of viewers and the resulting effects have barely been analyzed so far. Our paper tries to fill this gap.

3 The Model

There are three different types of players in our model, TV stations, viewers, and advertisers. We assume that all potential viewers have the necessary hardware to watch both TV. In the following we describe the three parties in turn along with the game structure of the model.

TV stations

There are two TV stations, indexed by $i \in \{0, 1\}$. Station 0 is located at point 0 on a line, station 1 at point 1, and the potential viewers are distributed in between these two points. The two stations set prices for their commercials of p_0 and p_1, respectively. We do not allow stations to price discriminate, so they can only set linear prices. With these prices the number of advertisements on each station is determined. Let us denote this number of advertisements by a_0 and a_1. For simplicity, we assume that the costs of producing a programme are zero. Thus, we get a profit function of station i of $\Pi_i = p_i a_i$.

Viewers

There is a continuum of viewers with mass M. Viewers are uniformly distributed on $[0, 1]$. Viewers can potentially watch both channels. A viewer who is located at position x_j obtains a net viewing benefit of

$$s_0(\beta - \gamma a_0 - \tau x_j) + s_1(\beta - \gamma a_1 - \tau(1 - x_j))$$

. Here, s_0 and s_1 are binary variables, which either have a value of 1 or 0. $s_k = 1$ if the viewer watches channel k , and is 0 otherwise. β represents the net benefit from watching channel k, if channel k would display no commercials. Viewers dislike advertising and γ represents the nuisance cost parameter concerning commercials. As usual, τ is the transportation cost parameter and represents the degree of differentiation between both stations. Note that since viewers can watch both channels in our model, τ is not a measure for the degree of competition between the stations. The effect from a decrease in τ here is only that the channels become more attractive to viewers and so more viewers are watching TV, everything else unchanged.

The marginal viewer of channel i is the one who gets zero utility from watching channel i. She is given by $x_{m0} = \frac{\beta - \gamma a_0}{\tau}$ and by $x_{m1} = 1 - \frac{\beta - \gamma a_1}{\tau}$, respectively. Let n_0 and n_1 denote the number of viewers of stations 0 and 1 and by n_{01} the number of viewers who watch both channels, i.e. $n_k - n_{01}$ is the number of viewers who watch only station k. After watching the commercial of a producer a viewer probably wants to buy this producer's good if its price is lower than her reservation value. The probability that a potential consumer buys a product in the end is 0 if the producer does not advertise or the consumer does not watch a channel where it is advertised. If the consumer watches exactly one channel where product is advertised, then the probability of buying the good is ω. If the consumer watches both channels and the product is advertised on both the probability is $\omega + \omega'$ where $\omega' \in [0, \omega]$. Advertising in this model is informative.[10] The more often a viewer watches a commercial the higher is the probability that she in fact buys the advertised good. But this relationship is a concave one, so watching the advertisement for the first time has a higher influence then watching it a second time. A possible interpretation is that there is some chance that a viewer missed the advertisement in a first watching. Note that if $\omega' = 0$ then watching the commercial a second time has no additional effect on the buying probability. If instead $\omega' = \omega$, then watching a commercial a second time has the same influence as the first time.

Additionally, we assume that all producers are the same and that consumers have reservation price of 1 for the good of every producer.[11]

[10] For an in-depth overview on the different views on advertising, see Bagwell (2003).

[11] This assumption simplifies the analysis a lot. Since all producers are the same it is not important, exactly which producer is advertising on both channels but only the amount of producers doing so. If

Advertisers

There is a mass of $N \leq \frac{2\beta-\tau}{\gamma}$ producers.[12] Each producer is a local monopolist in its market. Therefore each producer sets a price of 1, because it cannot gain more consumers by lowering its price. Again, we assume for simplicity that there are zero costs for the production af goods or commercials. Taking the buying probabilities of potential consumers into account this gives a profit function for every producer of

$$\pi = \begin{cases} 0 \text{ if no advertising} \\ n_k\omega - p_k = M(\frac{\beta-\gamma a_k}{\tau})\omega - p_k \text{ if only advertise on station } k \\ (n_0 + n_1 - n_{01})\omega + n_{01}(\omega + \omega') - p_0 - p_1 = \\ = M(\omega + \omega'(\frac{2\beta-\gamma(a_0+a_1)}{\tau} - 1)) - p_0 - p_1 \text{ if advertise on both stations} \end{cases}$$

Game structure

The time line of the game is as follows. In stage 1 stations set their prices p_0 and p_1 simultaneously. After observing this prices producers in stage 2 choose whether to advertise or not and if yes then on which one of the channels or on both. After that, in stage 3 viewers decide which channel(s) to watch.

4 Equilibrium in Monopoly and Duopoly

In this section we solve the game both under the regime that the channels are owned by one firm (monopoly case) and that they are owned by two different firms and are in competition to each other (duopoly case). As usual, this is done by backwards induction.

producer were different, meaning some have more valuable products than others, the order of producers valuations might change, dependent on if the producer has already a commercial on the other channel or not. So it would be important *which* additional producer advertisers and not only *that* a producer advertises.

[12] By imposing an upper bound on N we guarantee that some advertisers multi-home in equilibrium. This is done to make the problem interesting. As will become clear later if advertisers do not overlap then the solution is trivial.

In stage 3 viewers are deciding which channels to watch given the amount of advertising on each channel. Calculating marginal consumers gives $x_{m0} = \frac{\beta - \gamma a_0}{\tau}$ and $x_{m1} = 1 - \frac{\beta - \gamma a_1}{\tau}$. Thus n_0, n_1, and n_{01} are given by $n_0 = \min(\frac{\beta - \gamma a_0}{\tau}, 1)M$, $n_1 = \min(\frac{\beta - \gamma a_1}{\tau}, 1)M$, and $n_{01} = \max(0, \frac{2\beta - \gamma(a_0 + a_1)}{\tau} - 1)M$. To safe on notation we assume for the moment that not all viewers are watching channel i which means $\frac{\beta - \gamma a_i}{\tau} \leq 1$. Later, we will check under which conditions this holds and also state the equilibrium for the case where the market is completely covered.

In stage 2 advertisers decide on which station to place a commercial taking into account viewership in stage 3. So an advertiser places a commercial on station k if

$$\omega M(\frac{\beta - \gamma a_k}{\tau}) \geq p_k.$$

It places a commercial on both stations, if the additional profit from the second station j is bigger than its price, or

$$\begin{aligned} \omega M(\frac{\beta - \gamma a_j}{\tau} - \frac{2\beta - \gamma(a_k + a_j)}{\tau} + 1) + M\omega'(\frac{2\beta - \gamma(a_k + a_j)}{\tau} - 1) = \\ = \omega M(1 - \frac{\beta - \gamma a_k}{\tau}) + M\omega'(\frac{2\beta - \gamma(a_k + a_j)}{\tau} - 1) \geq p_j. \end{aligned}$$

Now we solve for the stations' problem in stage 1. To maximize profits it is optimal for them to set prices in such a way that one of the last two inequalities holds with equality. So we have therefore a one-to-one mapping between a station's price p_i and its number of advertisers a_i. It is therefore possible to conduct the analysis by letting each station maximize over a_i instead of p_i. This property of the model is contrary to models which allow only for single-homing of viewers. The reason is that in such models a change in a_i or p_i has different effects on the number of viewers of station j. This is not the case in the model analyzed here because there is no competition for viewers.

First assume that $n_{01} = 0$. The profit function of station i is then given by

$$\Pi_i = p_i a_i = \omega M(\frac{\beta - \gamma a_i}{\tau})a_i.$$

Solving this for a_i yields

$$a_i^* = \frac{\beta}{2\gamma}$$

giving a profit of

$$\Pi_i^* = \frac{\omega M \beta^2}{4\tau\gamma}.$$

But this is only an equilibrium if $n_{01} = 0$ or after checking if this is fulfilled at the equilibrium values, if $\beta \leq \tau$. If $\beta > \tau$ then viewers overlap in the above equilibrium. But this is not a problem as long as advertisers do not multi-home. If all advertisers single-home which means that $N \geq \beta/\gamma$ the above equilibrium is still valid. To make the problem interesting we assume in the following that $N < \beta/\gamma$.

Let us now turn to the case where $\tau < \beta$.

First look at the monopoly case, i.e. the case where both stations are controlled by one firm. The profit of the monopolist, if viewers overlap, is

$$\Pi_M = Ma_0\left(\omega(1 - \frac{\beta - \gamma a_1}{\tau}) + \omega'(\frac{2\beta - \gamma(a_0 + a_1)}{\tau} - 1)\right)$$
$$+Ma_1\left((1 - \frac{\beta - \gamma a_0}{\tau}) + \omega'(\frac{2\beta - \gamma(a_0 + a_1)}{\tau} - 1)\right).$$

Solving this for a_0 and a_1 and using symmetry yields

$$a_0^* = a_1^* = \frac{\omega'(2\beta - \tau) - \omega(\beta - \tau)}{2\gamma(2\omega' - \omega}).$$

Because of our assumption on N advertisers do overlap at this a_i^*. The monopolist's profit is then

$$\Pi_i = \frac{M[\omega'(2\beta - \tau) - \omega(\beta - \tau)]^2}{2\gamma\tau(2\omega' - \omega)}. \tag{1}$$

But this is only valid as long as $n_{01} > 0$. Inserting the equilibrium number of advertiser gives that $(\frac{2\beta - \gamma(a_0 + a_1)}{\tau} - 1) > 0$ if $\omega'(2\beta - \tau) - \omega\beta > 0$. So if ω' is small the above cannot be the monopolist's optimal strategy. a_i^* would be too large so that viewers do not overlap.

In this case the monopolist has two possibilities. The first is to set a_i in such a way that viewers just don't overlap, namely that $x_{m0} = 1 - x_{m1} = 1/2$. To achieve this $a_0 = a_1$ is set at $\frac{2\beta - \tau}{2\gamma}$. The profit is then

$$\Pi_i = \frac{\omega M(2\beta - \tau)}{2\gamma}. \tag{2}$$

The second is to set $a_i = \frac{N}{2}$ so that advertisers do not multi-home. The profit is then

$$\Pi_i = \frac{\omega MN(2\beta - \gamma N)}{2\tau}. \quad (3)$$

Comparing these two profits yields that the profit from non-overlapping of viewers is always higher than the one from non-overlapping of advertisers if $N \leq 2\beta - \tau\gamma$ which holds true because of our assumption in section 2.

Thus, the monopolist therefore chooses non overlapping of viewers if the profit (2)is higher than the profit from overlapping (1). It remains to calculate under which conditions the profit form overlapping is higher. Comparing (2) with (1) yields that the profit from overlapping is higher if $\omega' \geq \frac{\omega\beta}{2\beta-\tau}$. But this is precisely the condition for which the overlapping profit function is valid.

As a last step we have to check another boundary condition, namely that $x_{m0} \leq 1$. Inserting a_i^* yields that this is satisfied if $\beta - \frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{2\gamma(2\omega'-\omega}) \leq \tau$ or $\tau \geq \frac{\beta(2\omega'-\omega)}{3\omega'-\omega}$. So if $\tau < \frac{\beta(2\omega'-\omega)}{3\omega'-\omega}$, then $x_{m0} = 1$ and the optimal a_0 is such that x_{m0} stays at 1 which is $a_i = \frac{\beta-\tau}{\gamma}$. So the monopolist's optimal profit is given by the following.

$$\Pi^*_{mon} = \begin{cases} \frac{\omega M\beta^2}{2\tau\gamma} \text{ if } \tau \geq \beta \\ \frac{\omega M(2\beta-\tau)}{2\gamma} \text{ if } \tau < \beta \text{ and } \omega' < \frac{\omega\beta}{2\beta-\tau} \\ \frac{M[\omega'(2\beta-\tau)-\omega(\beta-\tau)]^2}{2\gamma\tau(2\omega'-\omega)} \text{ if } \frac{\beta(2\omega'-\omega)}{3\omega'-\omega} \leq \tau < \beta \text{ and } \omega' \geq \frac{\omega\beta}{2\beta-\tau} \\ 2M\omega'(\frac{\beta-\tau}{\gamma}) \text{ if } \frac{\beta(2\omega'-\omega)}{3\omega'-\omega} > \tau \text{ and } \omega' \geq \frac{\omega\beta}{2\beta-\tau} \end{cases}$$

One can verify that this profit function is continuous. If $\beta = \tau$ the firm would always choose non-overlapping because $\omega' \leq \omega$ and $\frac{\omega M(2\beta-\tau)}{2\gamma}$ evaluated at $\beta = \tau$ equals $\frac{\omega M\beta}{2\ \gamma}$ which is $\frac{\omega M\beta^2}{2\tau\gamma}$ evaluated at $\beta = \tau$. If $\tau < \beta$ and $\tau$is decreasing further it might be possible that $\omega' \geq \frac{\omega\beta}{2\beta-\tau}$. At this point the profit function has a second kink but it is still continuous. From this condition it is obvious that the monopolist would never choose overlapping if $\omega' < \omega/2$.

Now we turn to the duopoly case, namely the case where the stations are controlled by different firms.

In case of $\tau \geq \beta$ there is no difference to the preceding analysis. Since the stations are local monopolists in this case we still get $a_i^* = \frac{\beta}{2\gamma}$ and a profit of $\Pi_i^* = \frac{\omega M\beta^2}{4\tau\gamma}$. The interesting case arises when $\tau < \beta$ and, of course, $N < \beta/\gamma$. So both viewers and advertisers would multi-home. The profit function of firm i is then given by

$$\Pi_i = Ma_i\left(\omega(1 - \frac{\beta - \gamma a_j}{\tau}) + \omega'(\frac{2\beta - \gamma(a_i + a_j)}{\tau} - 1)\right).$$

Solving this for a_i yields

$$a_0^* = a_1^* = \frac{\omega'(2\beta - \tau) - \omega(\beta - \tau)}{\gamma(3\omega' - \omega)},$$

which is smaller than a_i^* in the monopoly case.[13] So we get the result that if viewers overlap there are fewer commercials in duopoly than in monopoly. Inserting the equilibrium values of a_i back into the profit function yields equilibrium profits of

$$\Pi_i = \frac{M\omega'[(2\beta - \tau)\omega' - \omega(\beta - \tau)]^2}{\gamma\tau(3\omega' - \omega)^2}. \tag{4}$$

As in the monopoly case this profit function is only valid as long as viewers overlap at the equilibrium amounts of advertising. One can show that this is the case if $2\beta\omega' - \tau\omega' - \tau\omega \geq 0$. If this does not hold true at $a_0^* = a_1^*$ every stations has so many advertisers that viewers would not overlap in equilibrium anymore. But if they do not overlap, we are back in the case where each stations is a local monopolist. Here, $a_i^* = \frac{\beta}{2\gamma}$ was optimal but since $\tau < \beta$ viewers would overlap in this equilibrium. So the only possible equilibrium if $2\beta\omega' - \tau\omega' - \tau\omega < 0$ is therefore that stations choose a_i in such a way that viewers just do not overlap.[14] In this case we have multiple equilibria in each of which station i chooses a_i exactly so that viewers just do not overlap given that station j chooses a_j.

The complete characterization of these equilibria is given in Appendix A. Here we concentrate on the symmetric equilibrium. The symmetric equilibrium is also the most efficient one because all viewers choose their most preferred station. There both stations choose $a_i = \frac{2\beta - \tau}{2\gamma}$.

[13] As in the monopoly case we assume here that $N < 2a_i^*$, so advertisers multi-home.

[14] Precisely for the same reason as in the monopoly case it is never optimal for a station to set the number of advertisers such that they do no multi-home. Avoiding multi-homing of viewers is always more profitable.

The next step is to calculate under which conditions a stations wants to deviate from that non overlapping equilibrium. This means, calculating what the best reaction of station i is on station j setting $a_j = \frac{2\beta-\tau}{2\gamma}$. Calculating this yields $a_i^{dev} = \frac{2\beta\omega'+\tau(\omega-\omega')}{4\gamma\omega'}$. The question now is under which conditions viewers overlap if firms choose $a_j = \frac{2\beta-\tau}{2\gamma}$ and a_i^{dev}. This is only the case for $2\beta\omega' - \tau\omega' - \tau\omega \geq 0$. If this does not hold then viewers would not overlap if stations i chooses a_i^{dev}. But this means that it is better for station i to choose $a_i = \frac{2\beta-\tau}{2\gamma}$ so that viewers do just not overlap. But this is our proposed equilibrium. If instead the condition $2\beta\omega' - \tau\omega' - \tau\omega \geq 0$ is fulfilled firm i chooses a_i^{dev}. But this condition is exactly the same condition under which the above profit function was valid because at $a_i^* = \frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(3\omega'-\omega)}$. Viewers would overlap in this case.

As in the monopoly case we have to check the boundary condition that $x_{m0} \leq 1$ or $\beta - \frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(3\omega'-\omega)} \leq \tau$. This is satisfied if $\tau \geq \frac{\beta}{2}$. Thus we can now characterize the only symmetric equilibrium of the duopoly game.

$$\Pi^*_{duo} = \begin{cases} \frac{\omega M\beta^2}{4\tau\gamma} \text{ if } \tau \geq \beta \\ \frac{\omega M(2\beta-\tau)}{4\gamma} \text{ if } \tau < \beta \text{ and } \omega' < \frac{\omega\tau}{2\beta-\tau} \\ \frac{M\omega'[(2\beta-\tau)\omega'-\omega(\beta-\tau)]^2}{\gamma\tau(3\omega'-\omega)^2} \text{ if } \frac{\beta}{2} \leq \tau < \beta \text{ and } \omega' \geq \frac{\omega\tau}{2\beta-\tau} \\ M\omega'(\frac{\beta-\tau}{\gamma}) \text{ if } \frac{\beta}{2} > \tau \text{ and } \omega' \geq \frac{\omega\tau}{2\beta-\tau} \end{cases}$$

Comparing the restriction on ω' under which viewers overlap in monopoly ($\omega' \geq \frac{\omega\beta}{2\beta-\tau}$) with that in duopoly ($\omega' \geq \frac{\omega\tau}{2\beta-\tau}$) shows that in duopoly viewers overlap for more values of ω' because $\tau < \beta$. The intuition behind this is that reducing advertisers and with that increasing overlapping viewers cause a negative externality on the other stations' profit, because it gets less revenue from these overlapping viewers. The monopolist which controls both station can better internalize this externality than a duopoly.

As in the monopoly case it is easily verified that the profit function is continuous.

At this point of our analysis we are already able to state an interesting feature of the equilibrium. Since viewers can multi-home the variable τ is not a measure of competition in our model. Instead, a decrease in τ increases the reservation value of (almost) all viewers which should lead to higher profits for firms. A look at the profit function reveals that this is obviously true if $\tau \geq \beta$ and if $\tau < \beta$ and $\omega' < \frac{\omega\tau}{2\beta-\tau}$. If, on the other hand, $\tau < \beta$ and $\omega' \geq \frac{\omega\tau}{2\beta-\tau}$, then the result is not clear. Differentiating $\frac{M\omega'[(2\beta-\tau)\omega'-\omega(\beta-\tau)]^2}{\gamma\tau(3\omega'-\omega)^2}$ with respect to τ yields

$$\frac{M\omega'}{\gamma(3\omega'-\omega)^2}[(2(2\beta-\tau)\omega'-2\omega(\beta-\tau))(\omega-\omega')\tau-[(2\beta-\tau)\omega'-\omega(\beta-\tau)]^2], \qquad (5)$$

which can be bigger or smaller than zero. Simplifying this expression gives

$$\text{sign}\left(\frac{\partial\Pi_i}{\partial\tau}\right) = \text{sign}\left((\tau^2-4\beta^2)(\omega')^2+(4\beta^2\omega-2\tau^2\omega)\omega'+\omega^2\tau^2-\omega^2\beta^2\right).$$

Solving this for ω' shows that there are two roots, namely,

$$\omega' = \frac{\omega(2\tau^2-4\beta^2\pm 2\beta\tau)}{2\tau^2-8\beta^2}.$$

Since the function sign $\left(\frac{\partial\Pi_i}{\partial\tau}\right)$ is concave ($\frac{\partial^2}{\partial\omega'}(\text{sign}\left(\frac{\partial\Pi_i}{\partial\tau}\right)) = 2(\tau^2-4\beta^2) < 0$) for values of ω' in between the two roots $\frac{\partial\Pi_i}{\partial\tau}$ is positive, so the profit of a station is increasing in τ. But remember that this profit function is only valid if $\omega' \geq \frac{\omega\tau}{2\beta-\tau}$. Since $\frac{\omega\tau}{2\beta-\tau}$ can be bigger or smaller than $\frac{\omega(2\tau^2-4\beta^2-2\beta\tau)}{2\tau^2-8\beta^2}$ we have the following result.

Lemma 1

If

$$\max\left(\frac{\omega(4\beta^2-2\tau^2-2\beta\tau)}{8\beta^2-2\tau^2}, \frac{\omega\tau}{2\beta-\tau}\right) \leq \left(\frac{\omega(4\beta^2-2\tau^2+2\beta\tau)}{8\beta^2-2\tau^2}\right),$$

then the profit function is increasing in τ if

$$\max\left(\frac{\omega(4\beta^2-2\tau^2-2\beta\tau)}{8\beta^2-2\tau^2}, \frac{\omega\tau}{2\beta-\tau}\right) \leq \omega' \leq \left(\frac{\omega(4\beta^2-2\tau^2+2\beta\tau)}{8\beta^2-2\tau^2}\right).$$

What is the intuition behind this result? If τ becomes smaller and smaller viewers reservation values become higher and more and more viewers begin to multi-home (as long as ω' is not very small). So both stations have more viewers but these new viewers are less valuable than the old ones because they are already watching the other channel. Moreover, some of the old viewers who were formerly watching channel i exclusively now begin to multi-home and are therefore also less valuable to channel i. If ω' has some middle-range value the effect that multi-homing viewers are not so valuable dominates the effect of more viewers and the profit function is decreasing when τ gets smaller. If ω' would be close to zero or close to ω this does not happen. In the first case an additional viewer has a very low value for each channel and in equilibrium both stations expand their amount of advertising in such a way that there is no viewer overlap. In the second case, an additional viewer is very valuable and the effect that more viewers are watching dominates the negative effect from more multi-homing.

5 Welfare

Now we turn to the welfare analysis. Again, let us first look at the case where viewers do not multi-home and check under which parameters values this is valid. The welfare function is

$$WF = \omega M(\frac{\beta-\gamma a_0}{\tau})a_0 + \omega M(\frac{\beta-\gamma a_1}{\tau})a_1$$
$$+M\int_0^{\frac{\beta-\gamma a_0}{\tau}}(\beta-\gamma a_0-\tau x)dx + M\int_{1-\frac{\beta-\gamma a_1}{\tau}}^{1}(\beta-\gamma a_1-\tau(1-x))dx.$$

Differentiating this yields a FOC of

$$\frac{\partial WF}{\partial a_i} = \frac{1}{\tau}(\beta\omega - 2\gamma\omega a_i - \beta\gamma + \gamma^2 a_i) \leq 0.$$

The second order conditions are satisfied if $2\omega > \gamma$ which we asumme in the following. Solving the FOC for a_i and taking into account that a_i can never be negative yields that the optimal a_i is given by

$$a_i^* = \begin{cases} \frac{\beta(\omega-\gamma)}{\gamma(2\omega-\gamma)} & \text{if } \omega \geq \gamma \\ 0 & \text{if } \omega < \gamma \end{cases}$$

But this welfare function is only valid as long as viewers do not overlap. So we have to calculate if this is the case at the optimal advertisement numbers. This shows that this is true as long as $\frac{\beta-\gamma a_1}{\tau} \leq 1/2$ or $\frac{2\beta\omega}{(2\omega-\gamma)} \leq \tau$. If $\tau < \frac{2\beta\omega}{(2\omega-\gamma)}$ viewers would overlap. Welfare is then given by

$$WF = M\omega(N-a_1)(\frac{\beta-\gamma a_0}{\tau}) + M\omega(N-a_0)(\frac{\beta-\gamma a_1}{\tau})+$$

$$+M(a_0+a_1-N)((\omega+\omega')(\frac{2\beta-\gamma(a_0+a_1)}{\tau}-1)+\omega(1-\frac{\beta-\gamma a_1}{\tau})+\omega(1-\frac{\beta-\gamma a_0}{\tau}))$$

$$+M\int_0^{\frac{\beta-\gamma a_0}{\tau}}(\beta-\gamma a_0-\tau x)dx+M\int_{1-\frac{\beta-\gamma a_1}{\tau}}^{1}(\beta-\gamma a_1-\tau(1-x))dx.$$

The first two terms are the welfare from trade of goods of producers who only advertise on one of the two stations. The third term stems from trade of goods of multi-homing advertisers. Finally, the last two terms are viewers utility from watching TV. Building the first order condition leads to

$$\frac{\partial WF}{\partial a_0}=-\frac{\beta\gamma}{\tau}+\frac{\gamma^2 a_0}{\tau}-\frac{\gamma\omega(N-a_1)}{\tau}-\frac{\omega(\beta-\gamma a_1)}{\tau}-\frac{\gamma(\omega+\omega')(a_0+a_1-N)}{\tau}+\frac{\gamma\omega(a_0+a_1-N)}{\tau}$$

$$+\omega(1-\frac{\beta-\gamma a_1}{\tau})+\omega(1-\frac{\beta-\gamma a_0}{\tau})+(\omega+\omega')(\frac{2\beta-\gamma(a_0+a_1)}{\tau}-1)=0.$$

Checking the second order conditions shows that this is a maximum if $\gamma-2\omega'<0$. Solving for a_i yields, because of symmetry,

$$a_i^*=\frac{\beta(2\omega'-\omega-\gamma)-(\omega-\omega')(\gamma N-\tau)}{\gamma(4\omega'-2\omega-\gamma)}.$$

We have to check under which conditions the viewers overlap if $a_i=a_i^*$. This is the case if $2\beta-2\left(\frac{\beta(2\omega'-\omega-\gamma)-(\omega-\omega')(\gamma N-\tau)}{(4\omega'-2\omega-\gamma)}\right)\geq\tau$ or

$$\omega'\geq\frac{\omega\gamma N+\tau\gamma-2\beta\omega}{2(\gamma N+\tau-2\beta\omega)}.$$

If this is not fulfilled then There are three possibilities. The first is to choose $a_i=0$ which gives welfare of $WF=\frac{\beta^2}{\tau}$. The second possibility is to set $a_i=\frac{N}{2}$ so that advertisers do not overlap. This yields to a welfare of $WF=\frac{(2\beta-\gamma N)(2\beta-\gamma N+2\omega N)}{4\tau}$. The third possibility is to set $a_i=\frac{2\beta-\tau}{2\gamma}$ so that viewers do not overlap giving a welfare of $WF=\frac{\tau}{4}+\frac{\omega(2\beta-\tau)}{2\gamma}$. Comparing these three possibilities now gives the following:

Let us first assume that $\frac{\gamma(4\beta-\gamma N)}{2(2\beta-\gamma N)}<\frac{\gamma(2\beta+\tau-\gamma N)}{2(\tau-\gamma N)}$. If this holds then $a_i^*=0$ if $\omega\leq\frac{\gamma(4\beta-\gamma N)}{2(2\beta-\gamma N)}$. If $\frac{\gamma(4\beta-\gamma N)}{2(2\beta-\gamma N)}<\omega\leq\frac{\gamma(2\beta+\tau-\gamma N)}{2(\tau-\gamma N)}$ then $a_i^*=\frac{N}{2}$ is optimal, so viewers multi-home, while advertisers single-home. If instead $\omega>\frac{\gamma(2\beta+\tau-\gamma N)}{2(\tau-\gamma N)}$ then $a_i^*=\frac{2\beta-\tau}{2\gamma}$ is welfare maximizing, which means that viewers single-home while advertisers multi-home. So in this case every one of the three possibilities can be optimal dependent on the value of ω. If instead $\frac{\gamma(4\beta-\gamma N)}{2(2\beta-\gamma N)}\geq\frac{\gamma(2\beta+\tau-\gamma N)}{2(\tau-\gamma N)}$ then $a_i=\frac{N}{2}$ can never be optimal. The solution then is that if $\omega \leq \frac{\gamma(2\beta+\tau)}{2\tau}$ then $a_i^* = 0$ is optimal while if $\omega > \frac{\gamma(2\beta+\tau)}{2\tau}$, $a_i^* = \frac{2\beta-\tau}{2\gamma}$ is optimal.

It remains to calculate what happens if $\omega' \geq \frac{\omega\gamma N+\tau\gamma-2\beta\omega}{2(\gamma N+\tau-2\beta\omega)}$. In this case it is easy to check that setting $a_i^* = \frac{\beta(2\omega'-\omega-\gamma)-(\omega-\omega')(\gamma N-\tau)}{\gamma(4\omega'-2\omega-\gamma)}$ which means multi-homing of viewers and advertisers gives a higher welfare than any of the three above mentioned alternatives. Thus, we can now state the welfare maximizing outcome.

If $\tau \geq \frac{2\beta\omega}{(2\omega-\gamma)}$ then

$$a_i^* = \begin{cases} \frac{\beta(\omega-\gamma)}{\gamma(2\omega-\gamma)} & \text{if } \omega \geq \gamma \\ 0 & \text{if } \omega < \gamma \end{cases}$$

If $\tau < \frac{2\beta\omega}{(2\omega-\gamma)}$ and $\omega' < \frac{\omega\gamma N+\tau\gamma-2\beta\omega}{2(\gamma N+\tau-2\beta\omega)}$, and $\frac{\gamma(4\beta-\gamma N)}{2(2\beta-\gamma N)} < \frac{\gamma(2\beta+\tau-\gamma N)}{2(\tau-\gamma N)}$,

$$\text{then } a_i^* = \begin{cases} 0 & \text{if } \omega \leq \frac{\gamma(4\beta-\gamma N)}{2(2\beta-\gamma N)} \\ \frac{N}{2} & \text{if } \frac{\gamma(4\beta-\gamma N)}{2(2\beta-\gamma N)} < \omega \leq \frac{\gamma(2\beta+\tau-\gamma N)}{2(\tau-\gamma N)} \\ \frac{2\beta-\tau}{2\gamma} & \text{if } \omega > \frac{\gamma(2\beta+\tau-\gamma N)}{2(\tau-\gamma N)} \end{cases}$$

If $\tau < \frac{2\beta\omega}{(2\omega-\gamma)}$ and $\omega' < \frac{\omega\gamma N+\tau\gamma-2\beta\omega}{2(\gamma N+\tau-2\beta\omega)}$, and $\frac{\gamma(4\beta-\gamma N)}{2(2\beta-\gamma N)} \geq \frac{\gamma(2\beta+\tau-\gamma N)}{2(\tau-\gamma N)}$,

$$\text{then } a_i^* = \begin{cases} 0 & \text{if } \omega \leq \frac{\gamma(2\beta+\tau)}{2\tau} \\ \frac{2\beta-\tau}{2\gamma} & \text{if } \omega > \frac{\gamma(2\beta+\tau)}{2\tau} \end{cases}$$

If $\tau < \frac{2\beta\omega}{(2\omega-\gamma)}$ and $\omega' \geq \frac{\omega\gamma N+\tau\gamma-2\beta\omega}{2(\gamma N+\tau-2\beta\omega)}$, then $a_i^* = \frac{\beta(2\omega'-\omega-\gamma)-(\omega-\omega')(\gamma N-\tau)}{\gamma(4\omega'-2\omega-\gamma)}$.

We have now solved for the welfare maximizing amount of advertisers and before for the optimal number in monopoly and duopoly. So we can now compare these three outcomes. Let us first start with comparison between monopoly and welfare.

Proposition 1

There can never be under-advertising in monopoly and there is normally over-advertising. The amount of advertising is efficient only if $\tau < \beta$, $\omega > \max(\frac{\gamma(2\beta+\tau-\gamma N)}{2(\tau-\gamma N)}, \frac{\gamma(2\beta+\tau)}{2\tau})$ and $\omega' < \frac{\omega\gamma N+\tau\gamma-2\beta\omega}{2(\gamma N+\tau-2\beta\omega)}$.

Proof

See the Appendix.

The intuition behind this result is simple. The monopolist is only concerned with the joint profit of both stations. In contrast, a social planner does not only look at the profit of both stations but in addition takes into account the rent of the viewers. But this viewer rent is decreasing in the number of commercials. Thus, there is normally too much advertising in monopoly. There is one exception to this result. It arises when ω' is low. This means that an overlapping viewer would only contribute little to social welfare and therefore a social planner (and also a monopolist) would avoid overlapping viewership at all.
We turn now to a comparison between the welfare maximizing outcome and duopoly

Proposition 2

There can never be under-advertising in duopoly and there is normally over-advertising. The amount of advertising is efficient only if $\tau < \beta$, $\omega > \max(\frac{\gamma(2\beta+\tau-\gamma N)}{2(\tau-\gamma N)}, \frac{\gamma(2\beta+\tau)}{2\tau})$ and $\omega' < \max(\frac{\omega\tau}{2\beta-\tau}, \frac{\omega\gamma N+\tau\gamma-2\beta\omega}{2(\gamma N+\tau-2\beta\omega)})$.

Proof

See the Appendix.

As in the monopoly case we find that there can never be under-provision of advertising in duopoly. The basic intuition is the same as in the monopoly case. Firms look at viewers' utility only to the extent that an additional commercial causes viewers drop out and so firms can only demand a lower price, but they are not concerned with the direct utility loss of viewers from an additional advertisement. Thus, there is too much advertising. This result stands in contrast to the previous literature, in which viewers can only single-home. In that case stations compete for viewers and if this competition is very harsh, advertising levels are lower than the social optimal optimum. The result of Proposition 2 shows that this outcome depends on the restriction of viewers to only watch one channel. If viewers can multi-home, which is probably more realistic, there can never be under-provision of advertising.

The last two results raise the question if a monopoly or a duopoly regime is more efficient. The two forms of competition are compared in the next proposition.

Proposition 3

Duopoly is always as least as efficient as monopoly. It is more efficient if $\tau < \beta$ and
(i) $\frac{\omega\beta}{2\beta-\tau} \geq \omega' > \frac{\omega\tau}{2\beta-\tau}$ or
(ii) $\omega' \geq \frac{\omega\beta}{2\beta-\tau}$ and also $\tau > \frac{\beta(2\omega'-\omega)}{3\omega'-\omega}$ holds.

Proof:

If $\tau > \beta$ then there is no difference between monopoly and duopoly, $a^*_{duo} = a^*_{mon} = \frac{\beta}{2\gamma}$. Now let us turn to the case when $\tau \leq \beta$. If $\omega' \leq \frac{\omega\tau}{2\beta-\tau}$ then $a^*_{duo} = a^*_{mon} = \frac{2\beta-\tau}{2\gamma}$.

If $\frac{\omega\beta}{2\beta-\tau} \geq \omega' > \frac{\omega\tau}{2\beta-\tau}$ then $a^*_{mon} = \frac{2\beta-\tau}{2\gamma}$ while $a^*_{duo} = \frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(3\omega'-\omega)}$. A comparison between these two values yields that $a^*_{mon} > a^*_{duo}$ only if $\omega' > \frac{\omega\tau}{2\beta-\tau}$, but this holds always true in this region.

Now let us look at the case of $\omega' > \frac{\omega\beta}{2\beta-\tau}$. If $\tau \leq \frac{\beta(2\omega'-\omega)}{3\omega'-\omega}$ then obviously the number of advertisers is the same in monopoly and in duopoly, namely $\frac{\beta-\tau}{\gamma}$.

If instead $\tau \geq \frac{\beta}{2}$ then $a^*_{mon} = \frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{2\gamma(2\omega'-\omega)}$ while $a^*_{duo} = \frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(3\omega'-\omega)}$. A comparison shows that $a^*_{mon} > a^*_{duo}$ if $\omega > \omega'$ which always holds true.

As a last case we have to look at intermediate values of τ namely at $\frac{\beta(2\omega'-\omega)}{3\omega'-\omega} \leq \tau < \frac{\beta}{2}$. In this region $a^*_{mon} = \frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{2\gamma(2\omega'-\omega)}$ while $a^*_{duo} = \frac{\beta-\tau}{\gamma}$. Comparing these values yields that $a^*_{mon} > a^*_{duo}$ if $\tau > \frac{\beta(2\omega'-\omega)}{3\omega'-\omega}$ which holds true in this region.

q.e.d.

Again, this result is in stark contrast to the previous literature which finds that joint ownership of both stations can lead to higher welfare than separate ownership and competition. The intuition behind the result of Proposition 3 is that either in monopoly and in duopoly there is too much advertising because firms neglect the viewers' loss from advertising. But in duopoly, if firm i is reducing its amount of advertising it gets more viewers and therefore more viewers are overlapping. But this in turn has a negative impact on the advertising price of firm j because it can only demand a lower price for multi-homing viewers. In monopoly were both stations are controlled by a single firm this effect is taken into account. By contrast, in duopoly each firm maximizes only profits of one station and therefore chooses a lower number of commercials. But since the overall amount of commercials is too high separate ownership does better than joint ownership. This result also points to the fact that the somewhat surprising result in the literature is an artefact of restricting viewers to watch only one channel.

6 Example from the German Broadcasting Market

In this section we give a short description of and some recent developments in the German broadcasting market. We argue that these developments fit the results of our model quite well.

The German broadcasting market consists of 6 major channels (ARD, Pro7, RTL, SAT1, ZDF, and the 'third programmes,' which are regional programmes in each state). Each of these programmes has a market share between 10% and 14%, leading with ARD with 13.9% and ending with SAT1 with 10.3%.[15] Three of these programmes are public channels (ARD, ZDF, the 'third programmes') and are financed partly by taxes and partly by advertising revenues. The other three channels (Pro7, RTL, SAT1) are private stations and make money only through advertising.

For these private stations there is no law regulating their broadcasting of commercials besides the 12 minutes limit per hour set by the media commission of the European

[15] The source of these data is AGF/GfK Fernsehforschung. See also www.agf.de

Union. The public stations on the other hand are not allowed to broadcast commercials after 8:00 p.m. The German government is currently discussing to abolish this law and to allow public stations to broadcast advertising after 8:00 p.m. Yet, the private stations are strongly against this abolishment arguing that the public stations have some social mission for their viewers and should therefore be not allowed to fill their viewing time with commercials. If one looks at existing models of broadcast competition this behavior of private stations can not be explained.[16] If viewers are single-homing then more advertising of a rival station should increase viewership of the other stations allowing them to set higher commercial prices. So private stations should be supporting such a new law. However, the behavior is in line with the results of our model. Since viewers can watch both channels the possibility the a viewer watches a commercial on channel a reduces the price that an advertiser is willing to pay for a commercial on channel b. If instead advertising on channel a is forbidden channel b would be a monopolist for bringing the commercials to its viewership and can therefore demand higher prices. This gives an explanation why private stations are opposed to the abolishment. It would bring an end to their monopoly power of commercials after 8:00 p.m. and thus reduce their prices.[17]

Another prevalent development in the broadcasting market is that is has grown steadily in the last 5 years. For example, the percentage of viewers in the population has increased from 71.9% to 74.1% and the minutes per week a viewer spends watching TV have increased from 185 to 209. But during this time commercial prices have dropped at around 20%. A possible explanation for that can probably be found if one looks at

[16] An exception is Schmidtke (2005). In his model stations play a quantity game and the market clearing price is set by an 'Walrasian auctioneer.' If more stations are allowed to advertise the market clearing price falls. This is detrimental for stations which were already allowed to advertise before.

[17] Another explanation for this behavior might be the fear that public stations would make higher profits which, if spend in their programme content, would lead to a better quality and induce some viewers to switch to to the public stations. Yet, the public stations are currently running a deficit and most of the new revenues would probably be spent on reducing this deficit. But still we think that the quality effect is an important one and our explanation should be seen as a complementary one to this effect.

the programm content of the stations. During this time period it is notable that the content of public stations has moved closer to the one of private stations, especially in the afternoon and late-afternoon programme. This move was made to attract more viewers, in particular younger ones which are the most profitable consumer group. In our model this development is in some sense equivalent to a decrease in τ which makes the channels more attractive and induces more potential viewers to watch. But this also increases the viewership who watches both channels and probably lowers advertising prices. As we have seen in Lemma 1 this can lead to lower profits although the channels are more attractive.[18]

7 Introduction of a viewer fee

In this section we analyze the case where stations can charge viewers a price for watching. We denote this viewer fee by f_i.[19] As in Section 3 we solve the game by backwards induction.

The utility of a viewer watching channel 0 is then given by $\beta - \gamma a_0 - \tau x - f_0$ and the utility of a viewer watching channel 1 by $\beta - \gamma a_1 - \tau(1-x) - f_1$.[20] Thus, $n_0 = M\frac{\beta-\gamma a_0-f_0}{\tau}$ and $n_1 = M\frac{\beta-\gamma a_1-f_1}{\tau}$.

In stage 2 an advertiser places an ad on station i if station i's price, p_i, is less than or equal to $\omega M(\frac{\beta-\gamma a_0-f_0}{\tau})$ in case there is no overlapping of viewers. In case of a viewer

[18] A second possible explanation, which is often given in newspapers, is the struggling German economy. This, of course, can explain part of that price drop. But it is hardly conceivable that prices have dropped in a growing market by such an amount because of this reason alone and our argument can be seen as another possible reason which all in sum amounts to that huge fall in commercial prices.

[19] Because of new encryption techniques viewer pricing becomes more and more important nowadays. For example, in the US many special interest channels, especially sports or movie channels, can only be watched by paying additional fees. But also in many European countries recent movies or popular sport events can only be watched via pay-TV.

[20] We require f_i to be positive here, so it is not possible for stations to pay viewers to watch their program.

overlap a producer who already advertises on the other station is willing to pay

$$p_i \leq \omega M(\frac{\beta - \gamma a_k}{\tau} - \frac{2\beta - \gamma(a_k + a_i) - f_i - f_k}{\tau} + 1) + M\omega'(\frac{2\beta - \gamma(a_k + a_j)}{\tau} - 1) =$$

$$= \omega M(1 - \frac{\beta - \gamma a_k}{\tau}) + M\omega'(\frac{2\beta - \gamma(a_k + a_j) - f_i - f_k}{\tau} - 1).$$

Now let us move on to stage 1. First we look at the case where stations are local monopolists meaning that viewers do not overlap. In this case the profit function of channel i is given by

$$\Pi_i = (\frac{\beta - \gamma a_i - f_i}{\tau})(\omega M a_0 + M f_0).$$

Differentiation with respect to a_0 and p_0 yields

$$\frac{\partial \Pi_i}{\partial a_i} = \omega(\frac{\beta - \gamma a_i - f_i}{\tau}) - \frac{\omega\gamma a_0}{\tau} - \frac{\gamma f_0}{\tau} = 0$$

and

$$\frac{\partial \Pi_i}{\partial f_i} = (\frac{\beta - \gamma a_i - f_i}{\tau}) - \frac{\omega a_0}{\tau} - \frac{\gamma f_0}{\tau} = 0.$$

The second derivative with respect to a_i is $-2(\omega\gamma)/\tau < 0$ and the second derivative w.r.t. f_i is $-(\omega + \gamma)/\tau < 0$. Solving the two FOCs gives $a_i = \frac{\beta}{\gamma - \omega}$ and $f_i = \frac{\beta\omega}{\omega - \gamma}$. The number of advertisers can never be negative and we require here also that viewers cannot subsidized. It follows that only one of these two values can actually be positive in equilibrium and the other one is zero.

Let us first start with the case, where $f_i = 0$. We are then back in the case without pricing, and $a_i^* = \frac{\beta}{2\gamma}$. The profit if firm i is $\Pi_i = \frac{\beta^2 \omega M}{4\tau\gamma}$. On the other hand, if $a_i = 0$, the optimal $f_i^* = \frac{\beta}{2}$ and $\Pi_i = \frac{\beta^2 M}{4\tau}$. Thus, the optimal solution for firm i is the one where the profit is higher. This gives that $f_i^* = 0$ and $a_i^* = \frac{\beta}{2\gamma}$ if $\omega > \gamma$, and $a_i = 0^*$ and $f_i^* = \frac{\beta}{2}$ if $\omega \leq \gamma$. After inserting of f_i^* and a_i^* in the equation for the marginal viewer, we find that in either case the above is only an equilibrium if $\beta \leq \tau$, because otherwise there would be overlapping of viewers.

Let us now turn to this overlapping case. We start with the monopoly situation where the two stations are controlled by one owner. The profit function of the monopolist is given by

$$\Pi_{mon} = Ma_0\left(\omega(1-\frac{\beta-\gamma a_1-f_1}{\tau})+\omega'(\frac{2\beta-\gamma(a_0+a_1)-f_0-f_1}{\tau}-1)\right)$$
$$+Ma_1\left((1-\frac{\beta-\gamma a_0-f_0}{\tau})+\omega'(\frac{2\beta-\gamma(a_0+a_1)-f_0-f_1}{\tau}-1)\right)$$
$$+Mf_0(\frac{\beta-\gamma a_0-f_0}{\tau})+Mf_1(\frac{\beta-\gamma a_1-f_1}{\tau}).$$

Differentiating with respect to a_i and f_i gives the two first order conditions

$$\omega(1-\frac{\beta-\gamma a_j-f_j}{\tau})+\omega'(\frac{2\beta-\gamma(a_i+a_j)-f_i-f_j}{\tau}-1)-\frac{\gamma\omega' a_i}{\tau}$$
$$+\frac{\gamma\omega a_j}{\tau}-\frac{\gamma\omega' a_j}{\tau}-\frac{\gamma f_i}{\tau}=0$$

and

$$-\frac{\omega' a_i}{\tau}+\frac{\omega a_j}{\tau}-\frac{\omega' a_j}{\tau}+\frac{\beta-\gamma a_i-f_i}{\tau}-\frac{f_i}{\tau}=0.$$

The second derivatives with respect to a_i is $-2\gamma\omega'/\tau<0$ and with respect to f_i is $-2/\tau<0$. Solving these equations for a_i and f_i gives

$$a_i^*=\frac{\beta(\omega+\gamma-2\omega')-2\tau(\omega-\omega')}{(\omega+\gamma-2\omega')^2}$$

and

$$f_i^*=\frac{\beta(\omega-2\omega')(\omega+\gamma-2\omega')+\tau(\omega-\omega')(\gamma-\omega+2\omega')}{(\omega+\gamma-2\omega')^2}.$$

Assuming for the moment that both expression are positive and that viewers indeed overlap at these values we can calculate the profit for the monopolist. Inserting these back in the profit function yields

$$\Pi_{mon}^*=\frac{2(\omega-\omega'}{(\gamma+\omega-2\omega')^2}(\beta(\gamma+\omega-2\omega')-\tau(\omega-\omega'))M. \tag{6}$$

But in equilibrium a_i and f_i can never be negative. So if either one of these would be negative it is optimal for the monopolist to set it to zero. Let us look at this case now. First, if f_i would be 0 then we are back in the optimization problem of Section 3 where pricing of viewers wasn't possible. The optimal values of a_i would be $a_1^* = \frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{2\gamma(2\omega'-\omega})$ if $\omega' \geq \frac{\omega\beta}{2\beta-\omega}$ (up to the boundary condition $x_m \leq 1$) and $\frac{\beta-\tau}{\gamma}$ if $\omega' \geq \frac{\omega\beta}{2\beta-\omega}$. Second, if a_i is zero then calculating the optimal f_i yields

$$f_i^* = \frac{\beta}{2}.$$

But this does only hold if $x_m \leq 1$ or $\tau \geq \frac{\beta}{2}$. If $\tau < \frac{\beta}{2}$, then x_m would be greater than 1 at $f_i = \frac{\beta}{2}$ and the optimal $f_i^* = \beta - \tau$ then. Inserting this back in the profit function gives $\Pi_{mon} = \begin{cases} \frac{M\beta^2}{2\tau} \text{ if } \tau \geq \frac{\beta}{2} \\ M(2\beta - 2\tau) \text{ if } \tau < \frac{\beta}{2} \end{cases}$

But comparing $\frac{M\beta^2}{2\tau}$ with (6) reveals that the profit with $a_i = 0$ is higher than the profit at $a_i > 0$ if $\frac{(2\tau\omega-\beta\omega-\beta\gamma+2\omega'(\beta-\tau))^2}{2\tau(\omega+\gamma-2\omega')^2} > 0$ which is always true. This shows that it can never be optimal for the monopolist to set both a_i and f_i positive. Instead, $a_i = 0$ and $f_i > 0$ is always better.

But we still have to compare the profit under $a_i = 0$ and $f_i > 0$ with the profit at $a_i > 0$ and $f_i = 0$. Let us start with the case $\omega' \geq \frac{\omega\beta}{2\beta-\omega}$. The two profits in this case are given by $\frac{M[\omega'(2\beta-\tau)-\omega(\beta-\tau)]^2}{2\gamma\tau(2\omega'-\omega)}$ (for the case of $f_i = 0$) and by $\frac{M\beta^2}{2\tau}$ (for the case of $a_i = 0$). The difference between these profits is $\delta = \frac{M[\omega'(2\beta-\tau)-\omega(\beta-\tau)]^2}{2\gamma\tau(2\omega'-\omega)} - \frac{M\beta^2}{2\tau}$. Setting the function δ equal zero yields that there are two roots namely of ω' namely $\omega'_{1,2} = \frac{\omega(2\beta^2-\tau^2)+\beta^2\gamma-3\beta\tau\omega\pm\sqrt{\beta^2\gamma(\beta^2\gamma-\tau^2\omega-2\beta\omega\tau)}}{(2\beta-\tau)^2}$. Differentiating δ twice with respect to ω' yields $\frac{\partial^2\delta}{\partial\omega'^2} = 8\beta^2 - 8\beta\tau + 2\tau^2 > 0$. So if ω' is greater than the higher root or smaller than the lower root it is better for the monopolist to set $f_i = 0$. For values in between $a_i = 0$ is optimal. But this function is only valid if $\omega' \geq \frac{\omega\beta}{2\beta-\omega}$ so only the higher root is relevant. If $\omega' < \frac{\omega\beta}{2\beta-\omega}$ then we have to compare $\Pi_{mon}(f_i = 0) = \frac{\omega M(2\beta-\tau)}{2\gamma}$ with $\Pi_{mon}(a_i = 0) = \frac{M\beta^2}{2\tau}$. This gives us that $\Pi_{mon}(f_i = 0) > \Pi_{mon}(a_i = 0)$ if $\omega > \frac{\beta^2\gamma}{\tau(2\beta-\tau)}$.

We are now in a position to state the optimal policy for the monopolist.

(i) $\tau \geq \beta$

If $\gamma \geq \omega$ then $a_i^* = 0, f_i^* = \frac{\beta}{2}$.

If $\gamma < \omega$ then $a_i^* = \frac{\beta}{2\gamma}, f_i^* = 0$.

(ii) $\tau < \beta$

$$\text{If } \omega > \tfrac{\beta^2\gamma}{\tau(2\beta-\tau)} \text{ then} \begin{cases} a_i^* = \frac{2\beta-\tau}{2\gamma}, f_i^* = 0 \text{ if } \omega' < \frac{\omega\beta}{2\beta-\tau} \\ a_i^* = \frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{2\gamma(2\omega'-\omega)}, f_i^* = 0 \text{ if } \frac{\beta(2\omega'-\omega)}{3\omega'-\omega} \leq \tau \text{ and } \omega' \geq \frac{\omega\beta}{2\beta-\tau} \\ a_i^* = \frac{\beta-\tau}{\gamma}, f_i^* = 0 \text{ if } \frac{\beta(2\omega'-\omega)}{3\omega'-\omega} > \tau \text{ and } \omega' \geq \frac{\omega\beta}{2\beta-\tau} \end{cases}$$

$$\text{If } \omega \leq \tfrac{\beta^2\gamma}{\tau(2\beta-\tau)} \text{ and } \omega' \geq \tfrac{\omega(2\beta^2-\tau^2)+\beta^2\gamma-3\beta\tau\omega+\sqrt{\beta^2\gamma(\beta^2\gamma-\tau^2\omega-2\beta\omega\tau)}}{(2\beta-\tau)^2} \text{ then}$$

$$\begin{cases} a_i^* = \frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{2\gamma(2\omega'-\omega)}, f_i^* = 0 & \text{if } \frac{\beta(2\omega'-\omega)}{3\omega'-\omega} \leq \tau \\ a_i^* = \frac{\beta-\tau}{\gamma}, f_i^* = 0 & \text{if } \frac{\beta(2\omega'-\omega)}{3\omega'-\omega} > \tau \end{cases}$$

$$\text{If } \omega \leq \tfrac{\beta^2\gamma}{\tau(2\beta-\tau)} \text{ and } \omega' < \tfrac{\omega(2\beta^2-\tau^2)+\beta^2\gamma-3\beta\tau\omega+\sqrt{\beta^2\gamma(\beta^2\gamma-\tau^2\omega-2\beta\omega\tau)}}{(2\beta-\tau)^2} \text{ then}$$

$$\begin{cases} a_i^* = 0, f_i^* = \frac{\beta}{2} & \text{if } \tau \geq \frac{\beta}{2} \\ a_i^* = 0, f_i^* = \beta - \tau & \text{if } \tau < \frac{\beta}{2} \end{cases}$$

We can now compare if this outcome with the optimal policy for the monopolist when pricing is not allowed.

Proposition 4

In monopoly the possibility of viewer pricing can never reduce welfare. It is strictly welfare enhancing if either

(i) $\tau \geq \beta$ and $\gamma < \omega$ or

(ii) $\tau < \beta$, $\omega \leq \frac{\beta^2\gamma}{\tau(2\beta-\tau)}$, and $\omega' < \frac{\omega(2\beta^2-\tau^2)+\beta^2\gamma-3\beta\tau\omega+\sqrt{\beta^2\gamma(\beta^2\gamma-\tau^2\omega-2\beta\omega\tau)}}{(2\beta-\tau)^2}$.

Proof

There are obviously only two cases where welfare under pricing differs from welfare without pricing. The first one arises when $\tau \geq \beta$ and $\gamma < \omega$. In this case welfare without pricing is given by the sum of viewer rent and stations profits, i.e.

$$WF_{no} = 2M\left(\int_0^{\frac{\beta}{2\tau}} \left(\frac{\beta}{2} - \tau x\right)dx + \frac{\omega\beta^2}{4\tau\gamma}\right) = 2M\left(\frac{\beta^2}{8\tau} + \frac{\omega\beta^2}{4\tau\gamma}\right).$$

Welfare in case with pricing is also given by the sum of viewer rent and stations' profits and is

$$WF_{wp} = 2M(\frac{\beta^2}{8\tau} + \frac{\beta^2}{4\tau}).$$

But since $\gamma > \omega$ welfare in case of pricing is higher.

The second case is if $\tau < \beta$ and $\omega \leq \frac{\beta^2\gamma}{\tau(2\beta-\tau)}$ and $\omega' < \frac{\omega(2\beta^2-\tau^2)+\beta^2\gamma-3\beta\tau\omega+\sqrt{\beta^2\gamma(\beta^2\gamma-\tau^2\omega-2\beta\omega\tau)}}{(2\beta-\tau)^2}$. Since $\frac{\omega(2\beta^2-\tau^2)+\beta^2\gamma-3\beta\tau\omega+\sqrt{\beta^2\gamma(\beta^2\gamma-\tau^2\omega-2\beta\omega\tau)}}{(2\beta-\tau)^2} > \frac{\omega\beta}{2\beta-\tau}$ the amount of advertising without pricing is $a_i^* = \frac{2\beta-\tau}{2\gamma}$. Calculating welfare in this case yields

$$WF_{no} == 2M(\frac{\tau}{8} + \frac{\omega(2\beta-\tau)}{4\gamma}).$$

Welfare in case without pricing is as before

$$WF_{wp} = 2M(\frac{\beta^2}{8\tau} + \frac{\beta^2}{4\tau}).$$

But since $\omega \leq \frac{\beta^2\gamma}{2\tau(2\beta-\tau)}$ welfare in case without pricing is always higher.

q.e.d.

The intuition behind this result is that with the introduction of a viewer fee the amount of advertising is never higher than without this fee. But under the two cases in the proposition the monopolist substitutes profits from advertisers with profits from viewers. These are the cases where ω (and ω') is low compared to γ, i.e. where a producer's profit gain from advertising is low compared to the nuisance cost of viewers. Since this viewer fee is never so high that too many viewers will drop out, welfare increases in these cases.

Let us now turn to the duopoly case. If $\tau \geq \beta$ then the analysis is the same as before. But if $\tau < \beta$ the profit function of station 0 is now given by

$$\Pi_0 = Ma_0\left(\omega(1 - \frac{\beta-\gamma a_1 - f_1}{\tau}) + \omega'(\frac{2\beta-\gamma(a_0+a_1)-f_0-f_1}{\tau} - 1)\right)$$
$$+Mf_0(\frac{\beta-\gamma a_0 - f_0}{\tau}).$$

The first order condition for a_i gives

$$\omega(1-\frac{\beta-\gamma a_j-f_j}{\tau})+\omega'(\frac{2\beta-\gamma(a_i+a_j)-f_i-f_j}{\tau}-1)-\frac{\gamma\omega' a_i}{\tau}-\frac{\gamma f_i}{\tau}=0$$

and the one for f_i is

$$-\frac{\omega' a_i}{\tau}+\frac{\beta-\gamma a_i-f_i}{\tau}-\frac{f_i}{\tau}=0.$$

The principal minors of the Hessian are the same as in the monopoly case. The solution for a_i and f_i is then

$$a_i^*=\frac{\beta(\omega+\gamma-2\omega')-2\tau(\omega-\omega')}{(\omega+\gamma-2\omega')(\gamma-\omega')}$$

and

$$f_i^*=\frac{\beta\omega'(2\omega'-\gamma-\omega)+\tau(\omega-\omega')(\gamma+\omega')}{(\omega+\gamma-2\omega')(\gamma-\omega')}.$$

As in the monopoly case, instead of setting both variables in this way, the firms also have the possibility to set only one of them greater zero and the other one zero. We have calculated the equilibrium with $f_i=0$ already in Section 3. If $\omega'\geq\frac{\omega\tau}{2\beta-\tau}$ then $a_i^*=\frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(3\omega'-\omega)}$ (up to the boundary condition $x_m\leq 1$) while if $\omega'<\frac{\omega\tau}{2\beta-\tau}$ then $a_i^*=\frac{2\beta-\tau}{2\gamma}$. On the other hand, if $a_i=0$ there is no interaction between the two platforms. Even if viewers overlap this has no consequence because the number of advertisers is zero. So we get the same result as in the monopoly case, $f_i=\begin{cases}\frac{\beta}{2} & \text{if } \tau\geq\frac{\beta}{2}\\ \beta-\tau & \text{if } \tau<\frac{\beta}{2}\end{cases}$

A comparison between the profits in this case and the profits if both a_i and $f_i>0$ gives that $\Pi_i(a_i=0,f_i>0)-\Pi_i(a_i>0,f_i>0)=\frac{M\beta^2}{2\tau}-\frac{M\omega'[(2\beta-\tau)\omega'-\omega(\beta-\tau)]^2}{\gamma\tau(3\omega'-\omega)^2}=\frac{(2(\omega-\omega')\tau-\beta\omega-\gamma\beta+2\omega'\beta)^2}{\tau(\omega+\gamma-2\omega')^2}>0$. So setting $a_i=0$ is therefore always better than setting both variables greater 0. A comparison between the profits with $a_i=0$ and $f_i=0$ if $\omega'\geq\frac{\omega\tau}{2\beta-\tau}$ yields that the first one is higher if $\frac{(\beta-\tau)(2\gamma-\omega')}{\gamma}>0$ or $\omega'<2\gamma$. If instead $\omega'<\frac{\omega\tau}{2\beta-\tau}$ the profit with $a_i=0$ is only higher if $\omega\leq\frac{8\gamma(\beta-\tau)}{2\beta-\tau}$.

Derivation of the equilibrium still incomplete!

(i) $\tau\geq\beta$

$$\text{If } \gamma \geq \omega \text{ then } a_i^* = 0, f_i^* = \tfrac{\beta}{2}.$$

$$\text{If } \gamma < \omega \text{ then } a_i^* = \tfrac{\beta}{2\gamma}, f_i^* = 0.$$

(ii) $\tau < \beta$

$$\text{If } \omega > \tfrac{8\gamma(\beta-\tau)}{2\beta-\tau} \text{ then} \begin{cases} a_i^* = \frac{2\beta-\tau}{2\gamma}, f_i^* = 0 \text{ if } \omega' < \frac{\omega\tau}{2\beta-\tau} \\ a_i^* = \frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(3\omega'-\omega)}, f_i^* = 0 \text{ if } \frac{\beta}{2} \leq \tau \text{ and } \omega' \geq \frac{\omega\tau}{2\beta-\tau} \\ a_i^* = \frac{\beta-\tau}{\gamma}, f_i^* = 0 \text{ if } \frac{\beta}{2} > \tau \text{ and } \omega' \geq \frac{\omega\tau}{2\beta-\tau} \end{cases}$$

$$\text{If } \omega \leq \tfrac{8\gamma(\beta-\tau)}{2\beta-\tau} \text{ and } \omega' \geq 2\gamma \text{ then}$$

$$\begin{cases} a_i^* = \frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{\gamma(3\omega'-\omega)}, f_i^* = 0 & \text{if } \frac{\beta}{2} \leq \tau \\ a_i^* = \frac{\beta-\tau}{\gamma}, f_i^* = 0 & \text{if } \frac{\beta}{2} > \tau \end{cases}$$

$$\text{If } \omega \leq \tfrac{8\gamma(\beta-\tau)}{2\beta-\tau} \text{ and } \omega' < 2\gamma \text{ then}$$

$$\begin{cases} a_i^* = 0, f_i^* = \frac{\beta}{2} & \text{if } \tau \geq \frac{\beta}{2} \\ a_i^* = 0, f_i^* = \beta - \tau & \text{if } \tau < \frac{\beta}{2} \end{cases}$$

As in the monopoly case we can now compare the result with pricing with the outcome when pricing is not possible.

Proposition 5

In duopoly the possibility of viewer can never reduce welfare. It is strictly welfare enhancing if either
(i) $\tau \geq \beta$ and $\gamma < \omega$ or
(ii) $\tau < \beta$, $\omega \leq \frac{\beta^2\gamma}{\tau(2\beta-\tau)}$ and $\omega \leq \frac{8\gamma(\beta-\tau)}{2\beta-\tau}$, and $\omega' < 2\gamma$.

The proof is similar to the one in the monopoly case and is therefore omitted here. The intuition behind the result is the same as in the monopoly case. The possibility of pricing does never increase the amount of advertising and can therefore only be welfare improving. This result is also in contrast to previous literature where the welfare consequences of pricing can go either way. The explanation is that in previous models there can be too much or too little advertising dependent on the degree of competition if pricing is not possible. Since in our model there is always too much advertising the effects of pricing are clear-cut.

8 Comparison with Single-Homing of Viewers

Now we shortly analyze the single-homing case, i.e. the case where viewers can only single-home and compare it with our previous analysis. We begin with the duopoly case. In the case $\tau \geq \beta$ the equilibrium is the same as before since stations are local monopolists and there is no problem with overlapping viewers. If $\beta < \tau$ viewers would begin to overlap. So we have to calculate the marginal viewer at first. She is given by $x = \frac{1}{2} + \frac{\gamma(a_1 - a_0)}{2\tau}$. Profit of station 0 is then

$$\Pi_0 = (\frac{1}{2} + \frac{\gamma(a_1 - a_0)}{2\tau})M\omega a_0.$$

Building the reaction function for both firms and solving for a_i yields $a_i = \frac{\tau}{\gamma}$ and a profit of $\Pi_i = \frac{M\omega\tau}{\gamma}$. But this function can only be valid if the viewer at 1/2 receives a positive utility at a_i^*. Inserting this in the utility function shows that this is only the case if $\beta > 3/2\tau$. So for values of β with $\tau < \beta \leq 3/2\tau$ the stations set a_i in such a way that viewers just don't overlap, i.e. that the utility of the viewer at 1/2 is zero. This is the case at $a_i^* = \frac{2\beta - \tau}{2\gamma}$. If $N < \frac{2\beta - \tau}{2\gamma}$ stations set $a_i^* = N$. Summing up, the optimal number of advertisers in the single-homing case is given by

$$a_i^{duo} = \begin{cases} \frac{\beta}{2\gamma} & \text{if } \beta \leq \tau \\ \min(N, \frac{2\beta - \tau}{2\gamma}) & \text{if } \beta > \tau \geq 2/3\beta \\ \frac{\tau}{\gamma} & \text{if } \tau > 2/3\beta \end{cases}$$

As one can see if τ becomes smaller and smaller the number of advertisers is decreasing more and more because competition for viewers becomes more intense.

Conducting a similar analysis for the case where a monopolist controls both platforms, we get that a monopolist would never choose to let viewers overlap. So a_i is always high enough that the viewer at 1/2 receives a utility of zero (if this is possible, i.e. if N is large enough). Thus, the optimal number of advertisers for a monopolist is given by

$$a_i^{mon} = \begin{cases} \frac{\beta}{2\gamma} & \text{if } \beta \leq \tau \\ \min(N, \frac{2\beta-\tau}{2\gamma}) & \text{if } \tau < \beta \end{cases}$$

Now we turn to the welfare analysis. As in the case with multi-homing if $\frac{2\beta\omega}{(2\omega-\gamma)} \leq \tau$ the socially optimal number of advertisers is given by either 0 or $\frac{\beta(\omega-\gamma)}{\gamma(2\omega-\gamma}$, dependent on ω and γ. If instead $\frac{2\beta\omega}{(2\omega-\gamma)} > \tau$ viewers would overlap with the aforementioned number of advertisers, which is not possible in the single-homing case. The welfare function in such a case is

$$WF = (\frac{1}{2} + \frac{\gamma(a_1 - a_0)}{2\tau})M\omega a_0 + (\frac{1}{2} - \frac{\gamma(a_1 - a_0)}{2\tau})M\omega a_1 +$$

$$+M\int_0^{\frac{1}{2}+\frac{\gamma(a_1-a_0)}{2\tau}} (\beta - \gamma a_0 - \tau x)dx + M\int_{\frac{1}{2}+\frac{\gamma(a_1-a_0)}{2\tau}}^1 (\beta - \gamma a_0 - \tau(1-x))dx.$$

Building the FOC yields that a_i^{WF} is zero if $\gamma \geq \omega$, while it should be the maximal possible number if $\gamma < \omega$. So the result of the welfare analysis can be summarized by the following:

$$a_i^{WF} = \begin{cases} 0 & \text{if } \gamma \geq \omega \\ \frac{\beta(\omega-\gamma)}{\gamma(2\omega-\gamma)} & \text{if } \gamma < \omega \text{ and } \frac{2\beta\omega}{2\omega-\gamma} \leq \tau \\ \min(N, \frac{2\beta-\tau}{2\gamma}) & \text{if } \gamma < \omega \text{ and } \frac{2\beta\omega}{2\omega-\gamma} > \tau \end{cases}$$

We are now able to compare the socially optimal number of advertisers with the number of advertisers in monopoly and in duopoly.

Lemma 2

(i) $\omega > \gamma$

If $\tau \geq \beta$ a monopolist advertises too much compared to the socially efficient amount.

If $\tau < \beta$ the monopoly amount of advertising is efficient.

(ii) $\omega \leq \gamma$

A monopolist always advertises too much.

If $\omega > \gamma$ the socially efficient amount is given by $\frac{\beta(\omega-\gamma)}{\gamma(2\omega-\gamma)}$ if $\frac{2\beta\omega}{2\omega-\gamma} \leq \tau$ and by $\min(N, \frac{2\beta-\tau}{2\gamma})$ if $\frac{2\beta\omega}{2\omega-\gamma} > \tau$. Instead a monopolist advertises $\frac{\beta}{2\gamma}$ if $\beta \leq \tau$ and $\min(N, \frac{2\beta-\tau}{2\gamma})$ if $\tau < \beta$. Comparing this shows that in the region $\frac{2\beta\omega}{2\omega-\gamma} \leq \tau$ a monopolist advertises too much because $\frac{\beta}{2\gamma} > \frac{\beta(\omega-\gamma)}{\gamma(2\omega-\gamma)}$ since $\gamma > 0$. In the region $\beta \leq \tau < \frac{2\beta\omega}{2\omega-\gamma}$, the monopolist also advertises too much because comparing $a_i^{mon} = \frac{\beta}{2\gamma}$ with $a_i^{WF} = \frac{2\beta-\tau}{2\gamma}$ shows that $a_i^{mon} > a_i^{WF}$ because $\beta \leq \tau$ in this region. Lastly, for $\beta > \tau$ both advertising amounts are the same, namely, $\min(N, \frac{2\beta-\tau}{2\gamma})$.

Concerning the second case, $\omega \leq \gamma$, the socially optimal number od ads is zero while in monopoly there is always a positive amount of commercials.

Lemma 3

(i) $\omega > \gamma$

If $\tau \geq \beta$ in duopoly there is too much advertising compared to the socially efficient amount.

If $\beta > \tau \geq 2/3\beta$ the duopoly amount of advertising is efficient.

If $\tau < 2/3\beta$ there is too few advertising in duopoly.

(ii) $\omega \leq \gamma$

There is always too much advertising in duopoly.

Since the advertising amount in duopoly is the same as in monopoly for $\tau \geq 2/3\beta$, the result is the same as in monopoly. For $\tau < 2/3\beta$, $a_i^{duo} = \frac{\tau}{\gamma}$ while the socially efficient amount is $\min(N, \frac{2\beta-\tau}{2\gamma})$. Comparing these two equations shows that $\frac{\tau}{\gamma} < \frac{2\beta-\tau}{2\gamma}$ if $\tau < 2/3\beta$. But this true in this region.

Concerning the second case, $\omega \leq \gamma$, the socially optimal number of ads is zero while, as in monopoly, it is always positive in duopoly.

So as a result we have that the number of commercials in duopoly can be too high or too low compared to the socially efficient one. The intuition is the same as in the previous literature. Platforms compete for viewers by reducing their number of commercials. If competition for viewers is harsh, because τ is low, the number of commercials in equilibrium is low and lower than the socially optimal one.
We can also compare if a monopoly or a duopoly is more efficient.

Lemma 4

A duopoly is more efficient than a monopoly if $\tau < 2/3\beta$ and $\gamma \geq \omega$.
Instead, a monopoly is more efficient than a duopoly if $\tau < 2/3\beta$ and $\gamma < \omega$.
In all other cases monopoly and duopoly are equally efficient.

The result follows directly from the previous two results.
The intuition behind this result is the following. From a welfare perspective the amount of commercials is independent of τ while the degree of competition is the driving force in the duopoly case. If τ is low then a_i^{duo} is also very low. From a social point of view this is good, if $\gamma \geq \omega$, because this means that the nuisance for viewers from advertising is higher than the gains from possible trading of advertisers' goods. Thus a duopoly does better if $\gamma \geq \omega$ (and $\tau < 2/3\beta$). If instead gains from trade are more important than nuisance costs, $\gamma < \omega$, there should be maximal amount of advertising. This can be achieved in a monopoly because if a firm controls both stations there is no competition between both stations and this leads to the maximal commercial amount. Instead, in duopoly the number of commercials is too low because the two firms cannot internalize the negative effect competition has on the number of commercials.

Summing up, in the analysis with single-homing of viewers we find the welfare results are usually nut clear. Monopoly can be better than duopoly and in duopoly there can either be too much or too little advertising, dependent on parameter values. If we make the analysis more realistic by revoking the assumption of single-homing we find that these ambiguous results do not longer hold. Instead, duopoly is always better than monopoly and there is too much advertising in both regimes.

9 Conclusion

In this paper we analyzed competition in a media market in which both sides of the market, viewers and advertisers, can multi-home. We have shown that, in contrast to existing literature, this leads to clear cut welfare predictions. Both in monopoly and in duopoly there is over-provision of advertising, so an advertising ceiling would be welfare improving. If stations are controlled by different owners then welfare can never be lower than under joint ownership and often is higher. The possibility of a viewer price can only enhance welfare because the amount of advertising is decreasing. Finally, stations have a strong incentive that viewers watch their programme exclusively, because these viewers are the most valuable ones. This can lead to the effect that the profits of the stations decrease, although the programme content becomes more attractive. We provided some anecdotal evidence that the results of our model fit well with the stations' behavior in the German broadcasting market.

An extension of the model might be the introduction of a quality dimension. It is often argued that advertising regulation might not be welfare improving because it would lead to a lower quality of the programmes. The stations provide quality only to attract viewer. But if these viewers are less valuable because of the advertising cap then quality would go down. It would be interesting to analyze how this effect interacts with the effect that channels generally advertise too much under to identify conditions and which either effect dominates.

10 Appendix

10.1 Appendix A

Characterization of asymmetric equilibria in the duopoly case

In this appendix we characterize the asymmetric equilibria if both stations choose non-overlapping of viewers. The stations choose their number of advertisers in such a way that viewers just don't overlap, or $2\beta - \gamma(a_i + a_j) = \tau$ which gives $a_i^{no} = \frac{2\beta-\tau-\gamma a_j}{\gamma}$. This is the number of advertisers of station i given that station j chooses a_j and viewers just do not overlap.

On the other hand, the optimal reaction of station i if viewers would overlap is given by the first order condition

$$\omega\left(1 - \frac{\beta - \gamma a_j}{\tau}\right) + \omega\prime\left(\frac{2\beta - \gamma(a_i + a_j)}{\tau} - 1\right) - \frac{\omega'\gamma a_i}{\tau} = 0,$$

which yields

$$a_i^{ov} = \frac{1}{2\gamma\omega'}(\tau(\omega - \omega') - \beta(\omega - 2\omega') + \gamma a_j(\omega - \omega')).$$

But a_i^{ov} is only the optimal reaction on a_j if viewers would indeed multi-home at these two values which is only the case if $2\beta - \gamma a_j - \frac{1}{2\omega'}(\tau(\omega-\omega') - \beta(\omega-2\omega') + \gamma a_j(\omega-\omega')) > \tau$ or $a_j < \frac{\beta}{\gamma}(\frac{\omega+2\omega'}{\omega+\omega'}) - \frac{\tau}{\gamma}$. Thus, if station j chooses $a_j = \frac{\beta}{\gamma}(\frac{\omega+2\omega'}{\omega+\omega'}) - \frac{\tau}{\gamma} + \delta$ with $\delta > 0$, station i reacts with $a_i = \frac{\beta\omega}{\gamma(\omega+\omega')} - \delta$.

So far, we have calculated the optimal reaction of firm i. But to determine under which conditions this is equilibrium we have to calculate the best response of firm j on $a_i = \frac{\beta\omega}{\gamma(\omega+\omega'} - \delta$. This is given by $a_j^{ov} = \frac{1}{2\gamma\omega'}(\tau(\omega-\omega') - \beta(\omega-2\omega') + \frac{\beta\omega(\omega-\omega')}{\omega+\omega'} - \delta\gamma(\omega-\omega')))$. But this leads to multi-homing of viewers only if

$$\delta > \frac{1}{\gamma}(\tau - \frac{2\beta\omega'}{\omega + \omega'})$$

.

Summing up, there can only be an asymmetric equilibrium if

$$0 \leq \delta \leq \frac{1}{\gamma}(\tau - \frac{2\beta\omega'}{\omega + \omega'}).$$

The term in the right hand side is weakly positive if $\tau(\omega + \omega' - 2\beta\omega') \geq 0$. But this is fulfilled since otherwise stations would choose overlapping of viewers. It is also easily verified that if $\delta = \frac{1}{\gamma}(\tau - \frac{2\beta\omega'}{\omega+\omega'})$ we are in the symmetric equilibrium.

We can now describe the asymmetric equilibria:
If $\tau < \beta$ and $\omega' < \frac{\tau\omega}{2\beta-\tau}$ there exist multiple equilibria. In this equilibria

$$a_i = \frac{\beta\omega}{\gamma(\omega + \omega')} - \delta$$

and

$$a_j = \frac{\beta}{\gamma}(\frac{\omega + 2\omega'}{\omega + \omega'}) - \frac{\tau}{\gamma} + \delta$$

with $\delta \in [0, \frac{1}{\gamma}(\tau - \frac{2\beta\omega'}{\omega+\omega'})]$.

So the number of equilibria becomes larger as ω' shrinks. If instead $\omega' = \frac{\tau\omega}{2\beta-\tau}$ there exists only one equilibrium, in which viewers do not multi-home, namely the symmetric one.

10.2 Appendix B

Proof of Proposition 1

If $\tau \geq \frac{2\beta\omega}{2\omega-\gamma}$ there is too much advertising if $\gamma > \omega$. There is also too much advertising if $\omega \geq \gamma$ because $a^*_{mon} = \frac{\beta}{2\gamma} > \frac{\beta(\omega-\gamma)}{\gamma(2\omega-\gamma)} = a^*_{WF}$. If $\tau < \frac{2\beta\omega}{2\omega-\gamma}$ then there are different cases. First look at the case where $\omega' < \frac{\omega\gamma N+\tau\gamma-2\beta\omega}{2(\gamma N+\tau-2\beta\omega)}$ and $\frac{\gamma(4\beta-\gamma N)}{2(2\beta-\gamma N)} < \frac{\gamma(2\beta+\tau-\gamma N)}{2(\tau-\gamma N)}$.
(i) If $\omega \leq \frac{\gamma(4\beta-\gamma N)}{2(2\beta-\gamma N)}$ then $a^*_{WF} = 0$ and there is too much advertising in monopoly.
(ii) If $\frac{\gamma(4\beta-\gamma N)}{2(2\beta-\gamma N)} < \omega \leq \frac{\gamma(2\beta+\tau-\gamma N)}{2(\tau-\gamma N)}$ then $a^*_{WF} = \frac{N}{2}$ while $a^*_{mon} = \frac{2\beta-\tau}{2\gamma}$. But because of our assumption that $N \leq \frac{2\beta-\tau}{\gamma}$, $a^*_{WF} \leq a^*_{mon}$.
(iii) If $\omega > \frac{\gamma(2\beta+\tau-\gamma N)}{2(\tau-\gamma N)}$ then $a^*_{WF} = a^*_{mon} = \frac{2\beta-\tau}{2\gamma}$ and so a monopoly is efficient.
Next we turn to the case where $\omega' < \frac{\omega\gamma N+\tau\gamma-2\beta\omega}{2(\gamma N+\tau-2\beta\omega)}$ and $\frac{\gamma(4\beta-\gamma N)}{2(2\beta-\gamma N)} \geq \frac{\gamma(2\beta+\tau-\gamma N)}{2(\tau-\gamma N)}$.
(i) If $\omega \leq \frac{\gamma(2\beta+\tau)}{2\tau}$ then $a^*_{WF} = 0$ and there is too much advertising.
(ii) If $\omega > \frac{\gamma(2\beta+\tau)}{2\tau}$ then $a^*_{WF} = a^*_{mon} = \frac{2\beta-\tau}{2\gamma}$ and the monopolist is efficient.

If $\frac{\omega\gamma N+\tau\gamma-2\beta\omega}{2(\gamma N+\tau-2\beta\omega)} \leq \omega' < \frac{\omega\beta}{2\beta-\tau}$ then $a^*_{mon} = \frac{2\beta-\tau}{2\gamma}$ while $a^*_{WF} = \frac{\beta(2\omega'-\omega-\gamma)-(\omega-\omega')(\gamma N-\tau)}{\gamma(4\omega'-2\omega-\gamma)}$. But in this case viewers would overlap in the welfare optimal outcome while they do not overlap in the monopoly case. Thus, there are too much advertisers in the monopoly case.

Lastly we turn to the case where $\omega' > \frac{\omega\beta}{2\beta-\tau}$. In this case $a^*_{WF} = \frac{\beta(2\omega'-\omega-\gamma)-(\omega-\omega')(\gamma N-\tau)}{\gamma(4\omega'-2\omega-\gamma)}$ and $a^*_{mon} = \frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{2\gamma(2\omega'-\omega}$). Comparing these two values yields that a^*_{WF} can only be higher than a^*_{mon} if $N \leq \frac{\omega(\beta+\tau)-\omega'(2\beta+\tau)}{2(2\omega'-\omega)(\omega-\omega')}$. But still, $N \geq \frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{2\gamma(2\omega'-\omega)} = a^*_{mon}$ because otherwise a_{mon} would be N and not the given value here. So, for these both inequalities to hold it must be fulfilled that $\frac{\omega(\beta+\tau)-\omega'(2\beta+\tau)}{2(2\omega'-\omega)(\omega-\omega')} \geq \frac{\omega'(2\beta-\tau)-\omega(\beta-\tau)}{2\gamma(2\omega'-\omega)}$. But this can only hold if $\tau \geq \frac{\beta(\omega+\gamma-\omega')(2\omega'-\omega)}{(\omega'+\gamma-\omega)(\omega-\omega')}$. But since we know that in this region $\beta > \tau$ this can never be fulfilled. Thus, it follows that $a^*_{mon} > a^*_{WF}$.

q.e.d.

Proof of Proposition 2

to be written

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