On the equity principle and the balanced solution for cooperative TU-games^{*}

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Abstract:

The Shapley value of a cooperative transferable utility game distributes the dividend of each coalition in the game equally among its members. Given exogenous weights for all players, the corresponding weighted Shapley value distributes the dividends proportionally to their weights. In this contribution we define the *balanced solution* which assigns weights to players such that the corresponding weighted Shapley value of each player is equal to her weight. We prove its existence for all monotone transferable utility games, discuss properties of this solution, and provide an axiomatization using balanced transformations.

KEY WORDS: Balanced solution, Proportionality, Equity principle, Balanced transformation, Weighted Shapley value, Fixed point.

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